INTENSITY RULES FOR ELECTROMAGNETIC TRANSITIONS IN DEFORMED NUCLEI

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The intensity ratios of single-particle transitions in deformed odd nuclei are considered. It is shown that the corrections to the nuclear wave functions resulting from the interaction between the rotation and the intrinsic motion can explain the observed deviations from the Alaga rules and the difference in the behavior of the transitions with $\Delta K = 0$ and $\Delta K = \pm 1$. The theoretical calculations agree with experimental results.

IN axially deformed atomic nuclei the intensities of electromagnetic transitions between levels of two rotational bands should be calculable by simple rules established first by Alaga, Alder, Bohr, and Mottelson.[1] In a number of cases, in particular for collective and for allowed single-particle transitions, these rules are quite well fulfilled. However, in transitions which are forbidden because of the asymptotic quantum numbers, the deviations from the Alaga rules are quite considerable and reach two and even three orders of magnitude. At this time no satisfactory explanation exists for this discrepancy. In particular, it is not clear why for electric dipole transitions with $\Delta K = \pm 1$ these discrepancies are large while for $\Delta K = 0$ they are unimportant (10—20%).

It is natural to suppose that the deviations from the Alaga rules are caused by the interaction of the internal motion with the rotational motion, i.e., by the Coriolis forces. Since this interaction is small in strongly deformed nuclei it will influence the probability of allowed transitions but little. However, for the forbidden transitions the influence of this effect can be relatively large and can result in the above discussed deviations from the Alaga rules. The matrix elements of the forbidden transitions are very sensitive to the choice of the single particle level scheme. Therefore the calculation of this effect based on an arbitrary single-particle model of the nucleus [2] is not very convincing. If the deviations from Alaga's rules are due to the Coriolis forces then one can show that in first order of perturbation theory the intensities of transitions between levels of two rotational bands depend on two parameters, sometimes even only on one. These parameters, which depend on the internal motion, can naturally be determined from experiment.

Since it is not possible to exclude other reasons for the deviations from Alaga's rules, a comparison of experiment with the theoretical formulae will be able to determine whether the proposed effect is the main reason for the observed discrepancies. The actual parameter values which depend on the internal motion, can be computed on the basis of particular nuclear models.

The wave function of a deformed nucleus has the form [3]

$$|IMKn\rangle = \left(\frac{2I+1}{16\pi^2}\right)^{1/2}[D^I_{MK}(\theta)\varphi_{nK} + (-1)^{I-\hat{j}} D^I_{M-K}(\theta)\varphi_{n-K}], \quad (1)$$

where I, M, and K are the total angular momentum and its projections on the z axis in the laboratory system and on the symmetry axis in the intrinsic system respectively, $D^I_{MK}(\theta)$ is Wigner's function of the Euler angles θ which describes the rotation of the nucleus as a whole, φ_{nK} is the wave function of the intrinsic motion with n and K its quantum numbers, and $\hat{j}$ is the operator of the intrinsic angular momentum.

The separation into collective and intrinsic motion holds only in zeroth approximation in terms of the angular velocity. The Coriolis forces lead to an interaction between the rotation and the intrinsic motion. The Hamiltonian for this interaction has the form

$$H' = -(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)/2J,$$
$$\hat{I}_\pm = \hat{I}_x \pm i\hat{I}_y, \qquad \hat{j}_\pm = \hat{j}_x \pm i\hat{j}_y, \quad (2)$$

where $\hat{j}$ is the single-particle angular momentum operator and J is the moment of inertia of the nucleus.

We begin by considering qualitatively the conditions for which (2) can lead to significant deviations from the Alaga rules, and take E1 transitions as an example. To this end we have to

Table I. Asymptotic selection rules for El and Ml transitions

Multi-polarity of transition	ΔK	Transition operator	ΔN	Δn_z	$\Delta\Lambda$	$\Delta\Sigma$
$E1$	$+1$	$x+iy$	±1	0	$+1$	0
	-1	$x-iy$	±1	0	-1	0
	0	z	±1	±1	0	0
$M1$	$+1$	l_+	0	±1	$+1$	0
	-1	l_-	0	±1	-1	0
	±1	$s_\pm$	0	0	0	±1
	0	s_z; l_z	0	0	0	0

Note: ΔN, Δn_z, $\Delta\Lambda$ and ΔK are the changes of the principal quantum number, the oscillator quantum number along the z-axis, and the projections on the z-axis of the orbital and the total angular momentum respectively.

