CONTRIBUTION TO THE THEORY OF ANGULAR CORRELATION OF INTERNAL CONVERSION ELECTRONS

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An expression is obtained for the conversion parameters that define the angular correlation of internal-conversion electrons. Approximate Furry-Sommerfeld-Maue electron wave functions are employed.

A study of the correlation of the directions of successive nuclear radiations emitted in nuclear cascade transitions yields important information on the spins and parities of the nuclear levels. If the cascade includes conversion transitions, the analysis is complicated by the following circumstances. First, no electronic wave functions that are natural for the correlation problem are exact solutions of the Dirac equation with a Coulomb potential, and describe at the same time the propagation of the electron in a definite direction (and with definite polarization). Therefore it becomes necessary to use for the electron wave functions the solutions of the Dirac equation in spherical coordinates, i.e., the eigenfunctions of the angular momentum and of the parity. The angular dependence is introduced in this case into the correlation function with the aid of rotations of the coordinate systems, connected with the successive emissions in the cascade[1]. Such an approach is cumbersome and not elementary. Second, even in this approach, the exact calculations cannot be carried through to conclusion without the use of numerical integration.

We calculate here the probability of the correlation of the conversion-electron directions for pure multipole transitions, using Furry-Sommerfeld-Maue electron wave functions[2,3]. The use of these functions makes it possible to obtain in simple fashion results whose accuracy is determined by the inequality $(\alpha Z)^2 \ll 1$ (α—fine structure constant, Z—nuclear charge).

The probability of the correlation of the photon and internal conversion electron directions in a double cascade is [1]

$$W = \sum_s F_s(J_1L_1; J)\, F_s(J_2L_2; J)\, \xi_s(L_2\lambda_2)\, P_s(\cos\theta), \quad (1)$$

where J_1—nuclear spin in the initial state, J_2—in the final state, J—in the intermediate state, L and λ—multipolarity and parity of the corresponding transitions, θ—angle between the directions of radiation of the cascade, and the function

$$F_s(J_kL; J) = (-1)^{J-J_k+1}(2L+1) \times \sqrt{2J+1}\, C^{s0}_{L_1;L-1} W(JJLL; sJ_k) \quad (2)$$

is a characteristic quantity describing the γ-quantum correlation. The factor ξ_S reflects the fact that one of the transitions (in our case, the second) is via conversion. The quantities W and C in (2) are respectively the Racah and Clebsch coefficients.

Inasmuch as the quantities $F_S(J_k, L; J)$ are known, the determination of the correlation function (1) reduces to a determination of the conversion parameters ξ_S. These parameters can be calculated if one knows the nuclear density matrix following the conversion transition with emission of an electron in a given direction. The latter can be written in the form

$$\rho_k(mm') \equiv \frac{\alpha\omega\varepsilon p}{8\pi^2}(-1)^{J-J_k}\sum_{s\sigma}[2s+1]^{1/2} \times \sum_{MM'}(-1)^M W(JJLL; sJ_k) \times C^{s\sigma}_{L-M;LM'} C^{Jm'}_{Jm;s\sigma} \sum_{\mu\mu_0} \langle p\mu \mid B^{(\lambda)}_{LM} \mid k\rangle \langle p\mu \mid B^{(\lambda)}_{LM'} \mid k\rangle^* = \sum_{s\sigma}\left[\frac{4\pi}{2J+1}\right]^{1/2} F_s(J_kL; J)\, \xi_s(L\lambda)\, C^{Jm}_{Jm';s\sigma} Y_{s\sigma}\left(\frac{\mathbf{p}}{p}\right), \quad (3)$$

where $\langle p\mu \mid B^{(\lambda)}_{LM} \mid k\rangle$—matrix element for internal conversion on the K shell, ω—transition energy, and ϵ and p—energy and momentum of the electron.

To calculate the density matrix (3) we use the conversion matrix elements obtained in [4]. These matrix elements were calculated using Furry-Sommerfeld-Maue electron wave functions for the continuous spectrum, and the corresponding approximate function for the K shell. As a result

we obtain the following expression for the polarization sum:

$$\sum_{\mu\mu_0} \langle \mathbf{p}\mu \mid B_{LM}^{(\lambda)} \mid k\rangle \langle \mathbf{p}\mu \mid B_{LM'}^{(\lambda)} \mid k\rangle^* = \sum_{t\tau} \left[H^{(\lambda)} - (2L+1) \begin{Bmatrix} LLt \\ LL1 \end{Bmatrix} F^{(\lambda)}\right] \times \frac{2L+1}{[4\pi(2t+1)]^{1/2}} (-1)^{M'} C_{L0,\,L0}^{t0} C_{LM;\,L-M'}^{t\tau} Y_{t\tau}\left(\frac{\mathbf{p}}{p}\right); \quad (4)$$

in the case of transitions of the electric type ($\lambda = 1$)

$$F^{(1)} = \left| \frac{L}{2L+1} (R_2 + R_3) - P_1 + iP_3 \right|^2,$$

$$H^{(1)} = \frac{L}{L+1} \left| R_1 - i \frac{L}{2L+1} R_2 + i \frac{L+1}{2L+1} R_3 + iP_1 - P_2 \right|^2, \quad (5)$$

In the case of transitions of the magnetic type ($\lambda = 0$)

$$F^{(0)} = \left| Q_1 - \frac{L+1}{2L+1} Q_2 - \frac{L}{2L+1} Q_3 \right|^2,$$

$$H^{(0)} = \frac{L(L+1)}{(2L+1)^2} |Q_2 - Q_3|^2. \quad (6)$$

The values of R, P, and Q are given in [4] [formulas (40), (40a), and (20)].

Substituting (4) in (3) we obtain the following formula for the conversion parameter

$$\xi_s = \beta_L^{(\lambda)} b_s, \quad b_s = 1 + \frac{s(s+1)}{2L(L+1) - s(s+1)} \frac{1}{1 + T^{(\lambda)}},$$

$$T^{(\lambda)} = \frac{F^{(\lambda)}}{H^{(\lambda)}} \quad (7)$$

and $\beta_L^{(\lambda)} = F^{(\lambda)} + H^{(\lambda)}$—K-shell conversion coefficient.

We note that it is necessary to calculate by means of formula (7) only the quantity b_2, inasmuch as for other values of s we can use the relation

$$b_s = 1 + \frac{s(s+1)}{2L(L+1) - s(s-1)} \frac{L(L+1) - 3}{3} (b_2 - 1). \quad (8)$$

As follows from (7), the correlation function depends appreciably on the transverse and longitudinal parts of the polarization sum, $F^{(\lambda)}$ and $H^{(\lambda)}$, which enter in the form of a ratio. If the calculation error is such that it cancels out in this ratio, then the accuracy of the result may turn out to be larger than the accuracy of the employed matrix elements. This apparently explains the following coincidence: the calculations carried out by the formulas of the present paper for the concrete case Z = 40, $\omega/m = 1$, quadrupole magnetic transition, give values $b_2 = 1.11$ whereas in accordance with the tables of [1], calculated with exact Dirac wave functions, we have in this case $b_2 = 1.120$.

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[1] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

[2] W. Furry, Phys. Rev. **46**, 391 (1934).

[3] A. Sommerfeld and A. Maue, Ann. Physik **22**, 629 (1935).

[4] V. F. Boldyshev, JETP **44**, 717 (1963), Soviet Phys. JETP **17**, 486 (1963).

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