THE FOUR-BODY PROBLEM AT LOW ENERGIES

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Under the assumption of the dominance of two-body forces, an integral equation for the amplitude of four-nucleon interaction reactions has been derived by summing an infinite series of perturbation theory diagrams. It is shown that the equations for the reaction amplitudes, in which two pairs of interacting particles are formed in the final state, are coupled to those in which there are three interacting particles. The kernels of these equations contain the amplitude for three-nucleon interactions in the same manner as the kernel of the equation for the three-nucleon interactions contains the two-nucleon scattering amplitude. For simplicity of exposition the derivation of the equations is given for spinless particles. The generalization of the equations to the case of real particles which have spin does not involve difficulties of principle and can be carried out by means of the method utilized by the authors in solving the three-body problem.[2,4]

THE problem of the interaction of four nucleons at low energies is being considered on the basis of the method of summation of an infinite series of nonrelativistic diagrams, which had been used for the description of the two- and three-body problems [1-4], under the assumption that only two-body forces are essential. As in the investigation of nucleon - deuteron scattering [2-4], it has been possible to represent the amplitudes of different four-nucleon interaction processes as the contributions of an infinite series of diagrams and, analyzing the structure of these contributions, to write down integral equations for the amplitudes of the reactions under consideration.

In order not to introduce complications which are inessential from the point of view of the principles involved, it will be assumed in what follows that all nucleons are identical and possess neither spin nor isospin.

1. DERIVATION OF GRAPHICAL EQUATIONS FOR THE REACTION AMPLITUDE CORRESPONDING TO DIFFERENT PROCESSES OF FOUR PARTICLE INTERACTIONS

In describing reactions of interactions of four identical particles we will consider, for convenience of the exposition, only those cases of reactions in which groups of interacting particles exist before the reaction only in a bound state, i.e., two of the interacting particles are in a deuteron state d, or three interacting particles are in a triton state or in the form of the He^3 nucleus. Without restriction of generality we may call in what follows the three bound particles a triton. Then all reactions in which four nucleons participate can be grouped into two classes: one in which in the initial state there are two deuterons, and another in which in the initial state there is a triton and a nucleon. In each of these two groups two types of reactions are possible, characterized by the fact that in the final state there exist two pairs of interacting nucleons or one nucleon and three pairwise interacting particles. We note that the interacting nucleons in the final state may also form bound states.

We consider first the method of deriving graphical integral equations for the class of four-nucleon reactions which lead to the formation of four nucleons which interact in various ways. In this case the first group will include the reactions $d + d \rightarrow 2N + 2N$ and $d + d \rightarrow 3N + N$, and the second group the processes $T + N \rightarrow 2N + 2N$ and $T + N \rightarrow 3N + N$. Here the symbols 2N and 3N denote groups of interacting nucleons.

We denote by $\mathbf{k}_0$ and $\mathbf{k}$ the momenta of relative motion of the two groups of interacting nucleons in the initial and final states respectively, by α_d^2/m and $3\alpha_t^2/2m$ the binding energies of the deuteron and triton respectively, by $\mathbf{f}_1$ and $\mathbf{f}_2$ the values of the relative momenta of each of the pairs of particles scattered in the final state of the reactions $d + d \rightarrow 2N + 2N$ and $T + N \rightarrow 2N + 2N$ respectively, and by $\mathbf{q}$ and $\mathbf{f}_3$ the relative momenta characterizing the system of three pairwise interacting nucleons in the c.m.s. of these nucleons, so that **q** is the relative momentum of one of the nucleons with respect to the pair of nucleons with relative momentum $\mathbf{f}_3$.

The values of the indicated relative momenta of the particles before and after the reaction are connected by relations which are consequences of energy conservation:

$$\frac{k_0^2}{m} - 2\frac{\alpha_d^2}{m} = \frac{k^2}{2m} + \frac{f_1^2}{m} + \frac{f_2^2}{m}$$

for the reaction $d + d \to 2N + 2N$.

$$\frac{k_0^2}{2m} - \frac{2\alpha_d^2}{m} = \frac{2}{3}\frac{k^2}{m} + \frac{3}{4}\frac{q^2}{m} + \frac{f_3^2}{m}$$

for the reaction $d + d \to 3N + N$,

$$\frac{2}{3}\frac{k_0^2}{m} - \frac{3}{2}\frac{\alpha_t^2}{m} = \frac{k^2}{2m} + \frac{f_1^2}{m} + \frac{f_2^2}{m}$$

for the reaction $N + T \to 2N + 2N$.

$$\frac{2}{3}\frac{k_0^2}{m} - \frac{3}{2}\frac{\alpha_t^2}{m} = \frac{2}{3}\frac{k^2}{m} + \frac{3}{4}\frac{q^2}{m} + \frac{f_3^2}{m}$$

for the reaction $T + N \to 3N + N$.