Table II. Asymptotic selection rules for the operator $\hat{j}_\pm$.

ΔK	Operator	ΔN	Δn_z	$\Delta\Lambda$	$\Delta\Sigma$
$+1$	l_+	0	±1	$+1$	0
-1	l_-	0	±1	-1	0
±1	$s_\pm$	0	0	0	±1

Note: $\Delta\Sigma$ is the change of the projection of the spin on the z-axis.

choose a transition which is forbidden by virtue of the asymptotic quantum numbers. However, this condition is by no means sufficient. It is necessary that the operators $\hat{M}_{L\nu}$ and $\hat{j}$ have allowed matrix elements for the admixed transitions. This is not always the case, and as a result deviations from the Alaga rules are observed only in particular forbidden transitions. To understand this we consider the selection rules for dipole transitions associated with the asymptotic quantum numbers [4] and with the operator $\hat{j}_\pm = \hat{l}_\pm + \hat{s}_\pm$ (l is the orbital and s the spin angular momentum of the nucleon) which connects the intrinsic ground state wave function with the admixed wave functions [5] (see Tables I and II). The asymptotic wave functions $|Nn_z\Lambda\Sigma\rangle$ (where N is the principal harmonic-oscillator quantum number, n_z is the oscillator quantum number along the z-axis, Λ and Σ are the projections of the nucleon orbital and spin angular momentum on the symmetry axis of the nucleus) have been given by Nilsson [6]. They are the solutions of the wave equation for the Nilsson potential in the limit of large deformations.

The most common electric dipole transitions forbidden because of the asymptotic quantum numbers can be divided into the following groups, according to the changes in the quantum numbers:

Group 1:

$$\Delta K=0 \quad \Delta N=\pm1 \quad \Delta n_z=\pm2 \quad \Delta\Lambda=\mp1 \quad \Delta\Sigma=1$$

Group 2:

$$\Delta K=\pm1 \quad \Delta N=\pm1 \quad \Delta n_z=\pm2 \quad \Delta\Lambda=\pm1 \quad \Delta\Sigma=0$$

Group 3:

$$\Delta K=\pm1 \quad \Delta N=\pm1 \quad \Delta n_z=\pm1 \quad \Delta\Lambda=0 \quad \Delta\Sigma=\pm1$$

As can be seen from Table I, all these transitions are forbidden by two orders while the transition elements calculated in the Nilsson scheme are forbidden only in first order. In Table III are given the changes in the asymptotic quantum numbers in the matrix elements of the admixed transitions for each group of nuclei. The matrix elements of $\hat{j}$ are allowed for these transitions as far as the asymptotic quantum numbers are concerned. The listed transitions are forbidden not higher than in second order. One sees from this table that the admixed transitions of group 1 as well as the fundamental transition are doubly forbidden. The allowed admixed transitions will be of minor importance, since their matrix elements of the operator $\hat{j}$ are forbidden. Thus, here the contribution from the correction terms will be small and the deviations from the Alaga rules must be unimportant. For the transitions of groups 2 and 3 some transitions are always admixed which are allowed according to the asymptotic quantum numbers. Since the matrix element of a transition singly forbidden according to the asymptotic quantum numbers is decreased in magnitude by roughly a factor $A^{1/3}$, the ratio of

Table III. Changes of the asymptotic quantum numbers in the admixed matrix elements for electric dipole transitions

Group of transition	ΔK	ΔN	Δn_z	$\Delta\Lambda$	$\Delta\Sigma$	Order of forbiddenness	Transition operator
1	1	±1	±1	0 2	±1	2	l_+, l_-
	-1	±1	±1	-2 0	±1	2	
	±1	∓1	∓2	±1	0	2	s_+, s_-
2	0	±1	±1	0	0	allowed	l_+, l_-
	0	±1	±2	$+1$	-1	2	s_+, s_-
	0	±1	±2	-1	$+1$	2	
3	0	±1	±1	0	0	allowed	s_+, s_-
	0	±1	0, ±2	$+1$	-1	2	l_+, l_-
	0	±1	0, ±2	-1	$+1$	2	
4	±1	$+1, -1$	$+1, -1$	0	±1	2	l_+, l_-
	±1	$+1, -1$	$+1, -1$	±2	∓1	2	
	±1	$+1, -1$	0	±1	0	allowed	s_+, s_-
5	0	±1	0, ±2	1	-1	2	l_+, l_-
	0	±1	0, ±2	-1	1	2	

