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THEORY OF ELECTRON CAPTURE IN THE BETATRON

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Azimuthal repulsion of injected electrons is considered. The effect of the repulsion in short injection pulses is noted.

THE idea that the capture of electrons in betatrons and synchrotrons is due to the Coulomb interaction[1-4] has recently been rather dependably confirmed by experiment (see the work of Gonella[5] and references given therein). Rodimov[1] has discussed the effect of Coulomb interaction between electron beams which have completed different numbers of revolutions after injection. Matveev[2,3] has shown that an important role in the capture is played by the Coulomb interaction between electrons in the same beam. Kovrizhnykh and Lebedev[4] have given a method for calculating the fraction of captured electrons by introducing phenomenologically the mean lifetime of the electrons in the accelerator chamber. In later papers Gonella,[5] Samoĭlov,[6] Seidl[7] and others have elaborated the formulation of the capture problem in various directions.

In all of the studies carried out up to the present time, the effect of the Coulomb interaction has been considered only on the radial and axial motion of the electrons. No attention has been given to the effect of the Coulomb interaction on the azimuthal motion. In the present note we show the importance of the azimuthal repulsion of the electrons in short injection pulses and indicate the effect of this repulsion.

It is well known that in a rather long right cylinder, uniformly charged throughout its volume, the electric field at the center of the base of the cylinder is approximately equal in magnitude to the field at the side surfaces. With a uniform increase in the cylinder dimensions, for a constant total charge, the field at the surface falls off inversely as the square of the linear dimensions. For an order-of-magnitude estimate we can take as an initial condition an electron beam in an accelerator chamber with an initial density ρ_0, considering it to have the shape of a cylinder, and discuss its change in length under the influence of the Coulomb repulsion of the electrons.

Usually the electron density at injection is much greater than the equilibrium density ρ_e determined from the formula

$$\rho_e = mv^2 / 4\pi e R^2, \tag{1}$$

where m, e, and v, are respectively the mass, charge, and velocity of the electron, and R is the radius of the equilibrium orbit. Under these conditions the strength of the Coulomb repulsion is much greater than the magnetic focusing forces and the latter can be neglected in the initial period of repulsion. Designating by y the length of a portion of the beam from its origin to points which were initially at a distance from the origin equal to the beam radius $r_0 = y_0$, for an order-of-magnitude determination we can write the following equation:

$$\frac{d^2y}{dt^2} = \frac{1}{2}\frac{1}{\tau^2}\frac{y_0^3}{y^2}, \quad \tau = \frac{T}{2\pi\sqrt{\alpha}}, \quad \alpha = \frac{\rho_0}{\rho_e} \gg 1, \tag{2}$$

where T is the time of a single revolution at injection. The solution of this equation for the appropriate initial conditions is given in implicit form by the formula

$$\sqrt{x(x-1)} + \frac{1}{2}\ln\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}} = \frac{t}{\tau}, \quad x = \frac{y}{y_0} \tag{3}$$

where t is the time after the beginning of the repulsion.

From Eq. (3) it follows that, for $\alpha \gg 1$, in a time less than the period of one revolution a noticeable lengthening of the beam in the azimuthal direction occurs which generally speaking must be taken into account in the theory of injection, since it is accompanied by a change of electron density in the beam, which leads to a change in the dynamics of the electrons in their radial and axial motions. The relative importance of the azimuthal repulsion becomes more important for shorter injection pulses.

Let us consider the dependence of the number of captured electrons N on the duration of the injection pulses t_i for a fixed injection current density. It is clear that as t_i increases from 0, N begins to

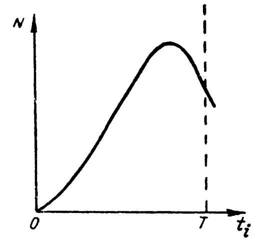

FIG. 1

increase from 0. However, for a very high injection current density, as the result of azimuthal lengthening of the beam, the front part of the beam will overtake its "tail" and the entire chamber will be filled with electrons. Capture will occur as the result of processes previously described by Matveev,[3] and further increase of the pulse length will lead only to a decrease in the number of electrons captured into the acceleration regime. Therefore the variation of N with t_i has the form shown in the figure. The occurrence of the maximum capture at an injection pulse length less than one period of revolution is due entirely to the azimuthal repulsion and cannot be understood in terms of the radial and axial repulsions. With a reduction in the injection current density, this maximum is displaced to a time greater than the period of one revolution.

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