If one takes into account that for a pair of interacting nucleons at low energies only the s state is essential, it is easy to see that for a given value of the momentum $\mathbf{k}_0$ the amplitudes of the indicated reactions will be functions of three variables. We will denote

by the symbol $A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$

— the amplitude for the reaction $d + d \to 2N + 2N$,

by the symbol $B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$

— the amplitude for the reaction $d + d \to 3N + N$,

by the symbol $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2)$

— the amplitude for the reaction $T + N \to 2N + 2N$,

by the symbol $D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3)$

— the amplitude for the reaction $T + N \to 3N + N$

(cf. the diagrams A, B, C, D in Fig. 1).

If one considers the problem of the interaction of four nucleons on the basis of non-relativistic field theory with the use of second quantization techniques, where the interaction operator between a pair of nucleons has the form

$$V(t) = \sum_{\mathbf{k}, \mathbf{k}', \mathbf{p}} a^+(\mathbf{k}', t)\, a^+(\mathbf{p} - \mathbf{k}', t) \times V(\mathbf{k}', \mathbf{k})\, a(\mathbf{k}, t)\, a(\mathbf{p} - \mathbf{k}, t),$$

with $V(\mathbf{k}', \mathbf{k}) = \int \exp i(\mathbf{k} - \mathbf{k}')\mathbf{r} V(\mathbf{r})\, d\mathbf{r}$, then the amplitudes of the reactions under consideration, as well as the amplitudes of two- and three-nucleon interactions [1-4], will be determined by the contribution of an infinite number of non-relativistic perturbation-theory diagrams corresponding to the given process. In Fig. 1 the first among the infinite number of successively more complicated diagrams are represented for the reactions d + d → 2N + 2N, d + d → 3N + N, T + N → 2N + 2N and T + N → 3N + N. Here the diagrams b_0, c_0, and d_0 are the simplest diagrams, usually called pole diagrams.

An investigation of the chains of diagrams of Fig. 1 has allowed us to construct graphical equations for each of the reactions under consideration (Fig. 2a).

As can be seen from Fig. 2a, the integral equations for the reaction amplitudes of one group are coupled among themselves and do not contain the amplitudes of reactions of the other groups, and there is a common law for the construction of the integral equations for the amplitudes of these four reactions. Namely the amplitude m_i, for a reac-

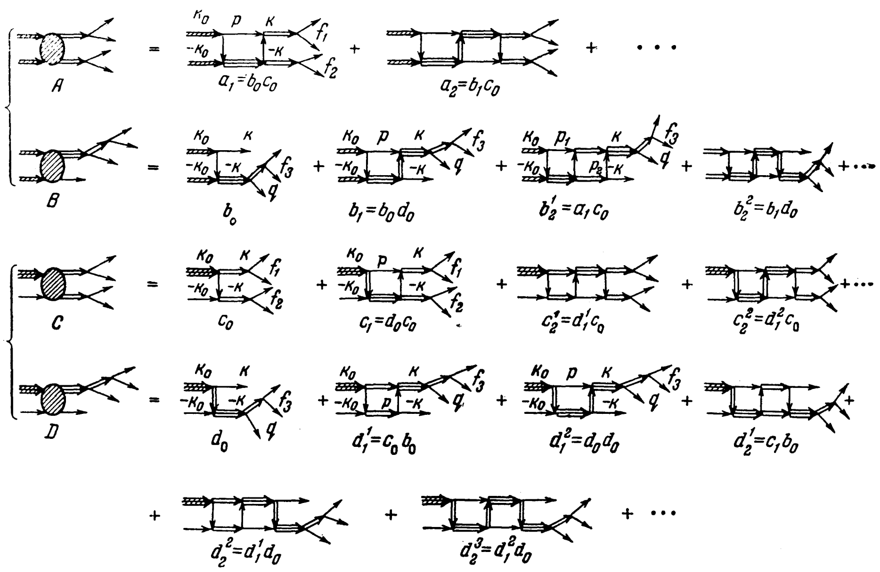

FIG. 1. Graphical representations of the four-nucleon interaction amplitudes $A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$, $B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$, $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2)$ and $D(\mathbf{k}_0\ \alpha_t; \mathbf{k}, \mathbf{q}, f_3)$ in the form of infinite series of diagrams, which become gradually more complicated.

