# EXPERIMENTAL STUDY OF THE PROPAGATION OF ELECTROMAGNETIC WAVES OF FINITE AMPLITUDE IN FERRITE-FILLED TRANSMISSION LINES

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In a coaxial transmission line filled with ferrite magnetized by a constant longitudinal magnetic field, shock waves are produced as the result of evolution of simple waves, whereas in unmagnetized ferrite their formation from the very beginning is related to dissipative processes. Comparison of the experimental and theoretical results showed that in the first case variation of the magnetization both on the front of the shock wave and in the region of comparatively slow variation of the field primarily proceeds coherently, whereas in the second case it is due to incoherent rotation. For a nonvanishing azimuthal component of the constant magnetic field, propagation of finite amplitude electromagnetic waves was found to be in accord with the theoretical predictions.

### 1. INTRODUCTION

IN previously published experimental studies of electromagnetic shock waves, <sup>[1-3]</sup> the waves have as a rule been studied in transmission lines with discrete parameters, for example, with lumped inductive nonlinear elements consisting of coils wound on ferrite cores. The systematic theoretical discussion of waves in such systems involves extraordinarily great difficulties (consideration of the spatial dispersion due to the discrete structure, calculation of the parameters of the inductive nonlinear elements on the basis of an assumed model of magnetization reversal of the ferrite, and so forth). This circumstance makes it practically impossible to compare quantitatively the experimental results with the conclusions of a rigorous theory.

In the present work we present the results of an experimental study of the propagation of the electromagnetic waves in a coaxial transmission line filled with ferrite. In view of the comparatively simple structure of the field in such a transmission line, certain quantities that characterize the propagation and structure of shock waves could be found theoretically with certain assumptions about the mechanism of the change of the average magnetization of the ferrite under action of an external field.

The coaxial transmission line in which we studied the formation and propagation of shock waves consisted of two pairs of sections 90 and 80 cm long. At the points where these sections were joined together, it was possible to connect a high resistance divider, the voltage from which was fed directly to the vertical plates of a fast oscilloscope of known sensitivity. The bandwidth of the entire system was sufficient for measurement of wave-front durations of 1 nsec or longer. Cylindrical tubes of F-1000 ferrite<sup>1)</sup> were placed over the center conductor of the coaxial line, which was first covered with a polyethylene film. The coaxial line sections were placed in two solenoids which produced a constant longitudinal magnetic field up to 300 Oe in strength. An azimuthal component of the constant magnetic field was produced by current flowing in the center conductor of the coaxial line. To reduce the demagnetizing field arising at the ends of each section in the presence of a constant magnetic field, the first and last ferrite tubes of each section of the transmission line were ground to a conical shape.

### 2. STUDY OF THE PROCESSES OF FORMATION AND PROPAGATION OF SHOCK WAVES

For a rather slow variation of the electromagnetic field, the structure of the field in the cross section of the transmission line has a static character. In this case, independently of the mechanism of variation of the magnetization and polarization, we can introduce the concept of a current J and voltage U between the conductors

<sup>&</sup>lt;sup>1</sup>)The length of the ferrite tubes was 55 mm, the internal diameter 8 mm, and the external diameter 16 mm. The gap between adjacent sections did not exceed 0.1 mm.

at a given point of the transmission line. If we neglect Joule losses, the variation of current and voltage is determined by the telegraphy equations<sup>2</sup>:

$$\partial J / \partial Z = -\partial Q / \partial t, \quad \partial U / \partial Z = -\partial \Phi / \partial t.$$
 (2.1)

Here Q is the instantaneous charge on one of the conductors of the transmission line,  $\Phi$  is the instantaneous flux (the magnetic induction through the area between the conductors of a coaxial line of unit length in the Z direction, divided by the velocity of light c).<sup>3)</sup> The relation between  $\Phi$  and J or Q and U depends on the processes of variation of the magnetization and polarization of the medium filling the transmission line.

