## STUDY OF THE REACTION $\pi^-$ + p $\rightarrow 2\pi^-$ + $2\pi^+$ + $k\pi^0$ + n INDUCED BY 3.5 BeV/c PRIMARY $\pi^-$ MESONS

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MANY recent experimental papers <sup>[1,2]</sup> point to resonances in four-pion systems. To search for four-pion resonances, we undertook a study of the reaction

$$\pi^- + p \to 2\pi^- + 2\pi^+ + k\pi^0 + n.$$
 (1)

The measurements were made with a liquidhydrogen bubble chamber of 25 cm diameter, placed in a 3.5 BeV/c  $\pi^-$ -meson beam from the proton synchrotron of the Institute of Theoretical and Experimental Physics. The chamber was placed in a magnetic field of ~ 14 kOe. Altogether ~ 80,000 photographs were obtained at an average count of 10-15  $\pi^-$  mesons per chamber expansion. Identification of reaction (1) was by excluding from the four-prong stars the events of the reaction

$$\pi^- + p \to 2\pi^- + \pi^+ + k\pi^0 + p.$$
 (2)

The protons were identified by the ionization they produced. This excluded from the number of four-prong stars reaction-(2) events with a proton momentum smaller than 800-900 MeV/c.

Figure 1 shows the effective-mass distribution of the four charged pions for the events of reaction (1). The average error in the determination of the

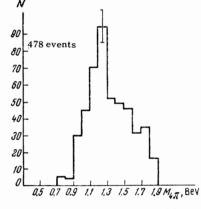


FIG. 1. Effective-mass distribution of four charged pions from reaction (1).

mass of the four pions was ~75 MeV. The distribution shows clearly a maximum at  $M_{eff} \sim 1200$  MeV. It is known from the published data<sup>[3]</sup> that the effective-mass distribution for reaction (1) with k = 0 is in good agreement with the phase curve. To separate the reaction-(1) events with k = 0 and  $k \ge 1$ , we calculated the residual mass  $M_{\Gamma}$  for the four pions. The average error in the determination of  $M_{\Gamma}$  was 120 MeV. From this value of  $M_{\Gamma}$  it follows that events with  $M_{\Gamma} \ge 1100$  MeV pertain predominantly to the reaction (1) with  $k \ge 1$ , whereas events with  $M_{\Gamma} < 1100$  MeV pertain predominantly to the reaction (1) with  $k \ge 1$ , whereas events with  $M_{\Gamma} < 1100$  MeV pertain predominantly to the reaction (1) with  $k \ge 0$ .

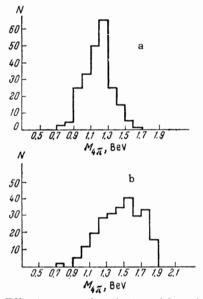


FIG. 2. Effective-mass distribution of four charged pions from the reaction (1): a – for the case  $M_r>1100$  MeV. b – for the case  $M_r<1100$  MeV.

Figure 2 shows the four-pion effective mass distributions with  $M_r \ge 1100 \text{ MeV}$  and  $M_r < 1100 \text{ MeV}$ . The distribution for  $M_r < 1100 \text{ MeV}$  is well described by the first curve, in agreement with the data of <sup>[3]</sup>. The four-pion effective-mass distribution for events with  $M_r \ge 1100 \text{ MeV}$  displays a maximum at  $M_{eff} \sim 1250 \text{ MeV}$ . The maximum observed on Fig. 1 is thus due to the events of reaction (1) for which  $k \ge 1$ .

A separate check was undertaken to show that the maximum on Fig. 1 is not due to contamination by reaction-(2) events with a proton momentum exceeding 900 MeV/c. The ionization of protons with momentum larger than 900 MeV/c differs insignificantly from that of  $\pi^+$  mesons, and such events could be mistaken for reaction (1).