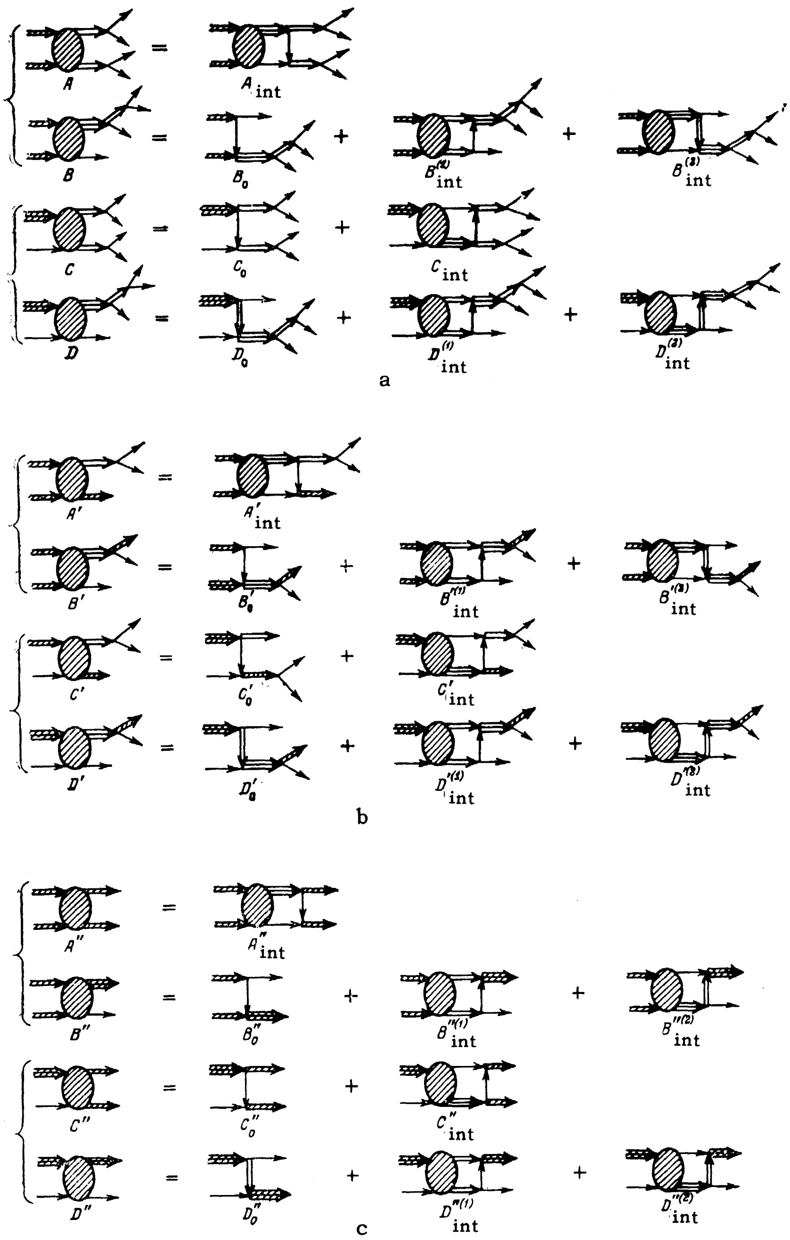

FIG. 2. Graphical integral equations for the amplitudes of four-nucleon interactions for the reactions in which: a – in the final state there are four interacting nucleons; b – in the final state there are a deuteron and two interacting or noninteracting nucleons; c – in the final state there are nucleons bound in nuclei.

tion in which in the final state two pairs of interacting nucleons are formed, is expressed in terms of the amplitude N_i (of the given group), where there are three pairwise interacting nucleons in the final state. The latter amplitude is in turn expressed in terms of the former and of itself. Consequently this law can be written in the form

$$M_i = M_i^0 + f(N_i), \qquad N_i = N_i^0 + f(M_i, N_i).$$

here the subscript i = 1, 2 characterizes the type of the group of reactions, and M_i^0 and N_i^0 are the contributions of the pole diagrams to the corresponding reactions.

Let us now derive the integral equations for the reaction amplitudes for the interaction of two deuterons, or of a triton with a nucleon, when in the final state there are two free nucleons and a deuteron interacting with one another. As in the case when free nucleons are formed, these four nucleon-interaction reactions can be divided into the same two groups, in which the initial states are the same. Within these groups there will be two types of reactions, so that in the first group there will be those reactions in which in the final state there is a deuteron and two free nucleons with momentum f_2, and in the other group there will be those reactions in which the final state consists of a nucleon and an interacting system, consisting of a nucleon and deuteron with relative momentum $\mathbf{q}$ in their c.m.s. Then, for a fixed value of the momentum $\mathbf{k}_0$, the reaction amplitudes of this type will be functions of two variables, in the same manner as the reaction amplitude for inelastic scattering of a nucleon on a deuteron.