For waves whose spectrum is mainly in the range  $10^2-10^3$  kcs to  $10^4-10^5$  Mcs, the charge Q is proportional to the voltage U:

$$Q = CU. \tag{2.2}$$

An expression for the capacitance C can be suitably written in the form

$$C = \varepsilon_{\text{eff}} / 2 \ln f, \qquad (2.3)$$

where  $\epsilon_{\rm eff}$  is the effective dielectric constant, which depends on the dielectric constants of the ferrite and dielectric filling the transmission line and on their geometry (in our case  $\epsilon_{\rm eff} \approx 9$ );<sup>4)</sup> f is the ratio of the radii (b/a) of the outer and inner conductors of the transmission line.

# 1. Formation and propagation of shock waves in a coaxial line with ferrite magnetized by a constant magnetic field

The mechanism of the variation of the average magnetization of the ferrite in the propagation of electromagnetic waves depends to a significant degree on the constant magnetic field in the transmission line. If the longitudinal component of the constant magnetic field is rather large, then, with a change in direction of the total magnetic field, only a reversal of the mean magnetization vector will occur. For small internal anisotropic fields

3)The Gaussian system of units is used.

we can assume that this process is described by the equation [6]:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma [\mathbf{M}\mathbf{H}] + \frac{\alpha}{M} \Big[\mathbf{M}\frac{\partial \mathbf{M}}{\partial t}\Big]. \qquad (2.4)^*$$

Here  $\gamma$  is the absolute value of the gyromagnetic ratio, M is the saturation magnetization of the ferrite,  $\alpha$  is a constant characterizing the dissipation accompanying a change in magnetization. For a rather slow change in the direction of the magnetic field, the magnetization vector will "follow" its changes and Eq. (2.4) can be approximately written in the form of the static coupling equations:

$$[\mathbf{MH}] = 0, \quad |\mathbf{M}| = \text{const.} \tag{2.5}$$

Taking into account that in the static approximation the longitudinal component  $H_Z$  of the magnetic field in the wave does not depend on the transverse coordinates, and that the azimuthal component is inversely proportional to the radius, in the case where Eq. (2.5) is valid we obtain for the flux  $\Phi$  the expression

$$\Phi = \frac{2(J+J_0)}{c^2} \ln f \Big[ 1 + \frac{4\pi M}{H_z} \frac{1}{\ln f} \ln \frac{f + \sqrt{f^2 + \operatorname{tg}^2 \theta}}{1 + \sqrt{1 + \operatorname{tg}^2 \theta}} \Big].$$
(2.6)<sup>†</sup>

Here  $\theta$  is the angle between the direction of the longitudinal component  $H_0$  of the constant magnetic field and the direction of the magnetization vector at the surface of the inner conductor

 $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$ , and  $J_0$  is the value of the dc current flowing in the inner conductor. The dependence of  $\theta$  and  $H_z$  on the current J is determined by the fact that, as a consequence of the assumption of infinite conductivity of the transmission line conductors, the magnetic flux through its cross section must remain constant. Considering Eq. (2.5), this dependence can easily be obtained in implicit form:

$$H_{z} = H_{0} + 4\pi M \left( \frac{f \sqrt{f^{2} + \mathrm{tg}^{2} \theta} - \sqrt{1 + \mathrm{tg}^{2} \theta}}{f^{2} - 1} - \frac{\mathrm{tg}^{2} \theta}{f^{2} - 1} \ln \frac{f + \sqrt{f^{2} + \mathrm{tg}^{2} \theta}}{1 + \sqrt{1 + \mathrm{tg}^{2} \theta}} \right) \Big|_{\theta}^{\theta_{0}},$$

$$\mathrm{tg} \theta = 2(J + J_{0}) / caH_{z}.$$
(2.7)

For the case where the static coupling equations (2.2), (2.5)—(2.7), are valid, it is easy to find [7,8] a partial class of solutions of the tele-

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*[MH] = M \times H.
†tg = tan.
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<sup>&</sup>lt;sup>2</sup>)It should be noted that, as shown by Ostrovskii <sup>[4]</sup> and Khokhlov, <sup>[5]</sup> in those cases where the Joule losses can be taken into account within the framework of the telegraphy equations (2.1) by introducing terms GU and RJ into the right-hand sides of these equations, these losses lead to some decrease in amplitude of the wave with propagation.

