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EFFECTS OF TRANSVERSE ELECTROMAGNETIC WAVE DECAY IN A PLASMA

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Equations are derived which describe three-plasmon induced scattering and transverse-wave decay in a plasma. Transformation of intense transverse waves into longitudinal plasma waves with nonrelativistic or relativistic phase velocities is considered. The transformation of the spectrum of transverse waves traveling through a turbulent plasma is discussed.

1. INTRODUCTION

IN a plasma, which is a system of weakly interacting particles, nonlinear wave-interaction effects can be considered by means of perturbation theory. Recently different approaches were used in several papers[1-3] to consider the interaction between various types of noise (waves with random phases), with analysis of nonlinear effects that are quadratic in the number of waves N. Attention was called to the important role of decay processes and wave-coalescence processes; it was noted in particular[4] that decay processes can constitute one of the dissipation mechanisms of collision-free shock waves[5].

The method proposed in[6] for the analysis of nonlinear effects is based on the fact that the wave transformation processes in a plasma, which is a system of weakly interacting particles, can be considered by using the quantum-electrodynamics diagram technique (see also[7]). The nonlinear effects which are produced when the number of waves N is large are determined by induced processes, wherein the external "photon" line of the emitted quantum corresponds to a factor N + 1, while that of the absorbed quantum corresponds to N. In the present paper we investigate transverse and longitudinal wave interactions that can be described by diagrams with three external photon lines. We shall consider the case when the frequencies of the transverse waves exceed greatly the frequencies of the longitudinal waves and the plasma is isotropic.

We know that the need for considering three-plasmon processes is connected with the fact that two-plasmon effects are possible for transverse high-frequency waves in the direct neighborhood of the optical band only in the presence of a sufficiently large number of superthermal particles which, generally speaking, have relativistic velocities[8,9]. Three-plasmon processes are possible for thermal plasma particles. The induced processes corresponding to a three-plasmon interaction result in the general case in effects that are cubic in the wave number, $\sim N^3$. However, in the case when the plasma particles do not acquire any momentum, the resultant effect is proportional to N^2. Indeed, for example, for the radiation of a longitudinal wave following collision of two transverse waves, the induced emission increases N^l by an amount proportional to $(N^l + 1)N^tN^t$, but absorption results in a decrease by $N^l N^tN^t$. The net effect is proportional to N^tN^t. It is therefore necessary to distinguish between three-plasmon decay processes, described by conservation laws[1)]

$$\omega_2 = \omega_2' + \omega_1; \qquad \mathbf{k}_2 = \mathbf{k}_2' + \mathbf{k}_1 \tag{1.1}$$

and contributing only to nonlinear effects proportional to N^2, and three-plasmon induced-scattering processes, described by conservation laws

$$\omega_1 + \omega_2' - \omega_2 = (\mathbf{k}_1 + \mathbf{k}_2' - \mathbf{k}_2)\mathbf{v}, \tag{1.2}$$

and contributing to the N^3 terms. The connection between the probabilities of these processes will be established later on.

The induced scattering processes (1.2) can occur also for thermal plasma particles, if the frequency condition for the decay is satisfied approximately so that the velocity v corresponding to (1.2) is of the same order as or smaller than the averaged thermal velocity of the plasma particles. The essential difference between (1.2) and (1.1) lies in the possibility of inexact satisfaction of the decay conditions. The first to consider the decay process (1.1) for optical waves was Kompaneets[10], but he did not consider induced decays.

Unlike Akhiezer et al.[11], we shall derive self-

[1)]The momenta and the frequencies of the transverse waves will henceforth be denoted by $\mathbf{k}_2$, ω_2, $\mathbf{k}'_2$, ω'_2; those of the longitudinal waves by $\mathbf{k}_1$, ω_1; $\omega_2 \approx |\mathbf{k}_2|$, $\omega'_2 \approx |\mathbf{k}'_2|$.

consistent equations for the decay processes. We consider not the coalescence of two waves into a transverse one that causes spontaneous radiation of the turbulent plasma at frequencies $\sim 2\omega_0$ [11], but the interaction between the plasma and radiation at high frequencies $\omega \gg \omega_0$. In addition, a connection is established between the decays and the three-plasmon scattering. In contrast to the widespread opinion that for wave decay to be possible it is necessary to satisfy the synchronization conditions (1.11) rigorously, we show below that if the decay conditions are not rigorously satisfied three-plasmon induced scattering can lead to effects of the same order as decays.

