SOVIET PHYSICS JETP VOLUME 20, NUMBER 6 JUNE, 1965

ON THE CRITERION FOR SUPERCONDUCTIVITY

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Submitted to JETP editor May 29, 1964

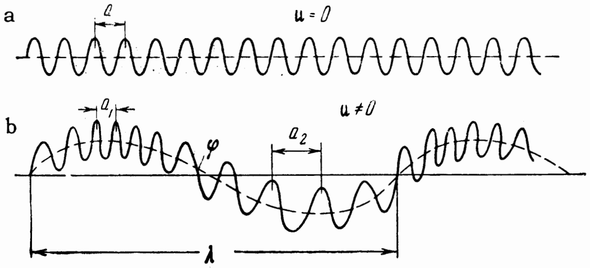
J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 2159-2167 (December, 1964)

We consider the interaction of electrons in a metal, arising both from exchange of virtual phonons and from longitudinal electromagnetic waves (Coulomb interaction), taking into account the influence of Coulomb effects on the phonon spectrum. We find a criterion for superconductivity which for an isotropic model is of the form $\rho s^2 < \frac{1}{3} N m v_F^2$, where ρ is the density of the metal, s the speed of longitudinal sound, N the electron concentration, m the electron mass, and v_F the Fermi velocity. This criterion is compared with the experimental data.

ACCORDING to contemporary theory, the reason for the absence of superconductivity in a number of metals is that the Coulomb repulsion exceeds in absolute magnitude the attraction between the electrons due to exchange of virtual phonons. The question of the criterion for superconductivity to occur has been the subject of a number of papers ([1-4], et al); however, in these papers the Coulomb effects are taken into account by introducing an effective interaction due to them. But the Coulomb interaction does not simply lead to an inter-electronic repulsion; when lattice vibrations occur, Coulomb effects also induce an almost total mutual cancellation of the electronic and lattice charges, thus causing a change (renormalization) of the phonon frequencies. Consequently, it is certainly not valid to treat the interaction between the electrons as the sum of two additive terms (phonon and Coulomb terms respectively). The actual resultant interaction has a very complex form, with a sign depending on the relation between the parameters of the lattice and of the electron gas. According to the ideas of Cooper, superconductivity can occur when the sign of this interaction is negative, indicating an attraction; electronic bound states then appear on the Fermi surface. The criterion for superconductivity obtained in this way—that is, that there should be an attraction between the electrons—is evidently in qualitative agreement with the experimental data (cf. Sec. 5).

1. A characteristic property of the lattice vibrations of a metallic crystal is that the adiabatic approximation cannot be applied to them. This fact has an important bearing on the nature of the interaction of the electrons with the lattice vibrations, which is responsible for superconductivity.

Suppose that an electron moves in the undeformed lattice in a periodic field V = V(**r**) (Fig. a).



Then when the lattice is deformed in a way characterized by the deformation vector **u**(**r**, t), it will move in the potential field indicated schematically in Fig. b, which we can write as $V' = V(\mathbf{r} - \mathbf{u}) + e\varphi$; here $V(\mathbf{r} - \mathbf{u})$ is a locally periodic function and φ is a slowly varying potential (we have assumed that the characteristic "wavelength" λ of the deformation is large compared to the lattice constant a). The quantity $\mathbf{E} = -\nabla\varphi$ describes the macroscopic field due to an inhomogeneous (or nonstationary) deformation of the lattice; its magnitude is proportional to the deformation tensor. As shown by Akhiezer, Kaganov, and Lyubarskiĭ[5] (cf. also[6,7]) the existence of this field follows directly from Maxwell's equations.

Thus the change $V' - V$ of the potential energy of the electron in the field of the sound wave consists of two terms:

$$-\mathbf{u}\nabla V + e\varphi.$$

The first term describes the "adiabatic" interaction of the electron with the lattice; by itself it leads to the usual Fröhlich electron-phonon interaction Hamiltonian (cf.[8]). The second term represents the "non-adiabatic"[1)] part of the inter-

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[1)] We use this term to denote effects connected with the influence on a given electron of the 'residual' electrons distant in space (in comparison with the lattice spacing).

action, which is not correctly treated by the Fröhlich model.