In distinction from the four-nucleon interaction which leads to the formation of four free nucleons, we will denote the reactions of the type under consideration by symbols with a single prime, so that the reactions of the first group $d + d \rightarrow 2N + d$ and $d + d \rightarrow Nd + N$ will be described by the amplitudes $A'(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_2)$ and $B'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q})$ and the reactions of the second group $T + N \rightarrow d + 2N$ and $T + N \rightarrow Nd + N$ by the amplitudes $C'(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2)$ and $D'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q})$, respectively.

The infinite series of non-relativistic diagrams which contribute to the amplitudes $A'(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_2)$, $B'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q})$, $C'(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2)$ and $D'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q})$ will differ from the series represented by the diagrams in Fig. 1 by the fact, that, for instance in the diagrams for the amplitudes $A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$ and $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2)$, in the final state the two interacting nucleons, which are emitted either from the decay vertex of the triton in the pole diagram or from the three-nucleon scattering block, have to be regarded as a deuteron, and in the diagrams corresponding to the amplitudes $B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$ and $D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3)$, the blocks of three-nucleon scattering and of scattering of one nucleon by a deuteron have to be replaced respectively by the blocks corresponding to the transition of three nucleons into a nucleon and a deuteron and to the elastic scattering of a nucleon on the deuteron. We then obtain the graphical equations represented in Fig. 2b for the reaction amplitudes $A'(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_2)$, $B'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q})$, $C'(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2)$ and $D'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q})$.

It is interesting to note that in this case the reaction amplitudes are expressed in terms of the unprimed amplitudes of the corresponding group of reactions, i.e.,

$$M_i' = M_i'^0 + f(N_i), \qquad N_i' = N_i'^0 + f(M_i, N_i).$$

By analogy with the preceding reasoning, it is easy to show that the amplitudes of the reactions $d + d \rightarrow d + d$ and $d + d \rightarrow T + N$, or $T + N \rightarrow d + d$ and $T + N \rightarrow T + N$ will be functions of only one variable, namely the scattering angle, and the graphical integral equations for these amplitudes, which will be denoted by doubly primed letters $[A''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_t)$, $B''(\mathbf{k}_0, \alpha_d; \mathbf{k}, d_t)$, $C''(\mathbf{k}_0, d_t, \mathbf{k}, \alpha_d)$ and $D''(\mathbf{k}_0, \alpha_t, \mathbf{k}, \alpha_t)]$, are given in Fig. 2c. From this figure it can be seen that the doubly primed amplitudes can be expressed in terms of the primed and unprimed amplitudes of the same group of reactions, so that

$$M_i'' = M_i''^0 + f(N_i'), \qquad N_i'' = N_i''^0 + f(M_i', N_i').$$

2. DETERMINATION OF THE CONTRIBUTIONS FROM DIFFERENT DIAGRAM BLOCKS, WHICH REPRESENT THE INTERACTION AMONG FOUR NUCLEONS

As can be seen from Figs. 1 and 2, the diagrams which correspond to the scattering of four nucleons will contain vertices describing the decay or formation of deuterons (Fig. 3a), vertices corresponding to the decay or formation of tritons (Figs. 3c, d), blocks corresponding to nucleon-nucleon scattering (Fig. 3d), blocks corresponding to elastic and inelastic nucleon-deuteron scattering (Figs. 3e, f), the block corresponding to the scattering of three free nucleons (Fig. 3g), and propagator lines for the nucleons.

We note that since in nonrelativistic perturbation theory diagrams the displacement of virtual particles from the mass shell is insignificant [4], in the zeroth and linear approximations with respect to r_0 $(kr_0 < 1)$ one may express the con-

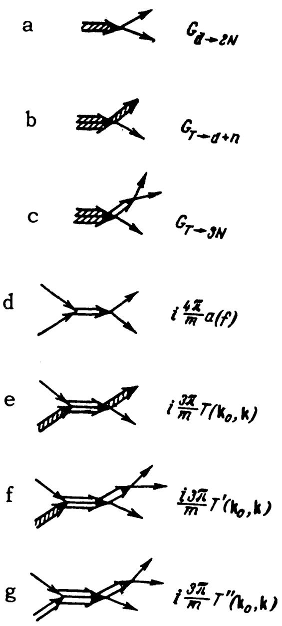

FIG. 3. The elements of four-nucleon non-relativistic diagrams: a – the vertex for the decay of a deuteron into two nucleons; b – the vertex for the decay of a triton into a nucleon and a deuteron; c – the vertex for the decay of a triton or He^3 into three nucleons, of which two interact with each other; d – the block for nucleon-nucleon scattering; e – the block for elastic nucleon-deuteron scattering; f – the block for inelastic nucleon deuteron scattering; g – the block for three-nucleon scattering.

tributions from the two-particle scattering blocks entering into complex diagrams in terms of the amplitudes for the scattering of real particles.