<sup>&</sup>lt;sup>4</sup>)Rather rough measurements give the dielectric constant of F-1000 ferrite as 16-20. In computing  $\epsilon_{\rm eff}$  we have taken into account the presence of the polyethylene film ( $\epsilon \approx 2.5$ ). The dispersion of  $\epsilon_{\rm eff}$  in the frequency range indicated can be neglected.

graphy equations (2.1) which have the form of simple waves:

$$J = F[Z \pm v(J)t], \quad U = \pm \int \sqrt[\gamma]{L(J)/C} \, dJ. \quad (2.8)$$

Here  $L(J) = d\Phi/dJ$ , F is a function determined by the boundary conditions at the transmission line input, and the velocity of propagation v(J) of a given point of the wave profile depends on the current at this point and on the properties of the function  $\Phi(J)$ :

$$v(J) = [L(J)C]^{-1/2}.$$
 (2.9)

a) Propagation of waves in the absence of an azimuthal component of the constant magnetic field. From (2.6) and (2.7) it is easy to see that when J = 0 and  $H_0 > 0$ , L(-J) = L(J) is a decreasing function of the absolute value of current in the wave, as a result of which the velocity of a given point of the profile of a simple wave is greater if the absolute value of current and voltage at this point is greater. Therefore, if we excite at the input to the transmission line a wave having the form of a single pulse in which the current and voltage do not change sign, the rate of variation of these quantities in the wave front increases with propagation, and the rate of change in the falling part of the wave decreases (Fig. 1). We can easily verify that in this case a solution of the form of (2.8) describes the propagation of the waves not only qualitatively but also quantitatively, if we note that in the propagation of a simple wave (2.8), the length  $\tau_p$  of the voltage pulse at a fixed



FIG. 1. Oscillograms of the voltage in a wave with  $V = 6 \ kV$  at the points: a)  $Z = 0 \ cm$ , b) 90 cm, c) 180 cm. Time marker interval is 10 nsec.



FIG. 2. Curve a: pulse length at different levels, at the points Z = 0, 90, and 180 cm for waves with amplitudes V = 6 and 12 kV; curves b, c, d: wave-front duration at the points Z = 0, 90, and 180 cm for a wave with an amplitude of 12 kV.

level must remain constant. Figure 2 shows results of a measurement of  $\tau_p$  at different voltage levels at the points Z = 0, 90, and 180 cm for waves of amplitude V = 6 and 12 kV (the amplitude  $H_{max}$  of the azimuthal component of the magnetic field in the wave is  $\sim 10$  and 20 Oe, respectively). The measured values of  $\tau_{\rm f}$ , the length of the wave front at different voltage levels at the same point of the line (Fig. 2), are characterized by an increase in the rate of change of voltage in the wave front and a distortion of the wave profile with V = 12 kV. From the data given in Figs. 1 and 2, it is evident that the propagation of waves of rather large amplitude in a ferrite magnetized by a constant longitudinal magnetic field is well described by a solution having the form of a simple wave (2.8), at least up to the point where the characteristic time intervals determining the rate of change of the field are in the range 10-200 nsec. With further propagation, regions of relatively rapid change of field are formed in the wave front, where the static coupling equations (2.2), (2.5)-(2.7) are invalid; in other words, electromagnetic shock waves are formed. These regions can be considered as discontinuities in the solutions of the telegraphy equations with quasistatic coupling equations. In the discontinuity the following boundary conditions must be satisfied (see [7,8]):

$$J_2 - J_1 = v_d C (U_2 - U_1), \quad U_2 - U_1 = v_d [\Phi(J_2) - \Phi(J_1)].$$
(2.10)

Here the indices 1 and 2 correspond to values before and after the discontinuity, and  $v_d$  is the velocity of the discontinuity. It follows from (2.10) that  $v_d$  depends on the values of current on both sides of the shock-wave front and on the properties of the function  $\Phi(J)$ :

$$v_{d^2} = (J_2 - J_1) / C[\Phi(J_2) - \Phi(J_1)]. \quad (2.11)$$

From (2.10) and (2.11) we can see that, by examining the change of the velocity  $v_d$  with a drop in the voltage in the shock-wave front, it is easy to verify the accuracy of any given expression for the dependence of the instantaneous flux  $\Phi$  on the current J.