We shall preface what follows with a classical treatment of lattice vibrations in metals according to the method of Akhiezer, Kaganov, and Lyubarskiĭ[5]. Silin[6] and Kontorovich[7] have derived equations to describe the propagation of sound waves in a metal. In an isotropic model with a quadratic electron dispersion law these equations read, for longitudinal phonons propagating parallel to the x axis:

$$\rho\ddot{u} = \lambda u'' - NeE + \Lambda \frac{\partial}{\partial x} \int \chi d\tau_p; \tag{1}$$

$$\frac{\partial\chi}{\partial t} + v_x \frac{\partial\chi}{\partial x} + \left(eE - \frac{\partial}{\partial x} \Lambda u' \right) v_x \frac{\partial f_0}{\partial \varepsilon} = 0,$$

$$\chi = f - f_0; \tag{2}$$

$$Nu' + \int \chi d\tau_p = 0 \qquad \left(\dot{u} = \frac{\partial u}{\partial t}, \quad u' = \frac{\partial u}{\partial x} \right). \tag{3}$$

Equation (1) above is the elasticity equation, which includes forces acting on the lattice due to the conduction electrons and to the electric field **E** which arises as a result of the lattice vibrations. (λ is the longitudinal modulus of elasticity of the lattice, Λ is the constant describing the interaction of the electrons with the deformed lattice and N is the electron concentration); (2) is the kinetic equation (χ is the deviation of the electron distribution from the equilibrium value $f_0(\epsilon)$); and (3) is the equation describing mutual cancellation of the lattice charge Neu′ and the electronic charge $e\int\chi d\tau_p$ ($d\tau_p$ is an element of phase space). In (2) we have dropped the collision integral, which is valid for $\omega\tau \gg 1$ (where τ is the electronic relaxation time and ω the frequency of the vibration); and in (3) we have dropped a term $E'/4\pi e$ corresponding to a lack of compensation between electronic and lattice charges. It can be shown that this term is small as a result of the smallness of the sound-wave frequencies in comparison with the electronic plasma frequency $\omega_0 \sim 10^{16}$ sec^{-1}. In fact, an uncompensated electron charge in a metal must be "dissipated" in a time of the order of the period of the plasma oscillations $T_0 = 2\pi/\omega_0$. In Sec. 6 we allow for the lack of total cancellation and give a more exact criterion for the applicability of Eq. (3) as it stands.

Notice that the above equations imply the existence of a nonzero field **E**. For if we assume **E** = 0, solve the kinetic equation (2) and substitute the solution into (3), we can check that charge cancellation does not occur. But then it follows from Maxwell's equations that there must be strong fields present, and these in the final analysis lead back to just such a cancellation. Accordingly, we must treat the field **E** in a consistent manner; if we leave it out of account, Eqs. (1)—(3) are mutually incompatible. Notice moreover that results corresponding to the Fröhlich model (cf. infra) are obtained by just this step of setting **E** = 0 [and neglecting (3)].

2. Equations (1)—(3) can be considerably simplified. In the kinetic equation (2) the first term $\partial\chi/\partial t \sim \omega\chi$ is small compared to the second term $v_x\partial\chi/\partial x \sim v_F k\chi$ (where k is the wave vector of the sound wave). Neglecting this term, dividing (2) by v_x and integrating over $d\tau_p$ and dx, we get

$$\eta + (e\varphi + \Lambda u')3N / mv_F^2 = 0, \tag{4}$$

where $\eta = \int\chi d\tau_p$ is the charge of electron concentration induced by the lattice vibrations. (The integration constant from the dx-integration has been included in the definition of the potential φ.)