We have therefore eliminated those reaction-(1) events for which at least one positive particle had a momentum larger than 800 MeV/c. The so-

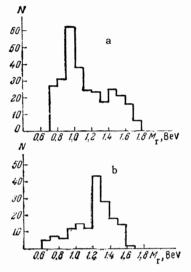


FIG. 3. Distributions of events with  $M_{eff}$  outside the interval 1100 to 1300 MeV(a) and events with  $M_{eff}$  within the limits 1100 - 1300 MeV(b).

selected reaction-(1) events certainly do not contain any contamination by reaction (2). We plotted the four-pion effective-mass distribution for these events. This distribution duplicates that shown in Fig. 2a. Thus, the maximum on Fig. 1 at  $M_{eff} \sim 1250$  MeV is not due to contamination by reaction (2).

Figure 3 shows the distribution with respect to  $M_r$  of events for which the four-pion mass lies both in the interval  $1100 < M_{eff} < 1300$  MeV and outside this interval. The second distribution (corresponding to events lying in the region of the maximum on Fig. 1) contains a maximum in the region of the  $(\frac{3}{2}, \frac{3}{2})$  isobar. There is no such maximum for events with values of  $M_{eff}$  lying outside the limits 1.1-1.3 MeV. If the maximum on Fig. 1 corresponds to a resonance in the fourpion system, then it follows from Fig. 3 that this resonance is predominantly produced simultane-ously with the  $(\frac{3}{2}, \frac{3}{2})$  isobar.

The investigations show that the maximum on Fig. 1 cannot be attributed to kinematic reflection of the known resonances in two- and three-pion systems. At the present time the question of the possibility of identifying the resonance which we have observed with four-particle decay of the B meson (resonance in the  $\omega$ - $\pi$  system at M<sub>eff</sub> ~ 1250 MeV) is under consideration.

In conclusion we consider it our pleasant duty to thank A. I. Alikhanov and V. V. Vladimirskiĭ for many useful discussions and also R. S. Guter for the calculations. <sup>2</sup> Chung, Dahl, Hess, Kalbfleisch, Miller, Smith, and Kirz, Sienna International Conference, 1963.

<sup>3</sup>L. Bondar, K. Bongartz, et al., Nuovo cimento **31**, 485 (1964).

Translated by J. G. Adashko 55

## REFLECTION SYMMETRY IN QUANTUM MECHANICS

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HE question of the symmetry of quantum mechanical quantities under a substitution of the type  $j \rightarrow \overline{j} = -j - 1^{[1]}$  is of considerable interest for practical applications. We shall show that this substitution can be regarded as a reflection of the coordinate system, and will give a procedure for finding the symmetry properties of the 3nj-coefficients under such a reflection, as well as a method for applying this symmetry to matrix elements of operators of physical quantities.

We earlier proposed the following property of the eigenfunctions of the angular momentum and its projection:

$$\psi(jm) = (-1)^{j-m} \psi(\bar{j}m).$$
 (1)

Using the assumption (1) and adopting the standard system of phases, [2] we get

$$\psi^*(\bar{j}m) = (-1)^{-j-m} \psi(\bar{j}-m).$$
(2)

We note that relation (1), when we replace  $l \rightarrow \overline{l}$  $\rightarrow -l - 1$ , holds for the spherical functions normalized, not to unity, as in Condon and Shortley, <sup>[3]</sup> but to  $[4\pi/(2l+1)]^{1/2}$ , like the  $C^l$  in Racah's paper, <sup>[4]</sup> which is easily seen by writing the function as a hypergeometric series. The phase factor  $i^l$ , which is attached to the spherical function to get the standard system of phases, changes to  $i^{-l}$ , according to (2), when we make the above substitution.

It is not difficult to see that Eq. (1) is related to a reflection of the coordinate system in the xy plane. The same statement applies to Eq. (4) in

<sup>&</sup>lt;sup>1</sup>Abolins, Lander, Mehlhop, Xuong, and Yager, Phys. Rev. Lett. **11**, 381 (1963).