FIG. 3. Theoretical variation of  $v_d \sqrt{\epsilon_{eff}}/c$  with  $\overline{H}$  in ferrite magnetized by a longitudinal magnetic field  $H_0$  (solid lines) and in an unmagnetized ferrite for  $m_0 = 0.7$  (dashed line). Experimentally measured values of  $v_d \sqrt{\epsilon_{eff}}/c$  for the following values of  $H_0: +: H_0 = 0$ ; •: 12 Oe;  $\times: 25$  Oe; 0: 75 Oe;  $\Box: 125$  Oe;  $\Delta: 188$  Oe.

The experimental data on the variation of the velocity of a shock wave with its amplitude, obtained at several values  $H_0$ , are shown in Fig. 3. The abscissa represents the value of  $\overline{H}$ , the amplitude of the azimuthal component of the magnetic field intensity during the shock-wave front at points a distance (b + a)/2 from the axis of the coaxial transmission line; the values of  $\overline{H}$ were calculated from the known value of current  $J_2$  =  $v_d \, C \, U_2$  (  $U_1$  = 0,  $J_1$  = 0). The same figure shows plots of  $v_d(\overline{H})\sqrt{\epsilon_{ff}}/C$  computed from formulas (2.6), (2.7), and (2.11) for  $4\pi M = 3200$  G. A systematic deviation of the experimental points from the theoretically computed values is noticeable for  $H_0 < 75$  Oe; this does not exceed 10% and may be due to inaccuracy in determining  $\epsilon_{eff}$  and  $4\pi M$ , and also to some systematic error in measurement of the velocity vd which, as a consequence of a different measurement technique, was different for values of  $H_0$  below and above 75 Oe.

b) Propagation of waves in a coaxial transmission line with a ferrite magnetized by a constant magnetic field having both longitudinal and azimuthal components. The oscillograms in Fig. 4 depict a change in the profile of a wave of rather large amplitude in the course of its propagation with  $H_0 = 25$  Oe and  $J_0 = -18$  A (the current in the wave whose direction is chosen as



FIG. 4. Oscillograms of voltage at the following points: a) Z = 0, b) 90 cm, c) 180 cm, for a wave of amplitude of  $V \approx 10 \text{ kV}$  with  $H_0 = 25 \text{ Oe}$ , J = 18 A. Time marker interval is 50 nsec.

positive flows in a direction opposite to the direction of the constant current  $J_0$ ). The profile change of the wave at the point Z = 180 cm with its amplitude at the input to the transmission line is illustrated by the oscillograms in Fig. 5 (the shape of the voltage pulse at the transmission line input is similar in all cases to that shown in Fig. 4).

The features of the propagation of waves in the case discussed can be explained by analyzing the variation of  $\Phi(J)$  determined by formulas (2.6) and (2.7) for  $J_0 \neq 0$ . A qualitative picture of the behavior of the function  $\Phi(J)$  is shown in Fig. 6a for

$$H_{0} > 4\pi M \left[ 1 - \frac{f \gamma f^{2} + tg^{2} \theta_{0} - \gamma 1 + tg^{2} \theta_{0}}{f^{2} - 1} + \frac{tg^{2} \theta_{0}}{f^{2} - 1} \ln \frac{f + \gamma \overline{f^{2} + tg^{2} \theta_{0}}}{1 + \gamma \overline{1 + tg^{2} \theta_{0}}} \right]$$
(2.12)

and in Fig. 6b for the case of the inverse inequality. In the latter case the dashed line indicates the region of the variation of  $\Phi_1$  where, as we can easily see from equations (2.5)-(2.7), the angle between the magnetization and magnetic field intensity vectors is equal to  $\pi$ . Such a state with a



FIG. 5. Oscillograms of voltage at the point Z = 180 cm for the following voltages at the transmission line input: a) V = 8 kV, b) 6 kV, c) 4 kV. Time marker interval is 50 nsec.

uniform or weakly nonuniform magnetization is known <sup>[9]</sup> to be unstable with respect to spin waves, and therefore the static variation of  $\Phi(J)$  can be considered to be a discontinuous function at  $J = -J_0$ .

If I, the current amplitude at the transmission line input, is less than  $|J_0|$ , then, since L(J) increases with increasing J in the region  $0 < J < |J_0|$ , the wave front will be lengthened and the rate of change of current and voltage in the tail of the wave will increase (Fig. 5c), so that a shock wave is eventually formed in this region.

