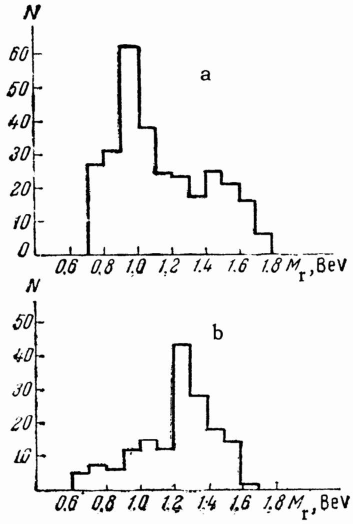

FIG. 3. Distributions of events with M_{eff} outside the interval 1100 to 1300 MeV (a) and events with M_{eff} within the limits 1100 – 1300 MeV (b).

selected reaction-(1) events certainly do not contain any contamination by reaction (2). We plotted the four-pion effective-mass distribution for these events. This distribution duplicates that shown in Fig. 2a. Thus, the maximum on Fig. 1 at $M_{eff} \sim 1250$ MeV is not due to contamination by reaction (2).

Figure 3 shows the distribution with respect to M_r of events for which the four-pion mass lies both in the interval $1100 < M_{eff} < 1300$ MeV and outside this interval. The second distribution (corresponding to events lying in the region of the maximum on Fig. 1) contains a maximum in the region of the ($^3/_2$, $^3/_2$) isobar. There is no such maximum for events with values of M_{eff} lying outside the limits 1.1–1.3 MeV. If the maximum on Fig. 1 corresponds to a resonance in the four-pion system, then it follows from Fig. 3 that this resonance is predominantly produced simultaneously with the ($^3/_2$, $^3/_2$) isobar.

The investigations show that the maximum on Fig. 1 cannot be attributed to kinematic reflection of the known resonances in two- and three-pion systems. At the present time the question of the possibility of identifying the resonance which we have observed with four-particle decay of the B meson (resonance in the ω-π system at $M_{eff} \sim 1250$ MeV) is under consideration.

In conclusion we consider it our pleasant duty to thank A. I. Alikhanov and V. V. Vladimirskiĭ for many useful discussions and also R. S. Guter for the calculations.

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REFLECTION SYMMETRY IN QUANTUM MECHANICS

A. A. BANDZAĬTIS, A. Yu. SAVUKINAS, and A. P. IUTSIS

Institute of Physics and Mathematics, Academy of Sciences, Lithuanian S.S.R.; V. Kapsukas Vilna State University

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THE question of the symmetry of quantum mechanical quantities under a substitution of the type $j \to \bar{j} = -j - 1$[1] is of considerable interest for practical applications. We shall show that this substitution can be regarded as a reflection of the coordinate system, and will give a procedure for finding the symmetry properties of the 3nj-coefficients under such a reflection, as well as a method for applying this symmetry to matrix elements of operators of physical quantities.

We earlier proposed the following property of the eigenfunctions of the angular momentum and its projection:

$$\psi\,(jm) = (-1)^{j-m}\psi(\bar{j}m). \tag{1}$$

Using the assumption (1) and adopting the standard system of phases,[2] we get

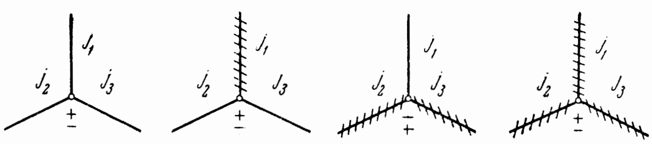
$$\psi^*(\bar{j}m) = (-1)^{-j-m}\psi(\bar{j}\,-m). \tag{2}$$

We note that relation (1), when we replace $l \to \bar{l} \to -l-1$, holds for the spherical functions normalized, not to unity, as in Condon and Shortley,[3] but to $[4\pi/(2l+1)]^{1/2}$, like the C^l in Racah's paper,[4] which is easily seen by writing the function as a hypergeometric series. The phase factor i^l, which is attached to the spherical function to get the standard system of phases, changes to i^{-l}, according to (2), when we make the above substitution.