For transverse waves of high frequencies ω_2 and $\omega_2' \gg \omega_1$, we confine ourselves to processes in which two external photon lines are transverse and the third longitudinal. Only such processes, together with the case when all three photon lines are transverse, can satisfy the conservation laws (1.1) and (1.2).

2. EQUATIONS OF THREE-PLASMON WAVE INTERACTION PROCESSES

1. To obtain the equations of three-plasmon interactions we make use of a diagram technique, introducing for the three-plasmon induced scattering process (Fig. 1a) and the decay process (Fig. 1b) the respective probabilities $w(k_1, k_2, k_2')$ and $w(k_1, k_2, k_2')$, which describe the absorption of transverse waves with momentum $k_2 = \{\mathbf{k}_2, \omega_2\}$, $k_2' = \{\mathbf{k}_2', \omega_2'\}$ and the emission of a longitudinal wave with momentum $k_1 = \{\mathbf{k}_1, \omega_1\}$. The sought equation describes all the possible transitions, including processes in which the absorption is replaced by emission either for k_2 or for k_2', or else for k_2 and k_2', and the processes inverse to these. In the limit of small momentum transfers $\Delta k \ll p$, the probabilities of these processes are obtained from those written out above by making the following substitutions of momenta: either $k_2 \to -k_2$, or $k_2' \to -k_2'$, or else $k_2 \to -k_2$ and $k_2' \to -k_2'$.

We write out the resultant equations for the waves [2)]

$$\frac{\partial N_{\mathbf{k}_1}{}^l}{\partial t} = \sum_\alpha \sum_{\varepsilon,\varepsilon'=\pm 1} \int d\mathbf{k}_2\, d\mathbf{k}_2'\, d\mathbf{p}_\alpha N_{\mathbf{k}_1}{}^l N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2'} w^{ltt}_{\mathbf{p}_\alpha}(k_1, \varepsilon k_2, \varepsilon' k_2')$$
$$\times (\mathbf{k}_1 - \varepsilon \mathbf{k}_2 - \varepsilon' \mathbf{k}_2') \frac{\partial f_{\mathbf{p}_\alpha}}{\partial \mathbf{p}_\alpha} + \sum_{\varepsilon,\varepsilon'=\pm 1} \int d\mathbf{k}_2\, d\mathbf{k}_2'$$
$$\times (N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2'} - \varepsilon' N_{\mathbf{k}_1}{}^l N^t_{\mathbf{k}_2} - \varepsilon N_{\mathbf{k}_1}{}^l N^t_{\mathbf{k}_2'}) w^{ltt}_{p_\alpha}(k_1, \varepsilon k_2, \varepsilon' k_2'); \quad (2.1)$$

[2)]Only three-plasmon scattering, described by a diffusion equation of the type given in [8], influences the kinetics of the particles.

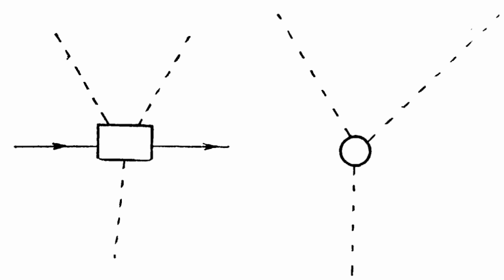

FIG. 1

$$\frac{\partial N_{\mathbf{k}_2}{}^t}{\partial t} = \sum_{\varepsilon,\varepsilon'=\pm 1} \int d\mathbf{k}_1\, d\mathbf{k}_2' \Big\{ (N_{\mathbf{k}_1}{}^l N^t_{\mathbf{k}_2'} + \varepsilon\varepsilon' N_{\mathbf{k}_2}{}^t N_{\mathbf{k}_1}{}^l - \varepsilon N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2'})$$
$$\times w^{ltt}(k_1, \varepsilon k_2, \varepsilon' k_2') - \sum_\alpha \int d\mathbf{p}_\alpha (\mathbf{k}_1 - \varepsilon \mathbf{k}_2 - \varepsilon' \mathbf{k}_2')$$
$$\times \frac{\partial f_{\mathbf{p}_\alpha}}{\partial \mathbf{p}_\alpha} w^{ltt}_{\mathbf{p}_\alpha}(k_1, \varepsilon k_2, \varepsilon' k_2') N_{\mathbf{k}_1}{}^l N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2'} \Big\} \quad (2.2)$$