the admixed transitions to the fundamental transition can reach a factor $A^{2/3}$. This can compensate for the smallness of the expansion coefficients of the admixed transitions which are here of the order $A^{-2/3}$ as will be shown below. Thus, with respect to the above-discussed transitions, the deviations from the Alaga rules will be large for transitions with $\Delta K = \pm 1$ while they will be small for the transitions with $\Delta K = 0$. A behavior of this kind in E1 transitions has been noted by Vergnes.[7] It should be emphasized that this difference exists only for the above discussed transitions. In principle doubly-forbidden E1 transitions with the following asymptotic quantum numbers are possible:

Group 4

$\Delta K = 0 \quad \Delta N = \pm 1 \quad \Delta n_z = 0 \quad \Delta\Lambda = \pm 1 \quad \Delta\Sigma = \mp 1$

Group 5

$\Delta K = \pm 1 \quad \Delta N = \pm 1 \quad \Delta n_z = \pm 1 \quad \Delta\Lambda = \pm 2 \quad \Delta\Sigma = \mp 1$

One sees from Table III that allowed transitions are admixed to group 4 but not to group 5. Thus here the deviations from the Alaga rules should be large for the transitions with $\Delta K = 0$ while they should be small for $\Delta K = \pm 1$.

For those transitions where large deviations from the Alaga rules are observed, it is meaningless to calculate the absolute transition probabilities without taking into account the contributions of the admixed terms. The quantities which are calculated in such a manner must differ considerably from the experimental values even when the intrinsic wave functions are chosen properly. One may hope that the strong forbiddenness of some E1 transitions is connected with the interaction between the single particle motion and the rotation.

We now turn to the quantitative consideration of the influence of the perturbation (2) on the ratio of the probabilities of the mentioned electromagnetic transitions. Using the known selection rules for the operator (2) one can find the wave function including the corrections up to first order in the perturbation H':

for $K \neq 1/2$

$$\Psi_{IMKn} = |IMKn\rangle - [(I+K)(I-K+1)]^{1/2} \times \sum_m \frac{\langle K-1m|\hat{j}_-|Kn\rangle}{2J(E_n-E_m)} |IMK-1m\rangle - [(I-K)(I+K+1)]^{1/2} \times \sum_m \frac{\langle K+1m|\hat{j}_+|Kn\rangle}{2J(E_n-E_m)} |IMK+1m\rangle, \tag{3}$$

for $K = 1/2$

$$\Psi_{IM^1/_2n} = |IM^1/_2n\rangle - (-1)^{I-1/2}(I+1/2) \times \sum_m \frac{\langle -1/_2m|\hat{j}_-|1/_2n\rangle}{2J(E_n-E_m)} |IM^1/_2m\rangle - [(I-1/_2)(I+3/_2)]^{1/2} \sum_m \frac{\langle 3/_2m|\hat{j}_+|1/_2n\rangle}{2J(E_n-E_m)} |IM^3/_2m\rangle. \tag{3'}$$

Here E_n is the energy corresponding to the state n of the intrinsic motion.

The corrections to the wave function which contain the allowed matrix elements of the operator $\hat{j}$ are small and are of the order of the ratio of the rotational energy to the distance between the mixing levels. Their order of magnitude is $I^{1/2} A^{-2/3}$ if $I \sim K$, and about $I\,A^{-2/3}$ if $I \gg K$.

When computing the reduced transition probability for a transition of multipolarity L,

$$B(L, IKn \to I'K'n') = \sum_{\mu M'} \left| \left(\Psi^*_{I'M'K'n'} \left| \sum_\nu D_{\mu\nu}{}^L(\theta) M_{L\nu} \right| \Psi_{IMKn}\right)\right|^2$$

with the wave functions (3) we obtain ($\Delta K = K' - K$)

$$B(L, IKn \to I'K'n') = B_0(L, IKn \to I'K'n')(1+b_{n'})^2 \times \left\{1 + [(I+K)(I-K+1)]^{1/2} \times \frac{\langle IL;\, K-1,\, \Delta K+1 | I'K'\rangle}{\langle IL;\, K\Delta K | I'K'\rangle} \frac{a_n^- + b_{n'}^-}{1+b_{n'}} + [(I-K)(I+K+1)]^{1/2} \times \frac{\langle IL;\, K+1\Delta K-1 | I'K'\rangle}{\langle IL;\, K\Delta K | I'K'\rangle} \frac{a_n^+ + b_{n'}^+}{1+b_{n'}}\right\}^2; \tag{4}$$

$$b_{n'} = b_{n'}^+ [(L+\Delta K)(L-\Delta K+1)]^{1/2} + b_{n'}^- [(L+\Delta K+1)(L-\Delta K)]^{1/2},$$