Similarly, (1) and (3) can be rewritten in the form:

$$\rho\ddot{u} = \lambda u'' + Ne\varphi' + \Lambda\eta', \tag{5}$$

$$Nu' + \eta = 0. \tag{6}$$

Equations (4)—(6) form a system of three equations for the three variables u, η, φ. Solution of these equations gives us the dispersion law of the vibrations, the amplitude of the electric field **E** and the electron density fluctuation η. However, we shall subsequently be interested in the Green functions rather than the actual solutions of the equations; for this reason it is more convenient for our purposes to obtain (4)—(6) from a Lagrangian formalism. We can easily verify that (4)—(6) are the Lagrange equations for a Lagrangian

$$L = \int d\mathbf{r} \left\{ -\frac{\lambda(u')^2}{2} + \frac{\rho\dot{u}^2}{2} - (\Lambda u' + e\varphi)\eta - Ne\varphi u' - \frac{1}{2}\frac{mv_F^2}{3N}\eta^2 \right\} \tag{7}$$

(the last term in (7) comes from the expansion of the energy of the electron gas, $\frac{3}{5}N\epsilon_F$, in a series in the deviation of the electron concentration from its equilibrium value N).

3. We can now use well-developed methods to find the electron-electron interaction potential. We know that in quantum field theory the interparticle interaction is expressed in terms of the Green functions of the fields which propagate the interaction. Qualitatively speaking, the relevant fields in our case are: (1) the lattice elastic deformation field u(x) representing an attraction be-

tween the electrons; (2) the longitudinal electromagnetic field $\varphi(x)$ representing the Coulomb repulsion; (3) the field of the electron density fluctuations $\eta(x)$ which tends to screen the Coulomb interaction. Actually the presence of cross-terms representing interactions between u, φ and η themselves makes such a decomposition purely formal.

For a rigorous treatment we need the Green functions

$$D_{\alpha\beta}(x-x') = -i\langle Tq_\alpha(x)q_\beta(x')\rangle, \qquad (8)$$

where $q_\alpha = (u, \eta, \varphi)$. The function representing the electron-electron interaction potential can be written

$$\mathcal{D}(x-x') = -i\langle T\Phi(x)\Phi(x')\rangle, \qquad (9)$$

where $\Phi(x) = e\varphi(x) + \Lambda\partial u/\partial x$ is the energy of the electron in the sound-wave field. We see from (2) that the force acting on the electron is

$$eE - \frac{\partial}{\partial x}\Lambda u' = -\frac{\partial\Phi}{\partial x}.$$

From (9) we have[2)]

$$\mathcal{D}(x-x') = e^2D_{\varphi\varphi}(x-x') + e\Lambda\frac{\partial}{\partial x}D_{u\varphi}(x-x')$$
$$+ e\Lambda\frac{\partial}{\partial x'}D_{\varphi u}(x-x') + \Lambda^2\frac{\partial^2}{\partial x\,\partial x'}D_{uu}(x-x'). \qquad (10)$$

To calculate the Green functions $D_{\alpha\beta}(x-x')$ we set up the equations satisfied by them. Lagrange's equations give us

$$\frac{\partial p_\alpha(x)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial q_\alpha'(x)}\right) - \frac{\partial L}{\partial q_\alpha(x)} = 0, \qquad (11)$$

where p_α is the momentum canonically conjugate to the coordinate q_α. Multiplying (11) by $q_\beta(x')$, applying the T-product operator to both sides of the equation and interchanging the order of the operations T and $\partial/\partial t$, we find

$$-i\frac{\partial}{\partial t}\langle Tp_\alpha(x)q_\beta(x')\rangle - i\left\langle T\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial q_\alpha'(x)}\right)q_\beta(x')\right\rangle$$
$$+ i\left\langle T\frac{\partial L}{\partial q_\alpha(x)}q_\beta(x')\right\rangle = -i[p_\alpha(x), q_\beta(x')]\delta(t-t')$$
$$= -\delta_{\alpha\beta}\delta(x-x'), \qquad (12)$$

since according to the general rules for quantization the commutator $[p_\alpha, p_\beta]$ is to be taken as $-i\{p_\alpha, q_\beta\}$ (we have set Planck's constant $\hbar$ equal to unity), where $\{p_\alpha, q_\beta\}$ are the classical Poisson brackets, equal to $\delta_{\alpha\beta}$ for any pair of canonical variables p_α, q_β.

