# PRODUCTION OF PION PAIRS FROM NUCLEI BY HIGH-ENERGY NEUTRINOS

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Differential and total cross sections for the process $\nu + A \to \mu^- + \pi^+ + \pi^0 + A$ for small transferred momenta are obtained on the basis of the Coulomb and diffraction mechanisms. It is shown that the diffraction mechanism dominates for neutrino energies $E < 60$ BeV. Comparison with experiment can yield in principle information on the magnitude of the cross sections of the processes $\nu + \pi^- \to \mu^- + \pi^0$ and $\gamma + \nu \to \mu^- + \pi^+ + \pi^0$.

EXPERIMENTS performed with high-energy neutrino beams at Brookhaven[1] and CERN[2] have led to many important conclusions concerning weak interactions. In addition to confirming the existence of two sorts of neutrinos (muonic and electronic), it was also observed that in the reaction

$$\nu(\bar{\nu}) + A \to \mu^\pm + A + n\pi \qquad (1)$$

with production of a single pion ($n = 1$) the cross section is equal in order of magnitude to the cross section for the elastic process ($n = 0$). It has been established that the process in which two pions are produced is suppressed by a factor approximately 4–5.[2]

In the processes considered here, the participating particles—pions and the nucleus—are strongly interacting. Since there is no strong-interaction theory at present, a consistent calculation of processes (1) in the entire region of momenta transferred to the nucleus is impossible. We can, however, estimate these cross sections in the region of small momentum transfer. In the present note we consider the Coulomb and diffraction mechanisms for the production of pion pairs in reaction (1). The corresponding mechanisms for the production of a single pion were considered earlier[3,4] 1). A study of reactions of the type (1) in the region of small momentum transfer is of interest in that it can yield information on the cross sections of the processes $\nu + \pi \to \mu + \pi$ and $\gamma + \nu \to \mu + \pi + \pi$, which are not observed under ordinary conditions. Comparison of the experimental data with the results of calculations based on the V – A weak-interaction theory would permit an additional verification of the theory and a clarification of the internal structure of vertices of the type $(\pi\pi)(\mu\nu)$.

## 1. COULOMB MECHANISM

A formalism based on using the Weizsäcker-Williams method in its covariant form[5,6] enables us to estimate the cross section of the process (1) ($n = 2$) in the Coulomb field of the nucleus, if we know the cross section of the process

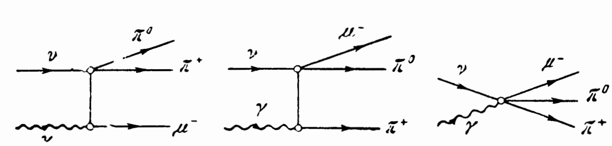
$$\nu + \gamma \to \mu^- + \pi^+ + \pi^0. \qquad (2)$$

This process is described by three Feynman diagrams (see, for example, Fig. 1), the contribution of the last diagram being determined from the gauge-invariance condition. If we take into consideration the experimental proof of the existence of two types of neutrinos[1,2], then the amplitude of the weak interaction in the vertex $(\pi^0\pi^+)(\nu_\mu\mu^-)$ is written in the form

$$fG[\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu]\left(\frac{\partial\varphi_{\pi^+}{}^*}{\partial x_\alpha}\varphi_{\pi^0} - \varphi_{\pi^+}{}^*\frac{\partial\varphi_{\pi^0}}{\partial x_\alpha}\right), \qquad (3)$$

where G—universal weak-interaction constant and f—form factor of the vertex $(\pi^0\pi^+)(\nu_\mu\mu^-)$, which depends on the square of the momentum transferred to the pions.

The exact expression for the cross section of the process (2) is quite cumbersome, but in limiting cases—near the threshold of the reaction and




a b c

FIG. 1. Diagrams of the photoprocess $\gamma + \nu \to \mu^- + \pi^+ + \pi^0$.