It is not difficult to see that Eq. (1) is related to a reflection of the coordinate system in the xy plane. The same statement applies to Eq. (4) in

[1]. Thus the phase relations under substitutions of the type considered here might be called reflection symmetries (in the plane of the angular momentum components that are not sharply determined).

The reflection symmetry properties of the 3nj-coefficients are subject to the following rule. If, in the graphs for these coefficients,[5] we hatch lines corresponding to reflected parameters, then any node in the diagram will belong to one of the four types shown in the figure; here only the signs of the nodes matter, while the directions of the lines are irrelevant. To get the phase factor for reflection, we write the following phases: for nodes of type a and d we have +1, while for nodes of type b and c we have $(-1)^{j_1 \pm (j_2 - j_3)}$. In addition, for each node of type b and c we add a factor i. If the hatched lines form closed contours, we add an additional factor (-1) to a power equal to the number of such closed contours.



a b c d

Using the reflection symmetry transformation, one can obtain expressions for quantities for certain values of the parameters (quantum numbers or tensor ranks) from the expressions for other parameter values by the substitution $j \to \bar{j}$. For example, let us consider the matrix of the electrostatic interaction between an l electron and the l_0^N shell, in the $l_0^N l$ configuration for $J_0 l$ coupling:

$$(L_0S_0J_0lKJ\,|\,H_{\mathrm{el}}\,|\,L_0'S_0'J_0'lK'J) = \Sigma A\,(L_0S_0J_0KJ,\, L_0'S_0'J_0'K'J,\, lk)\,(l_0\,\|\,C^k\,\|\,l_0)\,(l\,\|\,C^k\,\|\,l) \times F_k(n_0l_0,\, nl)\ {}^k + \sum_{k'} B\,(L_0S_0J_0KJ,\, L_0'S_0'J_0'K'J,\, lk') \times |\,(l_0\,\|\,C^{k'}\,\|\,l)\,|^2\, G_{k'}(n_0l_0,\, nl). \quad (3)$$

In addition to parentage coefficients or submatrix elements of the operator U^k, A contains three 6j-coefficients,[6] while B contains an 18 j-coefficient, whose type depends on whether the l_0^N shell is treated as partially filled[6] or as almost completely filled.[7] The parameters expressed in terms of l are given in the form $X = l + a_X$ $(0 \le a_X \le l)$, where $X = K, K', J, k'$. Having determined the symmetry property of the particular 3nj-coefficients under the substitution $l \to -l-1$ by the method given above, and having used the corresponding properties of the submatrix elements of the spherical function C^l, which follow directly from their expressions if one uses our remarks about their normalization, we find

$$(L_0S_0J_0ll \pm a_Kl \pm a_J\,|\,H_{\mathrm{el}}\,|\,L_0'S_0'J_0'll \pm a_{K'}l \pm a_J) = -(-1)^{L_0+L_0'}\,(L_0S_0J_0ll \mp a_Kl \mp a_J\,|\,H_{\mathrm{el}}\,|\,L_0'S_0'J_0'll \mp a_{K'}l \mp a_J)\,|_{l\to -l-1}. \quad (4)$$

Here the coefficient of F_k goes over into the coefficient of a similar integral, while the coefficient of the integral G for $l \pm a_{k'}$ goes over into the coefficient of the integral G for $l \mp a_{k'}$.

Thus in the expressions for these coefficients in terms of l, half of the table can be gotten from the other half by the substitution $l \to -l-1$. It is not hard to see that the use of the reflection symmetry properties makes it possible to shorten the computation of expressions of this type, and also to reduce the size of tables for matrix elements of arbitrary operators of any quantum system.

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