Letters to the Editor

RESONANCE ANNIHILATION OF POSITRONS IN COLLISIONS WITH NEUTRAL ATOMS AND MOLECULES

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In a recently published letter, Paul and Saint-Pierre [1] report discovery of an interesting phenomenon—the anomalously fast annihilation of positrons in polyatomic gases. The lifetimes of positrons in a number of hydrocarbons and in CCl_4 turned out to be several orders of magnitude (10—1000 times) smaller than those obtained from Dirac's formula

$$\tau_D = 1/\sigma_D v n, \qquad \sigma_D = Z_v \pi r_0^2 c / v, \tag{1}$$

where r_0 is the classical radius of the electron, n is the number of molecules per cm^3, Z_V is the number of valence electrons, and v is the positron velocity. We present here qualitative and quantitative interpretations of this phenomenon.

The positron slows down to an energy $E \sim 0.1$ eV in a time considerably shorter than the Dirac annihilation time (1). If a weakly bound positron-molecule state exists with a binding energy $\epsilon \sim 0.1$ eV, the positron will subsequently annihilate in the molecule as a whole in a resonant manner with a probability considerably greater than given by Dirac's formula.

In other words, in the scattering process there is formed a long-lived intermediate positron-molecule state, one of whose decay channels is annihilation. The cross section σ_a is given by a formula of the Wigner type (see Landau and Lifshitz [2], page 644):

$$\sigma_a = \sigma_e \frac{\tau_e}{\tau_a} = \frac{2\pi\hbar^2}{m} \frac{1}{E+\varepsilon} \frac{\Gamma_a}{\Gamma_e}, \tag{2}$$

where $\tau_e \sim R/v$ is the effective collision time of the positron with an atom (R is the size of the atom); τ_a is the annihilation time of the positron in the atom; Γ_a and Γ_e are the annihilation and elastic widths of the quasistationary level, $\Gamma_a = \hbar/\tau_a$, $\Gamma_e = \hbar/\tau_e$, i.e., $\Gamma_e = \sqrt{\epsilon_0 T}$, $\epsilon_0 \sim \hbar^2/mR^2$; m is the positron mass. Formula (2) is valid in the region $E \lesssim \sqrt{\epsilon_0 \epsilon}$. The ratio of the resonance and Dirac annihilation cross sections, which is equal to the ratio of the experimentally observed and Dirac annihilation rates, can be represented according to (1) and (2) by the formula

$$\frac{\sigma_a}{\sigma_D} = 4\left(\frac{\hbar c}{e^2}\right)^2 \frac{1}{Z_v\beta_0} \frac{\Gamma_a}{E+\varepsilon} = \frac{7.55\cdot 10^4}{Z_v\beta_0} \frac{\Gamma_a}{E+\varepsilon}, \tag{3}$$

where $\beta_0 = c^{-1}\sqrt{2\epsilon_0/m}$. Assuming $\epsilon_0 = 1$ eV, $E + \epsilon = 0.1$ eV, $\tau_a = 10^{-10}$ sec, and $Z_V = 10$, we obtain from (3), in agreement with the experimental results of Paul and Saint-Pierre, [1] $\sigma_a/\sigma_D = 240$. The assumption of the existence of the bound positron state can be justified in the case of approximately spherically symmetrical molecules by postulating that in these cases the interaction of the positron with the molecules is described by some centrally symmetric potential V(r). The value of a shallow level in such a potential can be determined from reasoning similar to that used in the theory of scattering of slow particles ([2], page 578).

Specifically, let the weakly bound state be described by the equation

$$\frac{d^2\chi}{dr^2} + (-\varkappa^2 - U(r))\chi = 0, \quad \varkappa^2 = \frac{2m\varepsilon}{\hbar^2}, \quad U = \frac{2mV}{\hbar^2} \tag{4}$$

and let $1/\kappa$ be much larger than the effective radius R of the potential. Then in the region $r \ll 1/\kappa$ we can neglect κ^2 in (4) and consequently in this region the solution χ is obtained from the simplified equation $\chi'' - U\chi = 0$, which has the asymptote $\chi = C(1 - Ar)$ as $r \to \infty$, where C is a normalizing factor and A is a constant which depends only on the properties of the potential V(r). In the intermediate region $R \ll r \ll 1/\kappa$, this asymptote must coincide with the asymptote of equation (4), i.e., $\chi = Ce^{-\kappa r} \approx C(1 - \kappa r)$. In other words, the level ϵ is determined by the equation $\kappa = A$. For the potential $V = -\alpha/(r^2 + R^2)^2$ used by Moussa, [3] for which the equation $\chi'' - U\chi = 0$ has a simple solution ($\alpha = pe^2a_0^3$, where a_0 is the Bohr radius and p is the polarizability), the level depth ϵ found in the above manner is

$$\varepsilon = \frac{\pi^2}{32} \frac{\hbar^2}{mR^2}\left(\left[1 + 2p\left(\frac{a_0}{R}\right)^2\right]^{1/2} - 2\right)^2. \tag{5}$$

