ANALYSIS OF ELASTIC SCATTERING OF He^3 *AND ALPHA PARTICLES ON THE BASIS OF THE OPTICAL MODEL OF THE NUCLEUS*

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Differential cross sections for the elastic scattering of He^3 by a number of nuclei have been calculated on the basis of the optical model of the nucleus. It is shown that with a single set of parameters for the optical potential one can satisfactorily describe experimental data over a broad range of nuclei from Be^9 up to Bi^{209}. A comparison is given of the parameters of the potentials describing elastic scattering of He^3 and of α particles by Al^{27}.

RECENTLY the optical model has been widely used for the analysis of elastic scattering of α particles and He^3. In the majority of cases satisfactory agreement has been obtained between the results of calculations and the experimental data. Evidently the use of the optical model for such an analysis can be justified only if the parameters of the potential do not vary appreciably both in going from one nucleus to another and also as the energy of the bombarding particles is varied. Moreover, of equal importance is the uniqueness of the set of parameters which gives satisfactory agreement of calculations and experiment. Also one should expect sensible correlation of parameters of the potential in describing the scattering of different particles by the same nucleus.

A detailed analysis of the elastic scattering of α particles by a number of nuclei from C^{12} up to Th^{232} has been given by Igo and Thaler [1]. It is shown that a set of parameters which varies relatively little enables one to give a satisfactory description of the available experimental data. The question of the uniqueness of the set of parameters is a somewhat more complicated one. Using several nuclei as examples Igo has shown [2] that essentially different sets of parameters can lead to the same angular distributions of elastically scattered α particles. However, it is characteristic that these external potentials turn out to be practically identical in the external region. From this a conclusion has been drawn relative to the "surface" character of the interaction of α particles with a nucleus in the case of elastic scattering. At the present time there exists no similar analysis of the elastic scattering of He^3. Calculations which have been carried out on the basis of relatively limited experimental material have led to values of the parameters which vary erratically from nucleus to nucleus [3]. Therefore, it is of interest to attempt a more systematic analysis of data on the elastic scattering of He^3 by different nuclei with the aid of the optical model. For this analysis we have utilized new data on differential cross sections obtained in our laboratory [4], and cross sections published in the paper by Gonzales-Vidal et al [5].

1. THE METHOD OF CALCULATION

For the calculation we have used the Coulomb potential of a uniformly charged sphere of radius $R = r_0 A^{1/3}$, $r_0 = 1.3$ F:

$$V_K = {}^1/_2 Z_1 Z_M e^2 R^{-1}(3 - r^2/R^2) \quad \text{for} \quad r \leqslant R,$$
$$V_K = Z_1 Z_M e^2 / r \quad \text{for} \quad r > R.$$

The nuclear potential was taken to be of the form

$$(V + iW) f(r), \quad f(r) = \left(1 + \exp \frac{r - R_0}{a}\right)^{-1},$$

with $R_0 = (1.2A^{1/3} + 1.8)$ F.

The most significant parts of the program for an electronic computing machine are the integration of the radial wave equation and the calculation of the Coulomb wave functions and of their derivatives. The numerical integration is carried out by the method of Fox-Goodwin [6] with an automatic choice of the increment. Integration of equations not containing a first derivative is carried out by this method approximately ten times faster than by means of the commonly utilized method of Runge-Kutta and yields the same accuracy of results. The evaluation of the Coulomb wave functions was carried out basically in accordance with the method described in the paper of Buck et al [7]. The program allowed for the possibility of an automatic change of the parameters of the potential corresponding to a given

interval in the desired region and for the choice of their best values based on the minimization of the quantity

$$\delta_N^2 = \sum_N \left[\frac{(d\sigma/d\Omega)_{\text{exp}} - (d\sigma/d\Omega)_{\text{theor}}}{\Delta\,(d\sigma/d\Omega)_{\text{exp}}}\right]^2 .$$

In order to test the program we have repeated calculations of some published differential cross sections.

2. RESULTS AND THEIR DISCUSSION

The calculations began with the differential cross sections for elastic scattering of He^3 by Al^{27} at an energy of 29.1 MeV. In order to find the range of values of the parameters which would give acceptable agreement with experiment three parameters were varied within the following limits: -3 MeV $\leq V \leq -100$ MeV; -3 MeV $\leq W \leq -100$ MeV; 0.1 F $\leq a \leq$ 1.0 F. The fourth parameter R_0 remained constant. In the course of calculations we have successively varied in steps of 50% each of the parameters, while the other two parameters remained fixed. The results of varying each parameter were read out of the machine and were analyzed. Their comparison showed that the differential cross sections, as has been already noted earlier [8], are most sensitive to a change in the parameter a characterizing the diffuseness. The cross section depends much more weakly on the values of V and W. The nature of the variation in the angular distributions as a result of varying any parameter depends only weakly on the values of the other two. As a result of the calculations carried out in this manner we have found the only domain of values of the parameters which allows us to obtain satisfactory agreement between the calculated and the experimentally observed cross sections: V = ~ -15 MeV; W = ~ -20 MeV; a = $\sim$ 0.7 F. Then utilizing these parameters for the initial values we have carried out a fine adjustment based on the minimization of the quantity δ_N^2 varying the parameters by 5% steps. The optimum agreement of calculations with experiment was obtained for the following values:

$$a = 0.65\ \text{F}; \qquad V = -13\ \text{MeV}; \qquad W = -19\ \text{MeV}.$$

In order to check the stability of these parameters with respect to varying the energy they were used to calculate the differential cross sections for the elastic scattering by Al^{27} in the range 26–33 MeV. Figure 1 shows the curves obtained, while Table I gives the values of the parameters of the potential. It can be seen that in this range of energies they remain practically constant. The same values of the parameters were utilized to calculate the differential cross sections for the

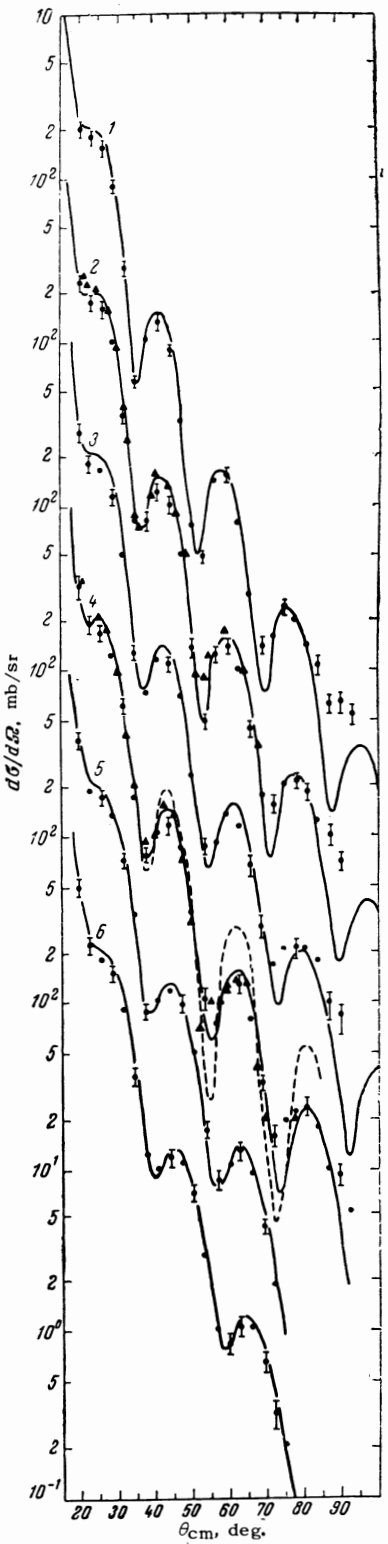

FIG. 1. Differential cross sections for the elastic scattering of He^3 by Al^{27}. The solid curves have been calculated by means of the optical model using parameters shown in Table 1. The dotted curve is the result of calculations using the parameters: V = -40 MeV; W = -30 MeV; a = 0.63 F; R = $1.60 \times A^{1/3}$ F.[9] The values of the energy of He^3 in MeV are as follows: 1 – 33; 2 – 31.4; 3 – 29.9; 4 – 29.1; 5 – 27.7; 6 – 26.1. Experimental data: ● from [4], ▲ on curves 2 and 4 from [5] and [9] respectively. The curves are displaced vertically by an order of magnitude.

Table I

Nucleus	E, MeV	−V, MeV	−W, MeV	a, F	R_0, F	σ_R in mb
Al^{27}	33.0	13	19	0.65	5.4	1320
	31.4	13	19	0.65		1324
	29.9	13	18.5	0.66		1326
	29.1	13.5	18	0.66		1358
	27.7	12	18.5	0.70		1365
	26.1	12	18	0.70		1360

elastic scattering of He^3 by Be^9, C^{12}, Na^{23}, Mg, Cu, Sn, and Bi^{209}. In each case we have carried out a variation of only the one parameter a in steps of 5% in order to obtain the best agreement of the calculated curve with the experimental values. In two cases a normalization was carried out. The experimental values of the cross sections for Bi^{209} were shifted by 0.5°, while the values for Sn^{120} were normalized for small angles to the Coulomb cross section (the normalization coefficient was 1.33). The results of this calculation are shown in Fig. 2 while the values of the energies and of the parameters of the potential are shown in Table II. In this table we have also given for comparison the values of the parameters and the corresponding values of the radius obtained earlier. It should be noted that variation of the parameters a up to 5, W up to 10, and V up to 20% does not lead to any significant deterioration in the agreement with experiment within the range of angles for which measurements have been carried out. As can be seen from Fig. 2 satisfactory agreement is obtained in all cases with the exception of C^{12}. For carbon we have not succeeded in obtaining better agreement even by varying all the parameters over a wide range. In this connection it should be noted that anomalies in the elastic scattering by C^{12} have been observed both for other charged particles [10] and also for neutrons [11].