1)We note that in[3,4] the factor 2 has been left out of the final expressions for the cross sections.

in the ultrarelativistic case—the formulas are quite manageable. Namely, for the process (2), assuming for simplicity that f = 1, we obtain in the nonrelativistic approximation

$$\sigma_2 \approx \frac{G^2\alpha}{2^4\pi}\frac{1}{W^{5/2}}(W-m_\mu)(W-2m_\pi)^{3/2}(W-m_\mu-2m_\pi)^2, \tag{4}$$

where $\alpha = 1/137$ and W—total energy of the produced particles in their center-of-mass system $[(W-m_\mu-2m_\pi)/W \ll 1]$.

The cross section of the process (1) (n = 2) is connected with the cross section $\sigma_2$ by the Weizsäcker-Williams formula[5] (the nucleus is assumed infinitely heavy):

$$\frac{d\sigma^{\text{Coul}}}{dWd\Delta^2} = \frac{2\alpha Z^2}{\pi}F(\Delta^2)\frac{\sigma_2(W)}{W\Delta^2}\left(1-\frac{W^4}{4\Delta^2E^2}\right), \tag{5}$$

where Z—charge of nucleus, $-\Delta^2 = t$—square of 4-momentum transfer to the nucleus, E—neutrino energy in the laboratory system, and $F(\Delta^2)$—electric form factor of the nucleus. The differential cross section $d\sigma^{\text{Coul}}/dWd\Delta^2$ has for fixed W a maximum at $\Delta^2 = 2\Delta^2_{\text{min}} = W^4/2E$ (we disregard here the rather weak dependence of F and of $\sigma_2$ on $\Delta^2$). The amplitude of the Coulomb maximum increases quadratically with the energy, as can be seen from (5).

In the next section we shall ascertain whether or not the pure Coulomb contribution to the cross section of process (1) can be separated from the background contributed by the strong interactions and, by the same token, information can be obtained on the cross section of process (2). We now estimate the total Coulomb cross section of the process (1). Choosing for the electric form factor of the nucleus

$$F(\Delta^2) = (1+\Delta^2R^2/6)^{-1},$$

where $R = A^{1/3}/m_\pi$—radius of the nucleus, confining ourselves to small momentum transfers $\Delta^2 \lesssim 6/R^2$, where a pure Coulomb effect is expected, and then integrating (5), we obtain the cross section of the process (1) in the Coulomb field of the nucleus. The results of numerical integration for lead and iron nuclei are listed in Table I. At large incident-neutrino energies the cross section increases asymptotically with the energy like $\sigma^{\text{Coul}} \sim E\ln E$[7].

If we substitute in place of f = 1 some reasonable form factor (such as $f = (1+q^2)^{-1}$, where q—momentum transferred to the pions), this would lead to a slower increase in the cross section $\sigma^{\text{Coul}}$ in the region of high energies (of the type $\sigma^{\text{Coul}} \sim (\ln E)^2$). However, at energies on the order of several BeV, such a form factor plays no important role in estimates.

**Table I.** Cross section for the production of two pions by a neutrino in the Coulomb field of a nucleus

| E, BeV | $10^{43}\sigma^{\text{Coul}}$, cm² | |
|---|---|---|
| | Pb | Fe |
| 2 | 0.01 | 0.009 |
| 3 | 0.2 | 0.1 |
| 4 | 1.0 | 0.3 |
| 5 | 2.5 | 0.7 |
| 10 | 27 | 5.8 |
| 25 | 260 | 47 |
| 50 | 1000 | 170 |

At a momentum transfer $\Delta^2 \gtrsim m_\pi^2$ reaction (1) can occur also with individual nucleons of the nucleus, i.e., incoherently. In this case the cross section on the nucleus is $\sigma^{\text{incoh}} \approx Z\sigma^{\text{nucl}}$, where $\sigma^{\text{nucl}}$—cross section of process (1) on the individual nucleon. Although $\sigma^{\text{nucl}}$ should be larger than $\sigma^{\text{Coul}}/Z^2$, $\sigma^{\text{incoh}}$ cannot be appreciably larger than $\sigma^{\text{Coul}}$ in the case of heavy nuclei, owing to the factor $Z^{-1}$. In addition, as will be shown below, the contribution to the cross section due to diffraction is much larger than the contribution due to the Coulomb mechanism.