For the quadratic Langrangian (7), Eq. (12) leads to a closed system of equations, which can be solved exactly, for the double-time Green functions (8). Actually, to find the Green functions in our case it is sufficient to multiply eqs. (4)—(6) by $u(x')$, $\eta(x')$ and $\varphi(x')$, take the time-ordered expectation value $-i\langle T\ldots\rangle$ and add on the right-hand side the inhomogeneous term $-\delta_{\alpha\beta}(x-x')$ which appears in (12). For instance, a typical equation is

$$\left(\rho\frac{\partial^2}{\partial t^2} - \lambda\frac{\partial^2}{\partial x^2}\right)D_{uu}(x-x') - \Lambda\frac{\partial}{\partial x}D_{\eta u}(x-x')$$
$$- Ne\frac{\partial}{\partial x}D_{\varphi u}(x-x') = -\delta(x-x'). \qquad (13)$$

Rather than writing down the other equations we proceed immediately to the solution. In Fourier-transformed form we have:

$$D_{uu}(k,\omega) = \frac{1}{\rho\omega^2 - (\lambda - 2N\Lambda + {}^1/_3 Nmv_F^2)k^2}; \qquad (14)$$

$$D_{u\varphi}(k,\omega) = -D_{\varphi u}(k,\omega)$$
$$= \frac{1}{e}\frac{ik(\Lambda - {}^1/_3 mv_F^2)}{\rho\omega^2 - (\lambda - 2N\Lambda + {}^1/_3 Nmv_F^2)k^2}; \qquad (15)$$

$$D_{\varphi\varphi}(k,\omega) = \frac{1}{e^2}\frac{\Lambda^2k^2 + (\rho\omega^2 - \lambda k^2)mv_F^2/3N}{\rho\omega^2 - (\lambda - 2N\Lambda + {}^1/_3 Nmv_F^2)k^2} \qquad (16)$$

(we omit the other Green functions, which do not appear in (10)).

Substituting the expressions (14)—(16) in (10), we find

$$\mathcal{D}(k,\omega) = \frac{mv_F^2}{3N}\frac{\rho\omega^2 - (\lambda - 2N\Lambda)k^2}{\rho\omega^2 - (\lambda - 2N\Lambda + {}^1/_3 Nmv_F^2)k^2}. \qquad (17)$$

Note that if we were to put $\varphi = 0$ in (4) and (5) and find the electron-electron interaction function, we should get

$$\mathcal{D}_F(k,\omega) = \frac{\Lambda^2k^2}{\rho\omega^2 - (\lambda - 3N\Lambda^2/mv_F^2)k^2}, \qquad (18)$$

which corresponds to the results of the Fröhlich model (cf., e.g.,[9]; $\lambda' = \lambda - 3N\Lambda^2/mv_F^2$ is the renormalized modulus of elasticity in the Fröhlich model and as such is positive definite). On the other hand, if we were to put u = 0 in (4)—(6), i.e., neglect the lattice vibrations,[3)] the electron-electron interaction would have the form

$$\mathcal{D}_C(k,\omega) = \frac{4\pi e^2}{k^2 + 3\omega_0^2/v_F^2}, \qquad \omega_0^2 = \frac{4\pi Ne^2}{m}, \qquad (19)$$

[2)] To simplify the notation we have used x to denote both the totality of the space and time coordinates (r,t) and the Cartesian coordinate x; this should not lead to any ambiguity.

[3)] In this case we must replace (6) by the equation $\phi'' + 4\pi e\eta = 0$. (See below in Sec.6.)

corresponding to the screened Coulomb potential of two charges. From (17)—(19) it is clear that it is quite illegitimate to write $\mathscr{D}(k, \omega)$ as the sum of the two terms $\mathscr{D}_F(k, \omega)$ and $\mathscr{D}_C(k, \omega)$.