and an analogous equation for $N^t_{k_2'}$, obtained from (2.2) by replacing k_2 with k_2'. The energy conservation law derived from (2.1) and (2.2) neglecting three-plasmon scattering, i.e., for the decay processes only, is of the form

$$\frac{d}{dt}\left[\int \frac{\omega_1{}^l N_{\mathbf{k}_1}{}^l\, d\mathbf{k}_1}{(2\pi)^3} + \int \frac{\omega_2{}^t N_{\mathbf{k}_2}{}^t\, d\mathbf{k}_2}{(2\pi)^3} + \int \frac{\omega_2'{}^t N^t_{\mathbf{k}_2'}\, d\mathbf{k}_2'}{(2\pi)^3}\right] = 0. \quad (2.3)$$

The equations written in the form (2.6)–(2.8) are convenient for the case of interaction of two transverse waves with essentially different frequencies. To investigate nonlinear effects in one wave, it is more convenient to use equations written out for the total distribution function of the transverse waves, $N^t_{k_2} + N^t_{k_2'}$, which can be readily obtained from (2.1) and (2.2).

2. Let us find the probability of three plasmon scattering. Following the previously employed calculation procedure [12], we indicate only the diagrams of Fig. 2, which describe the corresponding processes [3)]. The diagram technique of [12] enables us to write down with the aid of the diagrams of Fig. 2 the Fourier component of the charge density [or $\Lambda(k, k', k'')$ in (2.4)]:

$$\rho(k) = \int \Lambda(k, k', k'') \mathbf{E}_1{}^t(k') \mathbf{E}_2{}^t(k'')\, dk'\, dk''\, \delta((\mathbf{k} + \mathbf{k}' + \mathbf{k}'')\mathbf{v}$$
$$- \omega - \omega' - \omega''), \quad (2.4)$$

which determines the intensity Q^l of the longitudinal

[3)]The interference of these diagrams with the simplest diagram of Cerenkov radiation corresponds to corrections to the Cerenkov radiation [12], whereas the squares of the amplitudes of the diagrams of Fig. 2 describe three-plasmon scattering.

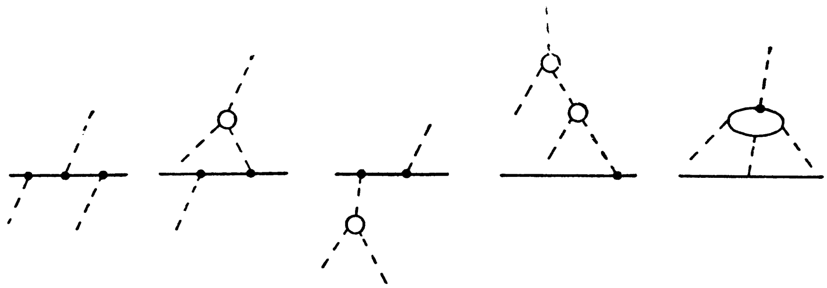

FIG. 2

waves generated when a trial charge with velocity v interacts with two transverse waves $\mathbf{E}_1^t(k_1)$ and $\mathbf{E}_2^t(k_2)$:

$$Q^l = \lim_{T\to\infty} \frac{(2\pi)^6}{T} \int \frac{dk d\omega\,|\,\omega\,|}{k^2}\,|\,\rho(\mathbf{k})\,|^2\,\delta(\varepsilon^l). \quad (2.5)$$

Leaving out cumbersome derivations, we present the result of the calculation carried in the approximation of interest to us $|\omega_2|$, $|\omega_2'| \gg \omega_1 \approx \omega_0$; $\omega_2' - \omega_2' - \omega_0 \ll \omega_2$:

$$w_{\mathrm{p}_\alpha}^{ltt}(k_1, k_2, k_2') = e^6 k_1^2 \left(1 + \frac{(\mathbf{k}_2\mathbf{k}_2')^2}{k_2^2 k_2'^2}\right) \delta(\omega_2 + \omega_2' - \omega_1 - (\mathbf{k}_2 + \mathbf{k}_2 - \mathbf{k}_1)\mathbf{v}_\alpha) \left\{\pi^2 m_e^4 \omega_0^4 \frac{\partial\varepsilon^l(\omega_1 \mathbf{k}_1)}{\partial\omega_1} \frac{\partial}{\partial\omega_2} \omega_2^2 \varepsilon^t(\omega_2, \mathbf{k}_2) \times \frac{\partial}{\partial\omega_2'} \omega_2'^2 \varepsilon^t(\omega_2', \mathbf{k}_2') \right\}^{-1}. \quad (2.6)$$