## 2. DIFFRACTION MECHANISM

The contribution of strong interactions to the cross section of the process (1) (n = 2) in the region of high energies and small momentum transfers to the nucleus can be estimated on the basis of the diffraction model of Landau and Pomeranchuk[8], which was used by one of the authors[4] for similar processes with n = 1. According to the diagram method developed by Zhizhin and one of the authors[9] for the calculation of inelastic diffraction processes, process (1) is described by the three diagrams shown in Fig. 2, where the small square denotes elastic scattering of the pion by the nucleus. In analogy with the procedure used for the $\gamma + A \to \pi^+ + \pi^- + A$ process by Pomeranchuk[10], it can be shown that in the diffraction approximation the contributions to the matrix element from the diagrams of Figs. 2b and 2c strongly

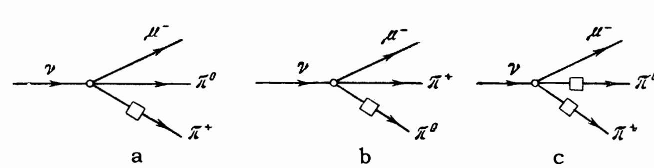



FIG. 2. Diagrams describing diffraction production of two pions by a neutrino and a nucleus.

cancel each other. Therefore to estimate the contribution of the strong interaction of the pions with the nucleus it is sufficient to use only a single diagram, Fig. 2a, which is the usual diagram of Chew and Low [11]. In the case of an infinitely heavy nucleus, the Chew and Low formula can be written in the form

$$d\sigma^{\text{dif}} = (16\pi^3E^2)^{-1}(s_1 - m_\pi^2)\sigma(s_1)\sigma_{\pi A}(\omega) \times (\omega^2 - m_\pi^2)^{1/2}(q^2 + m_\pi^2)^{-2}ds_1\, d\omega\, dq^2, \tag{6}$$

where E —energy of the incoming neutrino in the laboratory system, $s_1$—square of the total mass of the particles produced at the vertex of the weak interaction, $\omega$ —energy of virtual pion, $\sigma(s_1)$ — cross section of the $\nu + \pi^- \to \mu^- + \pi^0$ process, $-q^2$ —square of the 4-momentum of the virtual pion, and $\sigma_{\pi A}$ —cross section for the elastic scattering of the pion by the nucleus.

The cross section $\sigma(s_1)$ is calculated on the basis of the matrix element (3), and takes for f = 1 the form

$$\sigma(s_1) = (G^2/2\pi s_1^2)[(s_1 + m_\mu^2/4) \times (s_1 - m_\pi^2 + m_\mu^2) - 2m_\mu^2 s_1][s_1 - (m_\pi + m_\mu)^2]^{1/2} \times [s_1 - (m_\pi - m_\mu)^2]^{1/2}. \tag{7}$$

When $f = [1 - a(p_\pi - q)^2]^{-1}$, where q — 4-momentum of the virtual meson and $p_\pi$ —4-momentum of the real meson, the formula for $\sigma(s_1)$ is more cumbersome and will not be given here. We note, however, that in the case when $f = [1 - a(p_\pi - q)^2]^{-1}$, this cross section [unlike formula (7)] behaves asymptotically at large $s_1$ like a constant. For comparison, we have substituted in (6) the value of $\sigma(s_1)$ obtained for both f = 1 and $f = [1 - (p_\pi - q)^2/4m_\pi^2]^{-1}$.

Formula (6) for the cross section $\sigma^{\text{dif}}$ is approximate. Actually the integrand of each of the integrals with respect to $s_1$ and $\omega$ in (6) depends also on $q^2$. We start from the assumption that for small $q^2$ the difference cannot be appreciable, and confine ourselves in the integration with respect to $q^2$ to values $q^2 \lesssim m_\pi^2$. The results of the numerical integration for lead and iron nuclei are listed in Table II.