Here $M_{L\nu}$ is the electromagnetic transition operator in the intrinsic coordinate system, B_0 is the reduced transition probability without consideration of the perturbation (2), $\langle IL;\, K\Delta K \mid I'K'\rangle$ is a vector coupling coefficient, and the quantities $a_n^\pm$ and $b_{n'}^\pm$ are given by the expressions

$$a_n^\pm = -\sum_m \frac{\langle K\pm 1m|\hat{j}_\pm|Kn\rangle}{2J(E_n-E_m)} \frac{\langle K'n'|M_{L\Delta K\mp 1}|K\pm 1m\rangle}{\langle K'n'|M_{L\Delta K}|Kn\rangle}, \tag{5}$$

$$b_{n'}^\pm = -\sum_m \frac{\langle K'n'|\hat{j}_\pm|K'\mp 1m\rangle}{2J(E_{n'}-E_m)} \frac{\langle K'\mp 1m|M_{L\Delta K\mp 1}|Kn\rangle}{\langle K'n'|M_{L\Delta K}|Kn\rangle}. \tag{5'}$$

The coefficients $a_n^\pm$ and $b_{n'}^\pm$ do not depend on the total angular momentum of the nucleus. They are determined by the structure of the intrinsic states.

Equation (4) has been obtained for the case $L < K + K' - 1$ which, as a rule, is fulfilled for low multipolarity transitions. If $L \geq K + K' - 1$ one can write down analogous expressions. However, they contain a large number of constants.

As one can see from (4) the coefficients a and b can be of the order 1 because of the suppression of the matrix element of the main transition, even though the admixtures to the wave functions (3) are small. The ratios of the given probabilities between different numbers of a rotational band depend in the present approximation on two parameters. One sees from (4) that for $\Delta K = \pm L$ one of the vector coupling coefficients vanishes. Then the ratio of the reduced transition probabilities depends only on one parameter.

One can construct two ratios of reduced transition probabilities for E1 transitions orginating from a level with spin I:

$$\eta_1(I, K, \Delta K) = \frac{B(1, IKn \to IK'n')}{B(1, IKn \to I-1, K'n')},$$

$$\eta_2(I, K, \Delta K) = \frac{B(1, IKn \to I+1, K'n')}{B(1, IKn \to I, K'n')}.$$

Using (4) and the explicit form of the Clebsch-Gordan coefficients it is easy to obtain the general expressions for the quantities η_1 and η_2.

In the case $\Delta K = \pm 1$ the ratios of the probabilities depend on one parameter and have the form

$$\eta_1(I, K, \pm 1) = \eta_1^0(I, K, \pm 1)[1 - Iz_\pm]^{-2}; \qquad (6)$$

$$\eta_2(I, K, \pm 1) = \eta_2^0(I, K, \pm 1)[1 + (I+1)z_\pm]^2; \qquad (7)$$

$$z_\pm = \frac{\sqrt{2}(a_n^\pm + b_{n'}^\pm)}{1 - \sqrt{2}\, a_n^\pm \mp \sqrt{2}\, K(a_n^\pm + b_{n'}^\pm)}.$$

For $\Delta K = 0$

$$\eta_1(I, K, 0) = \eta_1^0(I, K, 0)\left[\frac{1 - I(I+1)u_-/2K}{1 - {}^1\!/_2 I u_+}\right]^2, \qquad (8)$$

$$\eta_2(I, K, 0) = \eta_2^0(I, K, 0)\left[\frac{1 + {}^1\!/_2 (I+1) u_+}{1 - I(I+1)u_-/2K}\right]^2; \qquad (9)$$

$$u_\pm(K) = \frac{\sqrt{2}(a_n^- \pm a_n^+ + b_{n'}^- \pm b_{n'}^+)}{1 - \sqrt{2}(a_n^+ + a_n^-) + K(a_n^- - a_n^+ + b_{n'}^- - b_{n'}^+)}.$$