In a non-isothermal plasma scattering can be accompanied by transformation of two transverse waves into plasma sound. The contribution of the plasma ions to ρ is in this case insignificant. The result of the calculation, in the approximation

$$|\omega_2'|, |\omega_2| \gg \omega_s; \quad |\omega_2 - \omega_2' - \omega_s| \ll |\omega_2|;$$
$$\omega_s \ll \omega_{0i}$$

takes the form

$$w_{\mathrm{p}_\alpha}^{stt}(k_s, k_2, k_2') = e^6 k_s^2 \left(1 + \frac{(\mathbf{k}_2\mathbf{k}_2')^2}{k_2^2 k_2'^2}\right) \delta(\omega_2 + \omega_2' - \omega_s - (\mathbf{k}_2 + \mathbf{k}_2' - \mathbf{k}_s)\mathbf{v}_\alpha) \left\{\pi^2 m_e^2 m_i^2 \omega_s^4 \frac{\partial\varepsilon^l}{\partial\omega_s} \left[\frac{\partial}{\partial\omega_2} \omega_2^2 \varepsilon^t(\omega_2, \mathbf{k}_2)\right] \times \left[\frac{\partial}{\partial\omega_2'} \omega_2'^2 \varepsilon^t(\omega_2', \mathbf{k}_2')\right]\right\}^{-1} \quad (2.7)$$

where

$$\omega_s = k_s v_s; \quad v_s = v_{Te}(m_e / m_i)^{1/2}.$$

3. We now find the probability of the decay processes corresponding to coherent emission induced by two transverse waves, of longitudinal waves by plasma electrons. Let $\mathbf{r}_i$ be the coordinate of some trial electron inside a unit volume occupied by the plasma[4)]. For the i-th charge, $\rho_i(k)$ has the form (2.4) with a factor $\exp\{i(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \cdot \mathbf{r}_i\}$ preceding $\Lambda(k, k', k'')$. The total of radiation is determined by the quantity

$$|\,\rho(k)\,|^2 = \sum \rho_i(k)\rho_j^*(k),$$

with the sum taken over all the electrons inside the unit volume. The result is a factor $\Sigma \exp\{i(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \cdot (\mathbf{r}_i - \mathbf{r}_j)\}$. The terms with $i = j$ yield incoherent radiation, the intensity of which is equal to the sum of the intensities of radiation of the individual charges, whereas the terms with $i \neq j$ correspond to coherent radiation, which is noticeable only if $\mathbf{k} + \mathbf{k}' + \mathbf{k}'' = 0$. Thus, coherent radiation differs from incoherent radiation by a factor $(2\pi)^3 n\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'')$, where n—number of particles per unit volume. From this we obtain, for example, for the probability of the decay of transverse waves into longitudinal waves

$$w^{ltt}(k_1, k_2, k_2') = \frac{e^2\omega_0^4 \mathbf{k}_1^2 (1 + (\mathbf{k}_2\mathbf{k}_2')^2 / k_2^2 k_2'^2)}{8\pi m_e^2 \omega_1^4\,|\,\mathbf{k}_2\,|\,|\,\mathbf{k}_2'\,|\,\partial\varepsilon^l/\partial\omega_1} \times \delta(\mathbf{k}_2 + \mathbf{k}_2' - \mathbf{k}_1)\,\delta(\omega_2 + \omega_2' - \omega_1). \quad (2.8)$$

By the same token we have established a connection between the decay probabilities and the three-plasmon scattering.