It is of interest to compare the elastic scattering of He^3 and He^4 by the same nucleus. On the basis of the structure of He^3 and He^4 one can expect in advance quite definite changes in the parameters of the potential in going over from the one kind of particles to the other. With this aim in mind we have calculated the angular distributions of α particles of energies 36, 38 and 40 MeV elastically scattered by Al^{27}. For the calculation we have utilized the experimental data obtained in the paper by Artemov et al [12]. Figure 3 shows the curves obtained, while Table III shows the corresponding values of the parameters of the potential. The data of Tables I and III show that the potentials for He^3 and He^4 have different ratios between their real and imaginary parts. Moreover, as should have been expected,

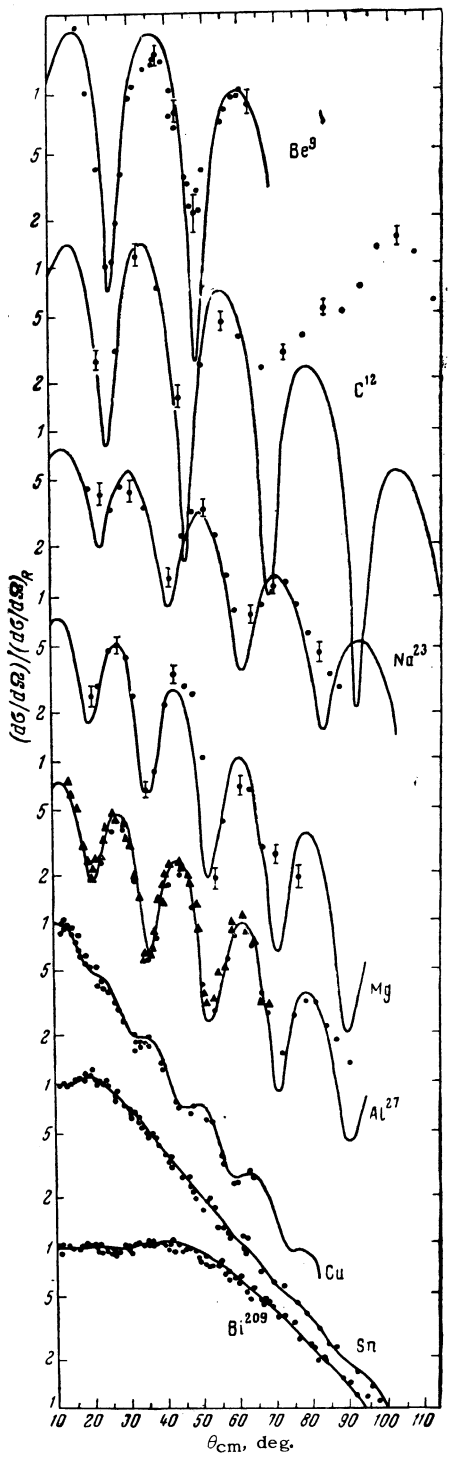

FIG. 2. The ratio $(d\sigma/d\Omega)/(d\sigma/d\Omega)_R$ for the elastic scattering of He^3. The solid curves have been calculated for the values of the energy and of the parameters given in Table II (data of this paper). The experimental data for C^{12}, Na^{23}, Mg and Al^{27} (●) have been taken from [4]; for Be^9, Al^{27} (▲), Cu, Sn, and Bi^{209} have been taken from [5].

the absolute values of the imaginary part of the potential W (the depth of the absorbing potential) are bigger in the case of He^3 than in the case of He^4.