Therefore the contribution from the blocks corresponding to nucleon-nucleon scattering for $kr_0 < 1$ [4] is the quantity $i4\pi m^{-1} a(f)$, where $a(f)$ is the exact nucleon-nucleon scattering amplitude for relative momentum f. Since the energies are small, it is considered here that only the s-wave is essential in the partial wave expansion of the nucleon-nucleon amplitude. Consequently the contribution from the nucleon-nucleon scattering block is determined by the selection of the character of the two-body forces. It is clear that in the zeroth order in r_0, the interaction radius of the forces, the value of this contribution has the form

$$i\frac{4\pi}{m}a(f) = -i\frac{4\pi}{m}(\alpha_d + if)^{-1}.$$

In the linear (first) approximation in r_0 it has the form

$$i\frac{4\pi}{m}a(f) = \frac{i4\pi}{m}\left[\frac{r_0}{2} - \frac{1+\alpha_d r_0}{\alpha_d + if}\right]. \quad (1)$$

In the case $kr_0 > 1$ one has to take as the contribution from the nucleon-nucleon scattering block a quantity proportional to the generalized nucleon-nucleon scattering amplitude, which depends on the energies and momenta of the particles before and after the reaction. With the blocks corresponding to elastic and inelastic nucleon-deuteron scattering or to three-nucleon scattering, one has to associate the magnitude of the amplitudes $T(\mathbf{k}_0, \mathbf{k})$ of elastic scattering of nucleons by deuterons, $T'(\mathbf{k}_0, \mathbf{k})$ - of inelastic nucleon-deuteron scattering or $T''(\mathbf{k}_0, \mathbf{k})$ the magnitude of the free three-nucleon scattering amplitude with the factor $i3\pi/m$ (which appears when one goes over from the amplitudes whose squares define the cross section to Feynman amplitudes), respectively. Here $\mathbf{k}_0$ and $\mathbf{k}$ are the momenta of relative motion of the pair of bound or unbound nucleons and of the third nucleon, before and after the interaction, respectively. For the determination of the values of these amplitudes there exist integral equations, depending on one variable, which have been obtained before.[4]

The contribution $G_{d\to 2N}$ from the decay vertex of a deuteron into nucleons, in non-relativistic diagrams for $kr_0 < 1$ does not depend on the relative energies of the nucleons which are formed and is only a function of the potential. As has been shown in [1], the contribution of this vertex (Fig. 3a) is determined by the residue $\operatorname{Res} a_{\text{pole}}(f)|_{f=i\alpha_d}$ of the pole part of the scattering amplitude for two nucleons at $f = i\alpha_d$ and has the form

$$G_{d\to 2N} = -im^{-1}[-8\pi\alpha_d \operatorname{Res} a_{\text{pole}}(f)|_{f=i\alpha_d}]^{1/2}. \quad (2)$$

In the zeroth approximation in r_0, obviously $G^0_{d\to 2N} = -i(8\pi\alpha_d)^{1/2}/m$ and in the linear approximation in r_0, $G^1_{d\to 2N} = -i(8\pi\alpha_d(1 + \alpha_d r_0))^{1/2}/m$.

We go over now to the determination of the contribution from the tritium decay vertex into three free, mutually scattering nucleons, or into a nucleon and a deuteron.

Let us see first what is the vertex for the decay of a triton into three free nucleons (Fig. 3c). Let one pair of nucleons out of the three which are formed in the decay have the energy of relative motion equal to E'. The energy of relative motion of the third nucleon with respect to the distinguished pair is E''. Then, as is well known, the three-nucleon scattering amplitude, considered as a function of the total energy of the given system $E = E' + E''$ and of the relative energy E', must have a pole at $E = E_0 = -(3/2)(\alpha_t^2/m)$, equal to the binding energy of the triton, and a pole at a value of the relative energy of the nucleon pair $E' = E_0' = -\alpha_d^2/m$, the binding energy of the deuteron. The pole term $T_0(E, E')$ of the three-nucleon scattering amplitude, as a function of E and E', must have the form

$$T_0(E, E') = \frac{g^2_{T-3N}}{(E-E_0)(E'-E_0')} = \frac{g^2_{T-3N}}{(E+3\alpha_t^2/2m)(E'+\alpha_d^2/m)}, \quad (3)$$

where g_{T-3N} is the coupling constant of the three nucleons in the triton.