4. Attention is called to the structure of the nonlinear current produced in the plasma under the influence of two transverse waves. In the approximation $k_\lambda v_{Te} \ll \omega_\lambda$ ($\lambda = 1, 2, 2'$) we have

$$\mathbf{j}_{\mathrm{nonlin}}(k_1) = -\frac{e^2\omega_0^2 \mathbf{k}_1}{8\pi m_e \omega_1 \omega_2 \omega_2'} \int d\mathbf{k}_2\, d\mathbf{k}_2' (\mathbf{E}^t(k_2)\mathbf{E}^t(k_2')) \times \delta(\mathbf{k}_2 + \mathbf{k}_2' - \mathbf{k}_1)\,\delta(\omega_2 + \omega_2' - \omega_1). \quad (2.9)$$

From (2.9) we see that the current $\mathbf{j}(k_1)$ is directed along $\mathbf{k}_1$ and cannot excite transverse waves. One might think that excitation of transverse waves could occur in the next higher order in v_{Te}^2/v_{ph}^2,

4)Inasmuch as we are interested in the radiation of an assembly of electrons, we cannot assume that $\mathbf{r}_i = 0$.

i.e., it would be v_{Te}^2 times smaller when $v_{ph} = \omega/k \sim 1$. However, the conservation laws for the process ttt, like for the process lll, are not satisfied in the plasma by virtue of the dispersive properties of the waves. On the other hand, the llt process is impossible only in the limit $\omega_2 \gg \omega_0$ considered here.

3. ANALYSIS OF LIMITING CASES AND SOME NUMERICAL ESTIMATES

1. For decay processes, the equations become much simpler if $|k_1| \ll |k_2|$ and $|k_2'|$. For transverse waves which are close to the optical band, this condition is not stringent and yields $v_{ph}^l = \omega_0/k_1 \gg \omega_0/\omega_2 \sim 10^{-4}$—$10^{-3}$ for a plasma density $\sim 10^{13}$ cm^{-3}. Equations (2.6), which describe the generation of longitudinal waves in the presence of two transverse waves, take the form

$$\frac{\partial N_{\mathbf{k}_1}{}^l}{\partial t} = \frac{e^2\omega_0\mathbf{k}_1{}^2}{2\pi m_e{}^2}\int\frac{1}{\mathbf{k}_2{}^2}N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2-\mathbf{k}_1}\,d\mathbf{k}_2\delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2}-\omega_0(\mathbf{k}_1)\right)$$

$$+N_{\mathbf{k}_1}{}^l\left\{\int\left[\frac{e^2\omega_0\mathbf{k}_1{}^2}{4\pi\mathbf{k}_2{}^2 m_e{}^2}\left(\mathbf{k}_2\frac{dN_{\mathbf{k}_2}{}^t}{d\mathbf{k}_2}\right)\delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2}-\omega_0(\mathbf{k}_1)\right)\right]d\mathbf{k}_2\right.$$

$$\left.+\,k_2\rightleftarrows k_2'\right\}. \qquad (3.1)$$

If N^t can be assumed as specified, then the solution of (3.1) can have the form

$$N_{\mathbf{k}_1}{}^l(t) = N_{\mathbf{k}_1}{}^l(0)\exp(\gamma_{\mathbf{k}_1}{}^l t) + N_{\mathbf{k}_1}^{l(0)}[\exp(\gamma_{\mathbf{k}_1}{}^l t)-1], \qquad (3.2)$$

$$N_{\mathbf{k}_1}^{l(0)} = \frac{1}{\gamma_{\mathbf{k}_1}{}^l}\frac{e^2\omega_0\mathbf{k}_1{}^2}{2\pi m_e{}^2}\int\frac{N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2-\mathbf{k}_1}}{k_2{}^2}\delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2}-\omega_0\right)d\mathbf{k}_2. \qquad (3.3)$$

To estimate $N_{k_1}^{(0)}$ and $\gamma_{k_1}^l$ we shall assume that N_k^t corresponds to almost monochromatic radiation in a narrow angle interval $\Delta\theta$. Under these conditions we have

$$N_{\mathbf{k}_1}^{l(0)} \approx \frac{2k_2}{k_1}\Delta\theta\frac{1}{1-v_{ph}^2}N_{\mathbf{k}_2}{}^t, \quad v_{ph} = \frac{\omega_0}{k_1};$$

$$\gamma_{k_1}{}^l = \frac{e^2\omega_0 k_1{}^2}{4m_e{}^2}\int_0^\infty\frac{dk_2}{k_2}\left(1-\frac{\omega_0{}^2}{k_1{}^2}\right)\frac{dN^t_{k_2,x}}{dx}\Bigg|_{x=\omega_0/k},$$

$$x = (\mathbf{k}_1\mathbf{k}_2)/k_1k_2, \qquad N^t_{k_2,x} = \frac{1}{2\pi}\int_0^{2\pi}N^t_{k_2,x,\varphi}\,d\varphi. \qquad (3.4)$$