The quantities η_1^0 and η_2^0 in (6) and (8) are the ratios of the reduced transition probabilities in zeroth order of the perturbation H′ found by Alaga [1]:

$$\eta_1^0(I, K, \Delta K) = \frac{\langle I1; K\Delta K | IK'\rangle^2}{\langle I1; K\Delta K | I-1K'\rangle^2},$$

$$\eta_2^0(I, K, \Delta K) = \frac{\langle I1; K\Delta K | I+1K'\rangle^2}{\langle I1; K\Delta K | IK'\rangle^2}. \qquad (10)$$

For transitions between levels where K or K′ equals ½, the formulae for the ratios of the transition probabilities become more involved and the number of parameters increases. Using the wave function (3′) one easily finds the ratios η_1 and η_2 for dipole transitions between levels of two rotational bands with $K = 3/2$, $K' = 1/2$ or $K = 1/2$, $K' = 3/2$:

for $K' = 1/2$

$$\eta_1(I, 3/2, -1) = \eta_1^0(I, 3/2, -1) \times \left[\frac{1 + (-1)^{I-1/2}(I + 1/2)t_-}{1 - Iz_-(3/2) + (-1)^{I+1/2}(I - 1/2)t_-}\right]^2,$$

$$\eta_2(I, 3/2, -1) = \eta_2^0(I, 3/2, -1) \times \left[\frac{1 + (I+1)z_-(3/2) + (-1)^{I+1/2}(I + 3/2)t_-}{1 + (-1)^{I-1/2}(I + 1/2)t_-}\right]^2; \qquad (11)$$

for $K' = 3/2$

$$\eta_1(I, 1/2, 1) = \eta_1^0(I, 1/2, 1) \times \left[\frac{1 + (-1)^{I-1/2}(I + 1/2)t_+}{1 - Iz_+(1/2) + (-1)^{I-1/2}(I + 1/2)t_+}\right]^2,$$

$$\eta_2(I, 1/2, 1) = \eta_2^0(I, 1/2, 1) \times \left[\frac{1 + (I+1)z_+(1/2) + (-1)^{I-1/2}(I + 1/2)t_+}{1 + (-1)^{I-1/2}(I + 1/2)t_+}\right]^2. \qquad (12)$$

In addition to the parameters $z_\pm(K)$ which are given by (7), these equations contain also the following quantities:

$$t_+ = \frac{\alpha_n(1/2) + \beta_{n'}(3/2)}{1 - (3/\sqrt{2})a_n^+ - (1/\sqrt{2})b_{n'}^+},$$

$$t_- = \frac{\alpha_n(3/2) + \beta_{n'}(1/2)}{1 + (1/\sqrt{2})a_n^- + (3/\sqrt{2})b_{n'}^-},$$

where

$$\alpha_n(K) = -\sum_m \frac{\langle K - 1m | \hat{j}_- | Kn\rangle}{2J(E_n - E_m)} \frac{\langle K'n' | M_{11} | -K + 1m\rangle}{\langle K'n' | M_{1\Delta K} | Kn\rangle},$$

$$\beta_{n'}(K') = -\sum_m \frac{\langle K'n' | \hat{j}_+ | K' - 1m\rangle}{2J(E_{n'} - E_m)} \times \frac{\langle -K' + 1m | M_{1,-1} | Kn\rangle}{\langle K'n' | M_{1\Delta K} | Kn\rangle}.$$

Equations (6), (8), and (11) can be easily checked by experiment. In Table IV the experimental values are compared with the values following from the Alaga rules and from the present approach. The transitions are emphasized which

Table IV. Comparison of the experimental ratios of the reduced dipole transition probabilities with the ratios computed from equations (6), (8) and (11)