The value of the constant ϵ_0 in (2) is given by the formula (see [2], page 590):

$$\varepsilon_0 = \frac{\hbar^2}{2m}\left(\int_0^\infty (1 - \chi_1^2)\, dr\right)^{-2} = \frac{\hbar^2}{2mR^2F^2(q)},$$

$$q = \left[1 + 2p\left(\frac{a_0}{R}\right)^2\right]^{1/2}, \tag{6}$$

where

$$F(q) = \int_0^{\pi/2} \left(1 - \frac{\sin^2 qs}{q^2 \sin^2 s}\right) \frac{ds}{\sin^2 s},$$

$$\chi_1 = \frac{[x^2+1]^{1/2}}{q} \sin\left[q\left(\frac{\pi}{2} - \tan^{-1} x\right)\right], \quad x = \frac{r}{R}$$

is the solution of the equation

$$\chi'' - U\chi = 0$$

normalized by the condition $\chi_1(\infty) = 1$.

According to Eq. (5), the condition for existence of the level is given by the inequality

$$p(a_0/R)^2 > 3/2. \tag{7}$$

Assuming as in [3] that R is approximately equal to the kinetic theory radius of the particle, and using the tabulated values of polarizability, we find that this inequality is not fulfilled in the case of argon (in agreement with the results of Paul and Saint-Pierre[1] where it is shown that for argon $\sigma_a/\sigma_D = 3$); i.e., the resonance effect is absent in argon. The condition (7), however, is fulfilled for xenon. Therefore we can assume the existence of the resonance effect in xenon.

For all the other molecules investigated by Paul and Saint-Pierre, the inequality (7) is fulfilled; i.e., the resonance effect exists for these molecules. For methane the value of the level given by formula (5) is $\epsilon = 0.05$ eV. Assuming for methane this value of the level, $\epsilon_0 = 0.6$ eV (for $F(q) = 1.5$), the experimental value $\sigma_a/\sigma_D = 20$, $Z_V = 8$, and $E = 0$, we find from (3) the annihilation time τ_a of the bound state in methane: $\tau_a = 3.6 \times 10^{-9}$ sec.

In conclusion it should be noted that a verification of the interpretations suggested would be the observation of elastic scattering of slow positrons ($E \sim 0.1$ eV) in the gases for which the increased annihilation rates have been observed. According to our suggestion, the cross section σ_e of the scattering process should considerably exceed the geometrical cross section of the molecule and should be given by the Wigner formula;

$$\sigma_e = (2\pi\hbar^2/m)/(E+\varepsilon).$$

The weakly bound state of the positron should be easily destroyed in a collision of the positron-molecule ion with another molecule. Therefore the scheme proposed could also be verified by observation of the pressure effect, i.e., the disappearance of the Paul–Saint-Pierre effect on increasing the gas pressure or on dilution by a gas which does not form a bound state with a positron (for example, argon).

[1] D. Paul and L. Saint-Pierre, Phys. Rev. Lett. **11**, 493 (1963).

[2] L. Landau and E. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), second edition, Fizmatgiz, 1963.

[3] A. Moussa, Proc. Phys. Soc. **74**, 101 (1959).

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ON THE NATURE OF THE "TAIL" IN THE INCOHERENT INELASTIC SCATTERING CROSS-SECTION OF SLOW NEUTRONS IN CRYSTALS

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1. In investigating the incoherent inelastic scattering cross-section of slow neutrons in crystals a number of authors have observed the differential cross-section to have a noticeable "tail" in a wide region of energy, which without any doubt lies above the upper end of the harmonic-approximation phonon spectrum ω_{max}. This was established with particular reliability in the case of vanadium by the work of Egelstaff and Turberfield[1], who specifically investigated the temperature dependence of the cross-section in this energy region. At first glance it would seem that "tails" of this kind must be connected with two- or many-phonon processes accompanying the scattering of the neutron from the crystal; however, even the most optimistic estimates (made, in particular, in [1]) show that the calculated values lie well below the experimentally observed values of the cross section. Thus, the question of the origin of the tail in the incoherent inelastic cross-section has remained open.

In this note we show that anharmonic effects in the crystal must necessarily produce a tail in the energy dependence of the cross section. Thus, to all appearances, the observed results are largely connected with anharmonicity.

2. To first order in the ratio of the recoil energy R from an individual nucleus to the characteristic energy ω_0 of the phonon spectrum, the expression for the differential scattering cross-section for scattering from an arbitrary crystal