In the case when  $I > |J_0|$ , the discontinuities are formed both in the front (in the region where  $J \ge |J_0|$ ) and in the tail (in the region where  $J \le |J_0|$ ) of the wave front<sup>5</sup>), whereas the remaining parts of the profile will be lengthened (Fig. 4). With the passage of time these discontinuities reach a maximum value. Here, according to the results of Belyantsev and Bogatyrev, <sup>[3]</sup>



it follows from the shock-wave stability conditions that the current jump at the discontinuity formed in the shock-wave front will occur from  $J'_1$  to  $J'_2$ (Fig. 6) and will be equal to the current amplitude in the wave I (t, Z), and the current jump formed in the tail of the shock wave will be from  $J''_1$  to the minimum current value in the wave (the values of  $J'_1$  and  $J''_1$  are determined by the method described in the work cited above). After this the jumps  $J'_2 - J'_1$  and  $U'_2 - U'_1$  begin to decrease because of energy dissipation in the region of the shock wave (Fig. 4).

Because of the limited length of the transmission line and reflections from the end, it has not been possible to trace the decrease in amplitude of the wave during further propagation. Some idea of this process can be obtained from the variation of the profile change, on propagation through a transmission line section of fixed length, with the amplitude of the wave at the input to the line (Fig. 5).

# 2. Formation and propagation of shock waves in unmagnetized ferrite

To investigate the process of shock-wave formation in a ferrite with no constant magnetic field, the first section of the coaxial transmission line, 90 cm long, was placed in a solenoid consisting of four identical sections each 30 cm long. The longitudinal magnetic field produced by the current in the solenoid was so great that the distortion of the wave profile in propagation through the first section was inappreciable. The value of this constant field was chosen so that the wave reflected from the boundary between the transmission line sections with ferrite magnetized by the constant field and the sections with no constant field was not reflected a second time from the input end of the transmission line.

In spite of the fact that the steepness of the

<sup>&</sup>lt;sup>5)</sup>For a sufficiently large current amplitude in the wave, the distinguishing feature of the case when  $\Phi(J)$  is a nonsingle-valued (or discontinuous) function lies primarily only in the fact that the shock waves both in the front and in the tail are formed, not as the result of the evolution of simple waves, but directly at the input to the transmission line (see Sec. 2) when, on passage of the current through a value equal to  $|J_0|$ , an irreversible change of magnetization begins, owing to the increase in amplitude of spin waves.



FIG. 7. From left to right: oscillograms of voltage at the point Z = 90 cm with successive disconnection of the four sections of the solenoid, starting with the last section; the last oscillogram is the voltage at the point Z = 180 cm with the solenoid disconnected.

initial part of the wave front increases with propagation of the wave in a ferrite not magnetized by a constant field (Fig. 7), the wave is essentially different from a simple wave (2.8) in this region. This can be seen from the fact that with successive disconnection of the four sections of the solenoid, starting with the last, points with the same value of voltage in the profile of a simple wave (2.8) should arrive at the output end of the first section of the transmission line at a time which should increase by the same amount, which is not actually observed (Fig. 7).

The wave propagation process illustrated in Fig. 7 agrees at least qualitatively with the shockwave formation process discussed by Ostrovskiĭ, <sup>[10]</sup> who assumed that the variation of the average azimuthal component of magnetization is due to incoherent rotation processes and is described by the equation<sup>6</sup>:

$$\frac{\partial M_{\varphi}}{\partial t} = \frac{\alpha \gamma}{(1+\alpha^2)M} \left(M^2 - M_{\varphi}^2\right) H_{\varphi}. \qquad (2.13)$$

It follows from the results of Ostrovskiĭ that with the passage of a certain time there is formed in the initially slow wave front, as the result of dissipation, a region of comparatively rapid change of field, a shock wave close in structure to a stationary wave, while the remaining parts of the wave are propagated without distortion of the profile. The velocity of propagation of the shock wave is determined by the boundary conditions (2.10), in which we must set  $J_1 = 0$ ,  $U_1 = 0$ , and

$$\Phi(J_2) - \Phi(0) = \frac{2J_2}{c^2} \ln f \left[ 1 + \frac{4\pi M (1 - m_0)}{\overline{H}_2} \frac{2(f - 1)}{(f + 1) \ln f} \right]. \quad (2.14)$$

Here  $m_0 = M_0/M$  is the ratio of the azimuthal component of magnetization before the wave front

to the saturation magnetization.