**Table II.** Cross section for the diffraction production of two pions by a neutrino on lead and iron nuclei

| E, BeV | $10^{42}\sigma$ dif, cm² (Pb) | | $10^{42}\sigma$ dif, cm² (Fe) | |
|---|---|---|---|---|
| 1 | 0.15 | (0.11)* | 0.062 | (0.048) |
| 2 | 2.2 | (1.4) | 1.0 | (0.6) |
| 3 | 5.7 | (4.2) | 2.4 | (1.8) |
| 4 | 10 | (5) | 4.4 | (2.2) |
| 5 | 15 | (7) | 6.4 | (3.0) |
| 10 | 44 | (15) | 19 | (6.2) |
| 25 | 140 | (29) | 60 | (12) |
| 50 | 310 | (41) | 140 | (17.6) |

*The parentheses contain the values of the cross sections with account of the form factor in the vertex $(\pi\pi)(\nu\mu)$.

Let us write out now a formula that is convenient for the extraction of information on the process $\nu + \pi^- \to \mu^- + \pi^0$ from the experimental cross section of the reaction (1):

$$\frac{d\sigma^{\text{dif}}}{ds_1 dq^2} = \frac{R^2(s_1 - m_\pi^2)\sigma(s_1)}{32\pi^2E^2(q^2 + m_\pi^2)^2}\left\{\omega_m(\omega_m^2 - m_\pi^2)^{1/2} - m_\pi^2 \times \ln\left[\frac{\omega_m + (\omega_m^2 - m_\pi^2)^{1/2}}{m_\pi}\right]\right\},$$

$$\omega_m = \frac{Eq^2}{(q^2 + m_\mu^2)}\left[1 - \frac{(q^2 + m_\mu^2)^2}{4q^2E^2}\right]. \tag{7'}$$

Comparing (7′) with the experimental data (on the basis of extrapolation to the point $q^2 = -m_\pi^2$ or else directly in the physical region with $q^2 \ll m_\pi^2$), we can obtain information on the cross section $\sigma(s_1)$ and clarify the role of the form factor of the vertex $(\pi\pi)(\mu\nu)$ and the virtuality of $(q^2)$.

In carrying out the integration in (6) we have assumed that the average value is $\sigma_{\pi A} = \pi R^2$, corresponding to the cross section for the diffraction scattering of a pion by an absolutely absorbing nucleus [12]. We can obtain in similar fashion as estimate of the process

$$\nu + A \to \mu^- + \pi^+ + \text{ star} \tag{8}$$

with formation of one fast pion. We recall that the summary cross section of all the inelastic interactions between a pion and a black nucleus is $\sigma_{in} = \pi R^2 = \sigma_{\pi A}$ [12] and consequently the cross section of the process (8) is equal to the cross section of the process (1) when n = 2, but of course only if we confine ourselves to the same region of pion virtuality $q^2 \lesssim m_\pi^2$.

Let us ascertain now when we can separate the pure Coulomb contribution to the differential cross section of the process (1). To this end we write out the diffraction cross section in a more convenient form, separating the variables W and $\Delta^2$ ($-\Delta^2$ —square of 4-momentum transferred to the nucleus). According to the rules developed in [9] we have

$$\frac{d\sigma^{\text{dif}}}{dW d\Delta^2} = \frac{G^2R^2J_1^2(q_\perp R)}{2^4\pi^5E^2q_\perp^2}W\int\frac{d^3p_\mu\, d^3p_{\pi^0}\, d^3p_{\pi^+}}{E_\mu E_{\pi^0}E_{\pi^+}}\left[\frac{(p_{\pi^+} + P)^2}{M^2} - m_\pi^2\right][8(p_\nu p_{\pi^0})(p_\mu p_{\pi^0}) - 4(p_\nu p_\mu)m_\pi^2 + 4m_\mu^2(p_\nu p_{\pi^0}) + m_\mu^2(p_\nu p_\mu)][(p_\nu - p_{\pi^0} - p_\mu)^2 - m_\pi^2]^{-2}\delta^{(4)} \times (p_\nu + P - p_{\pi^0} - p_{\pi^+} - p_\mu - P'), \tag{9}$$

where $q_\perp^2 \approx \Delta^2 - W^4/4E^2$, $J_1(x)$ —Bessel function, P and P′ —4-momenta of the nucleus before and after the interaction, and M —mass of nucleus. Integrating in (9) with $\Delta^2 \ll W^2 \ll E^2$ and $(W - 2m_\pi - m_\mu)/W \ll 1$ [i.e., near the threshold of the reaction (1)], we obtain

$$\frac{d\sigma^{\text{dif}}}{dW\,d\Delta^2} = \frac{G^2R^2J_1^2(q_\perp R)}{2^4\pi^2 q_\perp^2 W^{3/2}} \times (W - m_\mu)(W - 2m_\pi)^{3/2}(W - m_\mu - 2m_\pi)^2. \qquad (10)$$