We note that for directed radiation $dN^t/dx > 0$, with

$$\frac{\gamma^l}{\omega_0} \approx \frac{1}{(\Delta\theta)^2}\left(\frac{k_1}{k_2}\right)^2\frac{e^2}{mc^2}\lambda^{t\,2}\frac{\langle(E^t)^2\rangle}{mc^2}; \qquad \lambda_t = \frac{c}{\omega_2} \qquad (3.5)$$

If

$$[\langle(E^t)^2\rangle]^{1/2} \sim 10^3 \text{ cgs esu}; \quad \Delta\theta \sim 10^{-5}; \quad k_1/k_2 \sim 10^{-2};$$
$$\omega_2 \sim 10^{15}\,\text{sec}^{-1}; \qquad \omega_0 \sim 3\cdot 10^{11}\,\text{sec}^{-1}$$

($n \sim 10^{14}$ cm^{-3}) we have $\gamma^l \sim 3\times 10^7$ sec^{-1}. We note that the effect increases with increasing wavelength.

Depending on the angle between the momenta of the two transverse waves k_2 and k_2', either nonrelativistic plasma waves are generated, for which we can expect a noticeable absorption in the plasma, or plasma waves which have relativistic phase velocities. The latter can accelerate individual charged particles that are captured by the wave.

2. Let us consider the passage of relatively weak transverse waves through a turbulent plasma, in which there is a sufficiently high level of stationary longitudinal waves. Two cases are possible here, either $\omega_0 \ll \Delta\omega$ ($\Delta\omega$—width of the spectrum of the transverse waves), or $\omega_0 \gg \Delta\omega$. In the former case the deformation of the spectrum on passing through the turbulent plasma will be the result of the large number of decay and coalescence events, in each of which the energy is acquired or lost by the transverse waves in small batches. This intuitive picture agrees also with the structure of the equations obtained from (2.2). Discarding the terms N^tN^t, we obtain the diffusion equation

$$\frac{\partial N_{\mathbf{k}_2}{}^t}{\partial t} = \frac{\partial}{\partial k_{2i}}D_{ij}\frac{\partial N_{\mathbf{k}_2}{}^t}{\partial k_{2j}}; \quad D_{ij} = \int N_{\mathbf{k}_1}{}^l w(\mathbf{k}_1,\mathbf{k}_2)\,k_{1i}k_{1j}\,d\mathbf{k}_1; \qquad (3.6)$$

$$\frac{\partial N_{\mathbf{k}_1}{}^l}{\partial t} = \gamma_{\mathbf{k}_1}{}^l N_{\mathbf{k}_1}{}^l; \quad \gamma_{\mathbf{k}_1}{}^l = \int\left(\mathbf{k}_1\frac{dN_{\mathbf{k}_2}{}^t}{d\mathbf{k}_2}\right)w(\mathbf{k}_1,\mathbf{k}_2)\,d\mathbf{k}_2;$$

$$w(\mathbf{k}_1,\mathbf{k}_2) = \frac{e^2\omega_0}{8\pi m_e{}^2}\left(\frac{k_1}{k_2}\right)^2\delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2}-\omega_0\right). \qquad (3.7)$$

From (4.6) follows the conservation of the number of quanta of transverse waves, $\int N_{k_2}dk_2$ = const.

Under these conditions, the average energy of the waves per unit volume $\sim \int|k_2|\,N_{k_2}^t\,dk_2$ is conserved only if the quanta have an isotropic distribution. In the case when $N_{k_1}^l$ increases with increasing angle (k_1k_2), the energy of the transverse waves decreases, and vice versa. The order of magnitude of the spectrum deformation is characterized by the expression

$$\left(\frac{\Delta k_2}{k_2}\right)^2 \approx \frac{tD}{k_2{}^2} \approx \frac{\pi}{8}\left(\frac{k_1}{k_2}\right)^3\frac{e^2}{mc^2}\lambda_t\frac{\langle(E^l)^2\rangle}{mc^2}ct, \qquad (3.8)$$

where

$$\langle(E^l)^2\rangle \equiv 8\pi W^l = (2\pi)^{-3}\int\omega_0 N_{\mathbf{k}_1}{}^l\,d\mathbf{k}_1.$$