4. To apply these results to the theory of superconductivity we find the interaction function $\mathscr{D}(k, \omega)$ for $k = 0$. It is easy to verify that this is the quantity which corresponds to the interaction between the electrons of equal and opposite momenta which go to make up Cooper pairs.

From (17)—(19) we see that $\mathscr{D}_F(k)$ is everywhere negative (attractive), $\mathscr{D}_C(k)$ everywhere positive, while $\mathscr{D}(k)$ is given by

$$\mathscr{D}(k) = \mathscr{D}(k, 0) = \frac{mv_F^2}{3N} \frac{\lambda - 2N\Lambda}{\lambda - 2N\Lambda + {}^1/_3 Nmv_F^2}. \qquad (20)$$

This expression is valid for long wavelengths ($k \rightarrow 0$) (which is the region of validity of our whole approach); however, it seems plausible that the situation is much the same qualitatively even when we go to short wavelengths (large k).

The quantity in the denominator of (20) is the renormalized modulus of elasticity of the metal, and as such is positive; the speed of sound, indeed, is given by the expression (cf. also[10])

$$\rho s^2 = \lambda - 2N\Lambda + {}^1/_3 Nmv_F^2. \qquad (21)$$

Consequently, the sign of $\mathscr{D}(k)$ is determined by the sign of the numerator in (20).

For

$$2N\Lambda > \lambda \qquad (22)$$

$\mathscr{D}(k)$ is negative, indicating an attraction between the electrons; in this case superconductivity is possible at sufficiently low temperatures. Conversely, for $2N\Lambda < \lambda$ the metal cannot go superconducting however low the temperature, since then there is a repulsion between the electrons on the Fermi surface which inhibits the formation of bound states (Cooper pairs).

Thus, the inequality (22) is the criterion for superconductivity in our model. We see therefore that superconductivity sets in at some given critical strength of the electron-phonon interaction Λ, whereas the normal Fröhlich model leads to superconductivity for arbitrarily small Λ.

Using (21) we can rewrite the condition (22) for superconductivity to occur as

$$\rho s^2 < {}^1/_3 Nmv_F^2. \qquad (23)$$

The criteria (22) and (23) are equivalent, but (23) is the more convenient since the condition for superconductivity is expressed in terms of observable parameters—the renormalized (true) speed of sound s, the electron concentration N and so on.

It is important to notice that the electronic charge e does not appear in the criterion for superconductivity (23), nor in formulae (20)—(22), in spite of the fact that it is electrical effects which are most important in the derivation of these formulae. Actually, expressions (20)—(23) are valid in the limit $\omega_0 \rightarrow \infty$, i.e., $e \rightarrow \infty$; in this limit e drops out of the final expressions (cf. Sec. 6).

The inequality (23) means that the speed of sound in the metal s is less than the speed of sound in the electron system s_0 ($\rho s_0^2 = {}^1/_3 Nmv_F^2$). In this case the forces between electrons on the Fermi surface are attractive, leading to superconductivity. When $s = s_0$ the function $\mathscr{D}(k)$ goes to zero (for $k \rightarrow 0$); in this case it is necessary to investigate its behaviour for shorter wavelengths, i.e., large k. Abrikosov[11] has shown that in this region the interaction may be either attractive or repulsive, depending on the nature of the short-wave part of the phonon spectrum, so that in this case the metal may or may not be superconducting.

5. One might think that the simple model used here to describe superconductivity is much too crude for us to expect a good fit with the experimental data (we have assumed a quadratic electron dispersion law, used a Debye approximation for the phonons, neglected transverse phonons and the effects of anisotropy, etc.). However, it is reasonable to assume that in cases where the condition (23) (or its opposite) is fulfilled by a large margin, it should constitute a correct criterion for distinguishing superconductors from normal metals. The numerical data tabulated below illustrate this assumption (we have confined our attention to the nontransition metals).