We see from (10) that the effective momentum transferred to the nucleus is $\Delta \sim 1/R$. The ratio of the diffraction and Coulomb contributions, in accordance with (4), (5), and (10), is equal to

$$\frac{d\sigma^{\text{Coul}}}{d\sigma^{\text{dif}}} \approx \frac{2(\Delta^2 - W^4/4E^2)^2\alpha^2Z^2}{\Delta^4W^2R^2J_1^2(q_\perp R)} \qquad (11)$$

or in the region of the Coulomb maximum ($\Delta^2 = W^4/2E^2$)

$$\frac{d\sigma^{\text{Coul}}}{d\sigma^{\text{dif}}} = \frac{\alpha^2Z^2}{2W^2R^2J_1^2(RW^2/2E)}\,. \qquad (12)$$

The ratio (12) increases quadratically with the energy E at fixed W. However, the contribution of the Coulomb interaction is comparable with the diffraction contribution only when $E \approx 60$ BeV for the lead nucleus. It is possible to reach approximately the same conclusion by analyzing the total cross sections (see Tables I and II). Thus, the separation of the pure Coulomb effect is quite difficult at energies amounting to several times 10 BeV even in the region of exceedingly small momentum transfers.

## 3. DISCUSSION OF RESULTS

From a comparison of the Coulomb and diffraction cross sections for the production of pion pairs, listed in Tables I and II, we can conclude that the contribution of the strong interactions in the energy region up to 50 BeV greatly exceeds the contribution from the Coulomb interaction. Of course, we are comparing results for the case $f = 1$. The ratio of the diffraction and Coulomb contributions apparently remains the same for $f = (1 + ag^2)^{-1}$ (g —momentum transferred to the muon). We note that for $E \lesssim 10$ BeV the diffraction cross section in the case of $f = 1$ differs little from the case $f = (1 + ag^2)^{-1}$, $a = m_\pi^2/4$.

The estimate which we have obtained for $\sigma^{\text{dif}}$ increases strongly if we broaden the region of possible values of the momentum of the virtual pion. Thus, an increase of $q_{\max}^2$ to a value $q_{\max}^2 = 4m_\pi^2$ increases the cross section by one order of magnitude. Nonetheless, the value obtained in this case is still 1.5—2 orders of magnitude smaller than the experimentally observed total cross section[2]. This result is not surprising, since momentum transfers in most experimentally observed events are large, on the order of several pion masses[2], whereas the diffraction mechanism is realized in the region of much smaller momentum transfers $\Delta \sim 1/R$. Where the momentum transfer is $\Delta \gtrsim m_\pi$ the nucleus can no longer be regarded as a unit. Under such conditions the principal role is assumed by the interaction between the neutrino and the individual nucleons of the nucleus.

An experimental separation of the diffraction region, and in the case of ultrahigh energy also of the Coulomb region, is of considerable interest from the point of view of gathering of information on the processes $\nu + \pi^- \rightarrow \mu^- + \pi^0$ and $\nu + \gamma \rightarrow \mu^- + \pi^+ + \pi^0$. The rapid progress in experimental neutrino physics gives grounds for hoping that suitable experiments will become feasible in the nearest future.

In addition to processes of the type (1), we have considered the reaction $\nu + A \rightarrow \nu + \pi^+ + \pi^- + A$. The cross section of this reaction would be of the same order of magnitude as the cross section of reaction (1) with $n = 2$, if weak neutral currents having the same coupling constant were to exist. The experimentally measured cross section of the reaction $\pi + N \rightarrow N + \nu$ amounts to less than 4% of the cross section of the reaction $\nu + N \rightarrow \mu + N$[2]. In our case of charged-pion pair production, we should expect the same hindrance factor compared with process (11).

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100