Table II

Nucleus	Data of this paper: E, MeV	−V, MeV	−W, MeV	a, F	R_0, F	σ_R, mb	Data from [3]: E, MeV	−V, MeV	−W, MeV	a, F	R_0, F	$R=r_0A^{1/3}$ F	σ_R, mb
Be^9	31.2	13	19	0.65	4.3	1045							
C^{12}	31.4	13	19	0.60	4.6	1050	28.8	50	60	0.6	1.6	3.7	1036
Na^{23}	25.7	13	19	0.62	5.2	1227							
Mg	31.4	13	19	0.68	5.2	1370							
Al^{27}	31.4	13	19	0.65	5.4	1324	29.3	45	30	0.63	1.57	4.7	1258
Cu	31.2	13	19	0.75	6.6	1698	29.3	20	30	0.73	1.60	6.4	1739
Sn	31.2	13	19	0.67	7.7	1558	29.0	25	15	0.50	1.60	7.9	1384
Bi^{209}	31.2	13	19	0.63	8.9	1122							

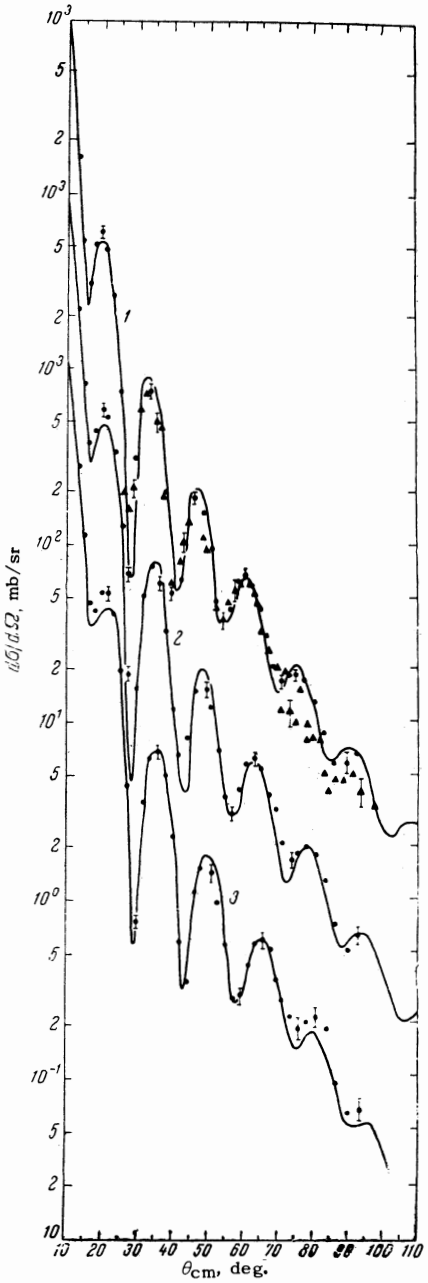

FIG. 3. Differential cross sections for the elastic scattering of He^4 by Al^{27}. The solid curves have been calculated with the aid of the optical model using the parameters shown in Table III. The values of the energy of He^4 in MeV are as follows: 1 – 40, 2 – 38, 3 – 36. The experimental values have been taken from [12]. The curves are displaced vertically by an order of magnitude.

From the results outlined above it follows that with practically the same set of parameters one can describe the differential cross section for the elastic scattering of He^3 of energy ~30 MeV over a broad range of nuclei from Be^9 up to Bi^{209}. The parameters of the potential vary insignificantly as the energy of He^3 is varied in the range 26–33 MeV. The values of the parameters obtained by us differ from those published previously [3]. Probably up to a certain point this can be explained by a somewhat different choice of the radius of the interaction and the small sensitivity of the differential cross sections within a limited range of angles to the values of V and W. In our calculations the choice of V and W was based exclusively on the correspondence between the results of calculations and of experiment without making use of any other possible estimates of the depth of the real and the imaginary parts of the potential describing the interaction of He^3 with nuclei. For example, in the paper by Greenless et al [9] values of the parameters have been taken which are closer to those obtained from the analysis of elastic scattering of nucleons. As can be seen from Fig. 1 (dotted curve) the agreement of the results of calculations and experiment is less satisfactory than in our case.

From a comparison of the parameters obtained in the analysis of the scattering of He^3 and He^4 by Al^{27} we can conclude that the nuclei are "blacker" for He^3 than for α particles. And the conclusion about the "surface" nature of the interaction of α particles with the nucleus in the case of elastic scattering can apparently be taken to apply to He^3 to an even greater degree. It is very probable that the non-uniqueness of the set

Table III

Nucleus	E, MeV	−V, MeV	−W, MeV	a, F	R_0, F	σ_R, mb
Al^{27}	40.0 38.0 36.0	25.5	10.3	0.5	5.35	1159 1165 1167

of parameters which was obtained in the work of Igo [2], and which specifically reflects the "surface" nature of the interaction of an α particle with a nucleus in the case of elastic scattering, also exists in the case of He^3.

Small changes in the parameters a in going over from one nucleus to another can hardly be related to peculiarities of the structure of individual nuclei. Of greatest interest apparently is the fact itself that it is possible to describe with a single set of parameters the differential cross sections for the elastic scattering of He^3 for a wide range of nuclei.

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