On the other hand, as has been derived in [2], the exact inelastic scattering amplitude for nucleon deuteron scattering with the formation of three free nucleons, $T'(\mathbf{k}_0, \varkappa)$ can be represented in the form:

$$T'(\mathbf{k}_0, \varkappa') = t'(\mathbf{k}_0, \varkappa') + t'(\mathbf{k}_0, -\tfrac{1}{2}\xi' + \varkappa') + t'(\mathbf{k}_0, -\tfrac{1}{2}\xi' - \varkappa'),$$

where, e.g. $t'(\mathbf{k}_0, \varkappa)$, is defined by an integral equation of the form

$$t'(\mathbf{k}_0, \varkappa') = \frac{8}{3}a(\xi')\Big\{[8\pi\alpha_d \operatorname{Res} a_{\text{pole}}(\xi')|_{\xi'=i\alpha_d}]^{1/2}\left(\frac{k_0^2}{4} - \alpha_d^2 - \frac{p^2}{2} - \frac{(\mathbf{k}-\mathbf{p})^2}{2}\right)^{-1} + 4\pi\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{{}^3/_4\, t'(\mathbf{k}_0, \mathbf{q})}{\varkappa'^2/4 - q^2/2 + \xi'^2 - (\varkappa'-\mathbf{q})^2/2}\Big\}. \quad (4)$$

From what was said above it follows that for the triton coupling constant g_{T-3N} one can obtain the following equation:

$$\frac{g^2_{T-3N}}{(E-3\alpha_t^2/2m)(E'+\alpha_d^2/m)} = T'_{\text{pole}}(\mathbf{k}_0, \varkappa')|_{E=-3\alpha_t^2/2m}.$$

From here it follows for g_{T-3N}:

$$g_{T-3N} = \left[T'_{\text{pole}}(\mathbf{k}_0, \varkappa')|_{E=-3\alpha_t^2/2m} \left(E + \frac{3}{2}\frac{\alpha_t^2}{m}\right)\left(E' + \frac{\alpha_d^2}{m}\right)\right]^{1/2}.$$

The decay vertex of a triton into three free nucleons will have the form

$$G_{T-3N}(\xi_0') = -i\sqrt{\frac{3\pi}{2}}\, g_{T-3N}. \quad (5)$$

It is clear that the vertex function corresponding to the decay of the triton into a nucleon and a deuteron will be determined from an investigation of the amplitude $T(\mathbf{k}_0, \varkappa)$ of elastic nucleon-deuteron scattering, which has a pole for $E = E_0$ and $E' = E_0'$ and consequently

$$G_{T-N+d} = -\frac{i}{m}\sqrt{\frac{3\pi}{m}} \times\left[T_{\text{pole}}(\mathbf{k}_0, \varkappa')|_{\substack{E=-3\alpha_t^2/2m \\ E'=-\alpha_d^2/m}} \left(E + \frac{3}{2}\frac{\alpha_t^2}{m}\right)\left(E' + \frac{\alpha_d^2}{m}\right)\right]^{1/2}. \quad (6)$$

We note that the propagator lines in the diagrams under investigation will be determined by the expression $i/(E - E(p) + i\tau)$, where E is the total energy of the given virtual particle, determined from the law of conservation of total energy at the vertex and $E(p) = p^2/2m$, where p is the momentum of the propagation line under consideration and m the mass of the virtual particle.

3. POLE DIAGRAMS FOR FOUR-NUCLEON INTERACTIONS

Knowing the expressions for the individual blocks which make up the diagrams for four-nucleon interactions, one can derive integral equations for the scattering amplitudes corresponding to a given graphical integral equation.

We determine first the contributions from the pole diagrams B_0, B_0', B_0'', C_0, C_0', C_0'', and D_0, D_0', D_0'' (Fig. 2). We note that going over from the Feynman amplitudes to the amplitudes the squares of which determine the reaction cross section it is necessary to multiply the contributions from the diagrams by $(-i)$ and by the factor $m/2\pi$.