If

$$[\langle(E^l)^2\rangle]^{1/2} \sim 10^2 \text{ cgs esu}; \quad k_1/k_2 \approx 1/3; \quad \lambda_t \sim 10^{-4}\text{cm};$$
$$t \sim 10^{-8}\text{ sec}$$

we have $\Delta\omega_2/\omega_2 \sim 10^{-4}$, i.e., a sufficiently large quantity. Such a change in the spectrum can be used, in principle, for plasma-noise diagnostics. We note the analogy between (3.6) and (3.7) which describe induced decays, and the quasilinear equations for plasma particles[13], which describe induced Cerenkov radiation[14].

3. We now consider effects arising when two relatively weak transverse waves pass through a turbulent plasma, with $\Delta\omega \ll \omega_0$ for each of the beams, and with the difference between the average frequencies of the transverse waves being approximately equal to ω_0. In this case the decay and coalescence effects lead only to a redistribution of intensities among the two beams of transverse waves. Neglecting terms quadratic in N^t and assuming that ω_2' is the smaller of the frequencies, i.e., $\omega_2' = \omega_2 - \omega_0$ and $k_2' = k_2 - k_1$, we obtain for $\omega_0/k_1 \ll 1$

$$\frac{\partial N_{\mathbf{k}_2}{}^t}{\partial t} = \frac{e^2\omega_0}{8\pi m_e k_2{}^2}\int d\mathbf{k}_1 k_1{}^2 \delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2} - \omega_0\right) N_{\mathbf{k}_1}{}^l\,(N^t_{\mathbf{k}_2-\mathbf{k}_1} - 2N_{\mathbf{k}_2}{}^t),$$

$$\frac{\partial N^t_{\mathbf{k}_2'}}{\partial t} = \frac{e^2\omega_0}{8\pi m_e k_2'^2}\int d\mathbf{k}_1 k_1{}^2 \delta\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_2} - \omega_0\right) N_{\mathbf{k}_1}{}^l\,(N^t_{\mathbf{k}_2'+\mathbf{k}_1} - 2N^t_{\mathbf{k}_2'}). \quad (3.9)$$

The value of $N^t_{k_2}$ is increased by the interaction of the longitudinal waves with $N^t_{k_2'}$, and decreased by the interaction with $N^t_{k_2}$ and $N^l_{k_1}$. If $N^t_{k_2}$ and $N^t_{k_2'}$ have approximately the same intensity, both $N^t_{k_2}$ and $N^t_{k_2'}$ attenuate. The reciprocal of the characteristic number of this process γ^t is of the order of

$$\frac{\gamma^t}{\omega_0} \approx \frac{\pi}{4}\left(\frac{k_1}{k_2}\right)^2 \frac{e^2}{mc^2}\lambda\!\!\!^-_0{}^2 \frac{\langle (E^l)^2\rangle}{mc^2}\frac{v_{\rm ph}{}^l}{c}, \quad (3.10)$$

where $\lambda\!\!\!^-_0 = c/\omega_0$. Thus, when

$$\frac{k_1}{k_2} \sim \frac{1}{3}; \qquad \frac{v_{\rm ph}}{c} \sim 10^{-2};$$

$$[\langle (E^l)^2\rangle]^{1/2} \sim 10^2 \text{ cgs esu}, \quad \lambda\!\!\!^-_0 \sim 10 \text{ cm}$$

we have $\gamma^t \sim 10^6\ \text{sec}^{-1}$, i.e., a sufficiently large quantity.

In the case when one of the beams of the transverse waves has an intensity much higher than the other, the lower-intensity beam increases in energy, with an increment of the order of (3.10), while the higher-intensity beam attenuates. This continues until the intensities of the beams become equal, after which both beams attenuate.