Nucleus	Type of transition	Spin of the initial state, I.	Single particle transition $K[Nn_z\Lambda] \to$ $\to K'[N'n_z'\Lambda']$	$\eta_1(I)$			$\eta_2(I)$			Parameters z, u, t	Reference
				Experiment	Alaga rule	Theory	Experiment	Alaga rule	Theory		
Tb^{157}	$E1$	$^5/_2$	$^5/_2\,[532] \to {}^3/_2\,[411]$	0.004	0,43	0.0035	29	0.17	—	$z=-4.02$	[8]
Tb^{159}		$^5/_2$	$^5/_2\,[532] \to {}^3/_2\,[411]$	0.014	0.43	0.013	5.7	0,17	—	$z=-1,94$	[9]
Yb^{169}	$M1$	$^5/_2$	$^5/_2\,[523] \to {}^5/_2\,[512]$				0.48	0,40	—	$u_+=2.6\cdot10^{-2}$, $u_-=9.5\cdot10^{-4}$	
		$^7/_2$	$^5/_2\,[523] \to {}^5/_2\,[512]$	2.3	1,86	—	1.25	0,98	1,22		[10]
	$M1$	$^7/_2$	$^7/_2\,[503] \to {}^5/_2\,[512]$	5.5	0.30	—	0,45	0,13	0.51	$z=0.22$	[10]
Yb^{171}	$M1$	$^7/_2$	$^7/_2\,[514] \to {}^5/_2\,[512]$	1.0	0,30	—	0.91	0.125	1,11	$z=0.44$	[10]
Yb^{173}	$M1$	$^7/_2$	$^7/_2\,[514] \to {}^5/_2\,[512]$	0,41	0.30	—	0.71	0.125	1,43	$z=0.53$	[10]
	$E1$	$^7/_2$	$^7/_2\,[633] \to {}^5/_2\,[512]$	83	0.30	—	0.55	0.125	0,56	$z=0.31$	[11]
Lu^{175}	$E1$	$^9/_2$	$^9/_2\,[514] \to {}^7/_2\,[404]$	2.0	0.23	—	0.72	0.10	0.69	$z=0,30$	[11]
Hf^{175}	$M1$	$^7/_2$	$^7/_2\,[514] \to {}^5/_2\,[512]$	2,86	0.30	—	0.5	0.125	0.91		
		$^9/_2$	$^7/_2\,[514] \to {}^5/_2\,[512]$	1.79	0,52	1,06		0,17	1.60	$z=0,38$	[10]
		$^{11}/_2$	$^7/_2\,[514] \to {}^5/_2\,[512]$	0,15	0,69	0.59		0.20	2.44		
Hf^{177}	$E1$	$^9/_2$	$^9/_2\,[624] \to {}^7/_2\,[514]$	175	0.23	—	0,37	0,10	0.46		[12]
		$^{11}/_2$	$^9/_2\,[624] \to {}^7/_2\,[514]$	15,8	0.41	17				$z=0.21$	
		$^{13}/_2$	$^9/_2\,[624] \to {}^7/_2\,[514]$	7.7	0,56	4,3					
		$^{15}/_2$	$^9/_2\,[624] \to {}^7/_2\,[514]$	20	0,70	2,3					
W^{183}	$M1$	$^3/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	14,3	0,80	—	0.46	0,25	—		
		$^5/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	0.11	1,14	0,51	100	0,31	137	$z=0,74$, $t=-0.26$	[13]
		$^7/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	0,77	1,33	0,99		0,35	0.77		
Os^{185}	$M1$	$^3/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	6,25	0,80	—	0.52	0,25	—		
		$^5/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	0,15	1,14	0,10	110	0,31	312	$z=0.84$, $t=-0,28$	[13]
		$^7/_2$	$^3/_2\,[512] \to {}^1/_2\,[510]$	0,67	1,33	0,79	0,90	0,35	0,89		

lead to the determination of the parameters z, u and t. As one sees from (6), (8) and (11), the ratio of the reduced transition probabilities depends quadratically on these parameters. Therefore for the case of transitions with $\Delta K = \pm 1$ [Eq. (6)] two values are found for the parameter z, while four pairs of values are obtained for the corresponding parameters (u_+, u_- or z and t) in the case $\Delta K = 0$ [Eq. (8)] and for transitions which are described by (11) and (12). Those values were selected for the parameters which gave the best agreement with experiment. It must be pointed out that the parameters are best determined from those ratios which have the largest deviations from the Alaga rules; as a rule these transitions are most sensitive to the admixtures.

The comparison of the theory with experiment shows that the experimentally observed intensity ratios are well described by the corrections introduced here. This agreement indicates that the observed deviations from the Alaga rules result basically from the interaction between the intrinsic motion and the rotation.

In order to estimate the limits of validity of the simple phenomenological description developed in this paper, more accurate experimental determinations of the ratios are needed, and a correct estimate of the experimental uncertainties of the measurements and of the analysis of the data is required. In particular, it is essential that the multipole composition of the radiation be found.

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