We consider the pole diagram corresponding to the amplitude $B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$ (diagram B_0 in Fig. 2a). It is obvious that the contribution from this diagram will be determined by the product of the three quantities corresponding to the vertex of the deuteron decay into two nucleons, the block of inelastic scattering of the deuteron with momentum $-\mathbf{k}_0$ on the nucleon with momentum $(\mathbf{k}_0 - \mathbf{k})$ and the propagation line of this virtual nucleon, so that

$$B_0(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = \frac{3}{2} imG_{d\to 2N}\, T'\left(\frac{2\mathbf{k}_0 - \mathbf{k}}{2};\ \mathbf{q}\right)\gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (7)$$

where

$$\gamma_1(\mathbf{k}_0, \mathbf{k}) = k_0^2/4 - \alpha_d^2 - k^2/2 - (\mathbf{k}_0 - \mathbf{k})^2/2.$$

It is clear that the contribution of the pole diagram $C_0(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2)$ (diagram C_0 in Fig. 2a) is determined by the product of the contributions from the decay vertex of tritium into three nucleons, of which two interact with relative momentum f_1, the contribution from the two-nucleon scattering block with relative momentum f_2 and the contribution of the propagation line of the virtual nucleon:

$$C_0(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2) = 2ima(f_2)G_{T-3N}(f_1)\gamma_2^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (8)$$

where

$$\gamma_2(\mathbf{k}_0, \mathbf{k}) = k^2/4 + f_2^2 - k_0^2/2 - (\mathbf{k} - \mathbf{k}_0)^2/2.$$

The contribution from the pole diagram $D_0(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3)$ (D_0 in Fig. 2a) will be determined by an integral over the product of the value of the vertex for the decay of tritium into three nucleons, of which two scatter with a relative momentum ξ' (which is the integration variable) the value of the block describing three-nucleon scattering and the value of the block describing the scattering of virtual nucleons with relative momentum ξ'. Consequently we will have

$$D_0(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3im}{4\pi^3}\int G_{T\to 3N}(\xi')\, T''\left(\frac{2\mathbf{k}_0 - \mathbf{k}}{2};\ \mathbf{q}\right) a^{-1}(\xi')\,\xi'^2 d\xi'. \quad (9)$$

Carrying out similar calculations, one can easily obtain the pole diagram contributions to the four nucleon interaction amplitude (Figs. 2b, c), which will have the form

$$B_0'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}) = \frac{3im}{3} G_{d\to 2N}\, T'\left(\frac{2\mathbf{k}_0 - \mathbf{k}}{2};\ \mathbf{q}\right)\gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (10)$$

$$C_0'(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2) = 2imG_{T\to 3N}\, a(f_2)\gamma_2^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (11)$$

$$D_0'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}) = -\frac{3im}{16\pi^3}\int G_{T\to 3N}(\xi')\, T'\left(\frac{2\mathbf{k}_0 - \mathbf{k}}{2};\ \mathbf{q}\right) a^{-1}(\xi')\,\xi'^2 d\xi', \quad (12)$$

$$B_0''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_t) = \frac{m^2}{2\pi} G_{d\to 2N} G_{T\to d+N} \gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (13)$$

$$C_0''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_d) = \frac{m^2}{2\pi} G_{d\to 2N} G_{T\to d+N} \gamma_3^{-1}(\mathbf{k}_0, \mathbf{k}), \quad (14)$$

$$D_0''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_t) = -\frac{m^2}{16\pi^4} \int G^2_{T\to 3N}(\xi') a^{-1}(\xi') \xi'^2 d\xi', \quad (15)$$

where

$$\gamma_3^{-1}(\mathbf{k}_0, \mathbf{k}) = k^2/4 - \alpha_d^2 - k_0/2 - (\mathbf{k} - \mathbf{k}_0)^2/2.$$

4. INTEGRAL EQUATIONS FOR THE AMPLITUDES OF FOUR NUCLEON INTERACTIONS

We have obtained expressions for the pole diagrams in the preceding section. In the present section we will write out the integral terms of the equations and will thus accomplish the construction of these equations.

The integral equations are written down in the following way. First the simplest diagrams a_1, b_1, b_2^1, c_1, d_1^1, d_1^2 are computed, diagrams which follow after the pole diagrams in the infinite series of gradually more complicated diagrams in Fig. 1. From the figure it is clear that all other diagrams, except the pole diagrams (b_0, c_0, d_0) contain the same elements as the six selected diagrams. Indeed, a_2 contains the same elements b and c as a_1; b_2^2 contain the same elements b and d as b_1; c_2^1 contains the same elements d and c as c_1, etc. After writing the expressions for the diagrams selected above we effect the summation over the infinite series of diagrams by replacing the first elements in a_1, b_1, b_2^1, c_1, d_1^1 and d_1^2 by the exact amplitudes (replacing b_0 in a_1 and b_1^1 by B, a_1 in b_2^1 by A, d_0 in c_1 and d_1^2 by D, c_0 in d_1^1 by C) (cf. Fig. 2).

We investigate first the group of reactions with the amplitudes $A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$ and $B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$ in the initial state of which there are two deuterons.

The contribution of the box diagram a_1 is determined by an integral over the momentum p and the energy ϵ of the intermediate particles. In [2] it has been shows that the computation of the integral over the energy reduces to the calculation of the residue of the integrand at the point $\epsilon = p^2/2m$.