In Table I we give the elastic moduli c_{11}, c_{12}, and c_{44} for most of the metals with cubic symmetry (Ag, Al, Au, Cu, K, Li, Na, Pb)[4)]. For an isotropic metal $c_{11} = \rho s^2$. In the last column of Table I we have tabulated the values of the quantities ${}^1/_3 Nmv_F^2$ calculated from the electron concentration in the metal (we have counted all the valence electrons as conduction electrons and taken their mass to be

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[4)] Most of the data refer to room temperature or to T = 78°K. However, the change in the elastic constants as we go to low temperatures is insignificant for most metals. For example, for Ag and Au[12] the elastic constants increase as we go from T = 300°K to T = 4.2°K by approximately 10% (for the alkali metals, admittedly, the change may apparently be greater). At the same time the quantity $1/3\ Nmv_f^2$ also increases somewhat when we go to low temperatures, as a result of the change in the lattice constant.

Table I

Metal	Character*	Lattice Structure	c_{11}	c_{12}	c_{44}	$\frac{1}{3}Nmv_F^2$
Ag	n	fcc	12.4	9.34	4.61	3.48
Al	s	fcc	10.82	6.13	2,85	22,6
Au	n	fcc	18.6	15.7	4.20	3,55
Cu	n	fcc	16.8	12.1	7.54	6,38
K	n	bcc	0,457	0.374	0,263	0,32
Li	n	bcc	1.48	1.25	1.08	2.32
Na	n	bcc	0.945	0.779	0.618	0,83
Pb	s	fcc	4.66	3.92	1.44	13,4

*Superconducting or normal.

that of a free electron)[5]. From this table we see that our criterion for superconductivity fits all the metals except Li, for which ρs^2 and $\frac{1}{3}Nmv_F^2$ have rather close values. We may notice, however, that the elastic properties of Li are strongly anisotropic; an isotropic model would require $c_{11} = c_{12} + 2c_{44}$, whereas for Li $c_{12} + 2c_{44} = c = 3.41 \times 10^{11}$ dynes/cm, i.e., more than twice c_{11}. Notice incidentally that c itself is greater than $\frac{1}{3}Nmv_F^2$, as it should be for a nonsuperconductor.

In Table II we tabulate the analogous data for the hexagonal (Be, Cd, Mg, Zn) and tetragonal (β-Sn, In) elements, also taken from the survey.[13] Examination of the data listed shows that these metals also show a correlation with our criterion. In the cases where agreement with formula (23) is lacking, c_{ij} and $\frac{1}{3}Nmv_F^2$ are close in magnitude. In most cases, of the quantities

$$c_{11},\quad c_{33},\quad (c_{12}+2c_{44}),\quad (c_{12}+2c_{66}),$$
$$(c_{13}+2c_{44}),\quad (c_{13}+2c_{66}),$$

which in an isotropic model should be equal (and equal to ρs^2), some are greater and some less than $\frac{1}{3}Nmv_F^2$. It appears therefore that for these metals the effect of anisotropy, which was not considered above, is an important factor.

6. In conclusion we obtain results that are independent of the assumption of charge cancellation in the lattice vibrations. This will also allow us to trace the manner in which the electronic charge e enters the expression for the electron-electron interaction energy.

Allowance for noncancellation of the charge affects only Eq. (6), which now reads

$$\varphi'' + 4\pi e(Nu' + \eta) = 0. \tag{24}$$

The corresponding change in the Lagrangian (7) consists in the addition of a term

$$\int \frac{E^2}{8\pi}\,d\mathbf{r} = \frac{1}{8\pi}\int (\varphi')^2\,d\mathbf{r}.$$

The calculation is now entirely analogous to the one already given; the Green function $\mathcal{D}(k, \omega)$ now becomes

$$\mathcal{D}(k,\omega)=\frac{mv_F^2}{3N}\left[\lambda k^2-\rho\omega^2-2N\Lambda k^2-\frac{N\Lambda^2k^4}{m\omega_0^2}\right]\times\left[(\lambda k^2-\rho\omega^2)\left(1+\frac{v_F^2k^2}{3\omega_0^2}\right)-2N\Lambda k^2+\frac{Nmv_F^2k^2}{3}-\frac{N\Lambda^2k^4}{m\omega_0^2}\right]^{-1}. \tag{25}$$