Comparison of the experimentally measured shock-wave velocities with the theoretical calculations carried out using formulas (2.10) and (2.14) shows that  $m_0 = 0.7$  (Fig. 3).

## 3. STUDY OF THE STRUCTURE OF SHOCK-WAVE FRONTS

In the coaxial line with F-1000 ferrite, the field in the region of the shock-wave front changed monotonically in all cases. This change of field can be roughly characterized by the length of the shock-wave front  $\tau_d$ . The variation of  $\tau_d$  with the amplitude of the shock wave for H<sub>0</sub> = 0 and 25 Oe (the dc component of the current J<sub>0</sub> = 0 in both cases) is shown in Fig. 8. For H = 25 Oe, waves were excited at the coaxial line input with a front duration  $\tau_f$  both shorter and longer than the front duration of a stationary wave  $\tau_d$ . The agreement of the measurements of  $\tau_d$  in the two cases (Fig. 8) indicates that the wave at the point Z = 180 cm, where the measurements were made, was close in structure to a stationary shock wave.



FIG. 8. Curve 1 – Dependence of  $\tau_d^{-1}$  on  $\overline{H}$  for  $H_0 = 0$ ,  $\alpha = 1$ ,  $m_0 = 0.7$ ; curve 2 – dependence of  $\delta_1/4$  on  $\overline{H}$  for H = 25 Oe,  $\alpha = 1$ ,  $4\pi M = 3200$  G. Experimental measurements of  $\tau_d^{-1}$ : O –  $\tau_f > \tau_d$ ,  $\times - \tau_f < \tau_d$  for  $H_0 = 25$  Oe; • – for  $H_0 = 0$ .

The fact that the length of the shock-wave front in ferrite magnetized by a constant magnetic field  $H_0$  is considerably less than for  $H_0 = 0$  (Fig. 8) indicates that there is a different mechanism of the change of the average magnetization in these two cases.

If we assume that for  $H_0 = 0$  the change in average magnetization occurs as the result of incoherent rotation processes and is described by equation (2.13), and if in addition we neglect the nonuniformity of the azimuthal component of the magnetic field in the wave, the shock-wave front

<sup>&</sup>lt;sup>6)</sup>The static dependence of  $\Phi(J)$  in this case is a discontinuous function for J = 0.

duration, as we have recently shown,<sup>[11]</sup> is determined by the expression:

$$\tau_{\rm d}^{-1} = 2\gamma \alpha \overline{H} / (1 + \alpha^2) f(m_0), \qquad (3.1)$$

where  $f(m_0)$  is a certain monotonic function which decreases from 15.5 at  $m_0 = -1$  to 4.5 at  $m_0 = 1$ . The linear dependence predicted by formula (3.1) for  $\tau_d^{-1}$  as a function of  $\overline{H}$  is well confirmed by the experimental data (Fig. 8). Measurement of the slope of this straight line allows us to determine that the quantity  $\alpha$  which characterizes the dissipation associated with a change in magnetization is roughly equal to unity (we recall that the quantity  $m_0$  is defined in the previous section).

In the case where the shock wave is propagated in the ferrite magnetized by a constant magnetic field  $H_0$ , with a coherent rotation of the magnetization there appears a demagnetizing field perpendicular to the conductors of the coaxial line. This qualitatively explains the substantially shorter duration of the shock-wave front in this case.