4. In Sec. 1 we considered the generation of longitudinal waves by transverse ones whose intensity is specified (for example, maintained from the outside). On the other hand, if the constancy of N^t is the result of the fact that during the initial instants of time the changes are small compared with the large initial values of N^t, then the subsequent change is determined, on the one hand, by expression (3.10), which is obtained when N^l is large, and on the other hand, by nonlinear effects of energy transfer from one beam to the other, determined by the terms in (2.2) proportional to N^tN^t. When $\omega_0/k_1 \ll 1$ and $k_1/k_2 \ll 1$ we have for the latter process

$$\frac{\partial N_{\mathbf{k}_2}{}^t}{\partial t} = -N_{\mathbf{k}_2}{}^t \int N^t_{\mathbf{k}_2'}\, d\mathbf{k}_2'\, \frac{e^2}{8\pi}\frac{\omega_0}{m^2}\frac{(\mathbf{k}_2 - \mathbf{k}_2')^2}{k_2{}^2} \times \delta(|\mathbf{k}_2'| - |\mathbf{k}_2| + \omega_0), \quad (3.11)$$

$$\frac{\partial N^t_{\mathbf{k}_2'}}{\partial t} = +N^t_{\mathbf{k}_2'} \int N_{\mathbf{k}_2}{}^t\, d\mathbf{k}_2\, \frac{e^2}{8\pi}\frac{\omega_0}{m^2}\frac{(\mathbf{k}_2' - \mathbf{k}_2)^2}{k_2'^2} \times \delta(|\mathbf{k}_2| - |\mathbf{k}_2'| - \omega_0). \quad (3.12)$$

The change in the total number of quanta is equal to zero:

$$\int N_{\mathbf{k}_2}{}^t\, d\mathbf{k}_2 + \int N^t_{\mathbf{k}_2'}\, d\mathbf{k}_2' = \text{const},$$

and the intensity of the wave of higher frequency decreases and the quanta are transformed into waves of lower frequency. The total energy of the transverse waves then decreases. The characteristic reciprocal energy-transfer time is

$$\frac{\tilde{\gamma}^t}{\omega_0} \approx \frac{\pi}{8}\left(\frac{k_1}{k_2}\right)^2 \left(\frac{\omega_2}{\Delta\omega_2}\right) \lambda\!\!\!^-_t{}^2 \frac{e^2}{mc^2}\,\frac{\langle (E^t)^2\rangle}{mc^2}, \quad (3.13)$$

which differs from γ^l by a factor $\sim (\Delta\theta)^2\omega_2/\Delta\omega_2$.

In the case of one smeared-out beam of transverse waves, such that $\Delta\omega \gg \omega_0$, the transfer from the higher frequencies to the lower ones of the type (3.11) decreases by a factor $\omega_0/\Delta\omega$. This is connected with the fact that, besides the transitions from the higher frequencies to the lower ones, transitions from lower to the higher frequencies are also allowed and only the difference in energy is subject to transfer.

5. In conclusion let us discuss briefly the effects of three-plasmon induced scattering. If ω_2, $\omega_2' \gg \omega_0$ and $\omega_2' \approx \omega_2$, then we obtain from (2.6) and (2.1) an equation that describes the changes of, say, N^l:

$$\frac{dN^l}{\partial t} = N^l\gamma_3^l;\ \gamma_3^l = \int N_{\mathbf{k}_2}{}^t N^t_{\mathbf{k}_2'} \frac{e^2\omega_0 f_{\rm p}\, d\mathbf{p}}{64\pi^4 n^2 v_{Te}{}^2}\frac{\mathbf{k}_1{}^2}{m^3 k_2{}^2}(\omega_2 - \omega_2' - \omega_0) \times \delta[\omega_2 - \omega_2' - \omega_0 - (\mathbf{k}_2 - \mathbf{k}_2' - \mathbf{k}_1)\mathbf{v}]. \quad (3.14)$$

In (3.14) f_p is the Maxwellian distribution function. If

$$|\omega_2 - \omega_2' - \omega_0| \lesssim |\mathbf{k}_2 - \mathbf{k}_2' - \mathbf{k}|\, v_{Te},$$

then γ_3^l is of the order of

$$\frac{\gamma_3{}^l}{\omega_0} \approx \frac{1}{16}\frac{e^2}{mc^2}\,\lambda_l^2\,\frac{\langle (E^t)^2\rangle}{mc^2}\,\frac{\langle (E^t)^2\rangle}{nmv_{Te}{}^2}. \quad (3.15)$$

It must be emphasized that the increments of the three-plasmon induced scattering can be quite appreciable and sometimes comparable with the values of the decay increments γ^l. It is easy to compare, finally, the obtained increments with the characteristic time of nonlinear interaction between the longitudinal waves themselves and their transformation into transverse waves[9,15,16].

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