The poles of this function define the phonon energies $\omega = \omega_k$. We find

$$\rho\omega_k^2=\left[\lambda+\left(\frac{1}{3}Nmv_F^2-2N\Lambda-\frac{N\Lambda^2k^2}{m\omega_0^2}\right)\left(1+\frac{v_F^2k^2}{3\omega_0^2}\right)^{-1}\right]. \tag{26}$$

We shall consider various limiting cases.

A. If there are no electric interactions ($e \to 0$) we get expression (18) for the Fröhlich-model Green function and the formula

$$\rho\omega_k^2=\left(\lambda-\frac{3N\Lambda^2}{mv_F^2}\right)k^2=\lambda'k^2 \tag{27}$$

for the elementary excitation dispersion law. Equation (27) describes the renormalization of the phonon frequencies in the Fröhlich model.

B. In the limit of strong electric interactions ($e \to \infty$) we get expression (17) for the Green functions and (21) for the spectrum. The additional condition necessary for (17) to follow from (25) is that

Table II

Metal	Character	Lattice Structure	c_{11}	c_{33}	c_{44}	c_{12}	c_{13}	c_{66}	$\frac{1}{3}Nmv_F^2$
Be	n	hcp	28.1	30.2	15.5	—2.5		(15,3)	38,6
Cd	s	hcp	11.0	4.69	1,56	4.04	3,83	(3,48)	7.64
Mg	n	hcp	5.97	6.17	1.64	2,62	2.17	(1,67)	6.64
Zn	s	hcp	16.1	6,10	3.83	3.42	5.01	(6,34)	13.5
β-Sn	s	tetr.	7.35	8.7	2.2	2,34	2.8	2.26	16,6
In	s	tetr.	4.45	4.44	0.65	3.95	4.05	1.22	10.4
Hg	s	tetr.	3.60	5.05	1.29	2,89	3.03	(0.35)	6.56

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[5] The quantities c_{ij} and $1/3\ Nmv_f^2$ listed in Table I (and also in Table II below) are expressed in units of 10^{11} dynes/cm.

$$kv_F \ll \omega_0 \quad \text{or} \quad \omega \ll \omega_0 s / v_F, \tag{28}$$

i.e., that the frequency is low enough (wavelength long compared with the lattice constant).

C. Finally, we get from (25) the equation (19) for the screened electron-electron interaction by setting $\rho = \infty$, i.e., neglecting the lattice vibrations and the possibility of an associated phonon-induced interaction.

Using (25), we can obtain a criterion for superconductivity which is more exact than (23) when charge compensation is not total; it reads

$$\rho s^2 < \left(\frac{1}{3} Nmv_F^2 + \lambda \frac{v_F^2 k^2}{3\omega_0^2}\right)\left(1 + \frac{v_F^2 k^2}{3\omega_0^2}\right)^{-1}, \qquad s = \frac{\omega_k}{k}. \tag{29}$$

Unfortunately, however, the presence in (29) of the ''bare'' constant λ makes it impossible to compare this criterion with experiment.

In conclusion I should like to express deep gratitude to I. M. Lifshitz for discussing this work, to A. A. Abrikosov and L. P. Gor'kov for a number of critical comments and to L. A. Fal'kovskiĭ for his interest in the work and for helpful discussions.

Note added in proof (31 October 1964): We notice that the electron-electron interaction considered in this work is essentially classical in nature (Eq. (17) does not contain Planck's constant) and can be obtained without using a quantum-mechanical formalism (i.e., without quantizing the Lagrangian (7)). The corresponding derivation is analogous to the derivation of Coulomb's law in classical electrodynamics and consists in solving the classical field equations (written in this case in terms of the variables μ, η, φ) with given external sources. Such a procedure leads to formula (25) which in the appropriate limits gives (17), (18), and (19).

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Translated by A. J. Leggett
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