We have not been able to find an analytic expression determing the structure of the shockwave front for  $H_0 \neq 0$  and  $\alpha \approx 1$ , even if we neglect the nonuniformity of the field over the cross section of the transmission line.<sup>[12]</sup> It is easy to evaluate the quantity  $\tau_d$  if we use the fact that, as one of us has shown,<sup>[13]</sup> before the shockwave front and after it, where the deviations of the



FIG. 9. Oscillograms of the voltage at the output of the helical coaxial transmission line with unmagnetized ferrite for the following amplitudes: a) 3 kV, b) 4 kV, c) 6 kV. Time marker interval is 5 nsec.

field components from the corresponding constant values are so small that their variation can be described by linear equations, waves are excited whose frequency is determined by the equations

$$\beta_{1,2}(\omega) / \omega = v_{\mathrm{d}}^{-1} \tag{3.2}$$

and by the requirement that the deviations decrease with increasing distance from the shockwave front. In (3.2),  $\beta_{1,2}(\omega)$  is the propagation constant of the principal wave (i.e., the wave critical frequency is zero), obtained for values of the field components and the magnetization which are constant in time and which correspond to the region before the shock-wave front (index 1) and after it (index 2). The least roots  $\omega_1$  and  $\omega_2$  of (3.2) which satisfy the above requirement also determine the order of magnitude of  $\tau_{d}$ .<sup>[13]</sup> The dispersion dependence of  $\beta_1(\omega)$  in the region before the shock-wave front is easy to obtain in the quasistatic approximation to the field structure of the wave<sup>7</sup>:

$$\beta_{1}(\omega) = \frac{\omega}{c} \sqrt{\varepsilon_{\text{eff}} \mu_{\perp 1}}. \qquad (3.3)$$

Here  $\mu_{|1}$  is the effective magnetic susceptibility:

$$\mu_{\perp 1} = \frac{\omega_B^2 - (1 + \alpha^2) \,\omega^2 - 2i\alpha\omega_B\omega}{\omega_H \omega_B - (1 + \alpha^2) \,\omega^2 - i\alpha(\omega_H + \omega_B) \,\omega} \tag{3.4}$$

where  $\omega_{\rm H}$  =  $\gamma {\rm H}_0$  and  $\omega_{\rm B}$  =  $\gamma ({\rm H}_0 + 4\pi {\rm M})$ . The dispersion dependence of  $\beta_2(\omega)$  after the shockwave front is determined, even in the quasistatic approximation, by rather unwieldy expressions, since the magnetic field and magnetization components which are constant in time are nonuniform over the cross section in this region. Evaluations made without taking this nonuniformity into account show that for  $\alpha \approx 1$ , both  $\omega_2$  and  $\omega_1$  are pure imaginary numbers whose modulus is approximately equal to the rise time increment  $\delta_1 = |\omega_1|$  of the field components before the shock-wave front. Consequently the shock-wave front duration should be equal to several units of  $\delta_1^{-1}$ . Comparison of the rise time increment  $\delta_1$ calculated for  $\alpha = 1$  from formulas (3.2)-(3.4) with the experimental results (Fig. 8) shows that in the case discussed  $\tau_d = 4\delta_1^{-1}$ .

To illustrate more graphically the dependence of the structure of a shock-wave front on the dispersion of the waves we carried out in the linear

<sup>&</sup>lt;sup>7</sup>)The quasistatic approximation to the field in the wave is valid if the inequalities  $|\kappa_{1,2}^2|$  (b - a)<sup>2</sup> << 1 are fulfilled, where  $\kappa_{1,2}$  are the transverse wave numbers determined from the formulas given by Gurevich. <sup>[14]</sup> Under the conditions existing in the experiment described above, these inequalities are clearly fulfilled.

approximation a study of the structure of the shock-wave fronts propagated in a transmission line whose inner conductor was a cylindrical tube cut along a generatrix, inside which was ferrite, and whose outer conductor was a helix with a pitch of 3 mm. In a coaxial transmission line without ferrite and with anisotropically conducting walls, the dispersion of the principal wave is already very important at relatively low frequencies.<sup>[15]</sup> In this case it turns out that the group velocity is greater than the phase velocity. In the region before the shock-wave front the average magnetization of the ferrite can be taken as zero. Therefore resonance effects in the change of the average magnetization due to incoherent rotation processes are absent, as a result of which the presence of ferrite in the transmission line does not change the relation between the group velocity and phase velocity. Consequently, according to Freidman.<sup>[13]</sup> before the shock-wave front, waves should be excited which are attenuated with increasing distance from the front. In this case, the ratio of the damping constant to the frequency of the excited waves should decrease with increasing frequency.<sup>[16]</sup> These qualitative conclusions are confirmed by our experimental results (Fig. 9).

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