Consequently, the contribution from the diagram a_1 has the form

$$A_1(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2) = 4\pi a(f_1) \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{B_0(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_3)}{\gamma_4(\mathbf{k}, \mathbf{p})}, \quad (16)$$

where

$$\gamma_4(\mathbf{k}, \mathbf{p}) = k^2/4 + f_1^2 - p^2/2 - (\mathbf{k} - \mathbf{p})^2/2,$$
$$\mathbf{q}' = (2\mathbf{k} - \mathbf{p})/2.$$

Since in the graphical equation for $A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$ the pole term is absent, the replacement in the expression (16) of $B_0(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_3)$ by $B(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_3)$ leads directly to the analytic form of the equation for the required amplitude:

$$A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2) = 4\pi a(f_1) \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{B(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_3)}{\gamma_4(\mathbf{k}, \mathbf{p})}. \quad (17)$$

The form of the integral terms in the equation for $B(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}, f_3)$ is determined from an analysis of the diagrams b_1 and b_2^1.

It is obvious that the contribution from the first diagram (b_1 in Fig. 1) is determined by an integral over the energy ϵ and over the momentum p of the virtual nucleon, and also over the momentum f′ of relative motion of the pair of nucleons, which are emitted from the block describing the inelastic scattering of the virtual nucleon on the initial deuteron. The contribution from the second diagram (b_2^1 in Fig. 1) is determined by a multiple integral over the energies ϵ_1, ϵ_2 and momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ of the intermediate particles in the two closed loops in this diagram, and also over the momentum f_2' of relative motion of the two nucleons emitted from the block of inelastic scattering of the virtual nucleon on the initial deuteron. The value of the relative momentum f_1' of the two virtual nucleons having momenta $\mathbf{p}_1$ and $(\mathbf{k} - \mathbf{p}_2 - \mathbf{p}_1)$ respectively is determined from the conservation of total energy for given values of $\mathbf{p}_1$, $\mathbf{p}_2$, and f_2'.

The integrals over $d\epsilon$, $d\epsilon_1$, and $d\epsilon_2$ are calculated via the residue of the integrands for the contributions from the diagram b_1 in Fig. 1 in the point $\epsilon = p^2/2m$, and for the contribution of the diagram b_2^1 in Fig. 1 in the points $\epsilon_1 = p_1^2/2m$ and $\epsilon_2 = p_2^2/2m$.

Then the values of the contributions from these diagrams are determined only by integrals over the momenta p and f′ in the first case, and over the momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and f_2' in the second case:

$$B_2^{(1)}(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2} \int \frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T''((\mathbf{k} - 2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times A_0(\mathbf{k}_0, \alpha_d; \mathbf{p} - \mathbf{k}, f_1', f_2'), \quad (18)$$

$$B_1(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2} \int \frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T''((\mathbf{k} - 2\mathbf{p})/2; \mathbf{q})}{a(f')} \times B_0\left(\mathbf{k}_0, \alpha_d; \mathbf{p}, \frac{\mathbf{p} - 2\mathbf{k}}{2}, f'\right). \quad (19)$$

It follows therefore that the integral terms in the equation for the amplitude we are looking for have the form (after substituting in (18) and (19) for

$A_0(\mathbf{k}_0, \alpha_d; \mathbf{p}-\mathbf{k}, \mathbf{f}_1', \mathbf{f}_2')$ and $B_0(\mathbf{k}_0, \alpha_d; \mathbf{p}, (\mathbf{p}-2\mathbf{k})/2, \mathbf{f}')$ the exact values of these amplitudes):

$$B^{(1)}_{\text{int}}(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times A(\mathbf{k}_0, \alpha_d; \mathbf{p}-\mathbf{k}, f_1', f_2'), \tag{20}$$

$$B^{(2)}_{\text{int}}(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times B\left(\mathbf{k}_0, \alpha_d; p, \frac{\mathbf{p}-\mathbf{k}}{2}, f'\right), \tag{21}$$

and the equation itself can be written in the form

$$B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = B_0(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) + B^{(1)}_{\text{int}}(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) + B^{(2)}_{\text{int}}(\mathbf{k}_0, \alpha_d, \mathbf{k}, \mathbf{q}, f_3). \tag{22}$$

Thus we obtain the following system of integral equations for the amplitudes of four-nucleon interactions, in the initial state of which there are two deuterons:

$$A(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2) = 4\pi a(f_1)\int\frac{d\mathbf{p}}{(2\pi)^3} B(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_3)\gamma_4^{-1}(\mathbf{p}, \mathbf{k}),$$

$$B(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3) = \frac{3im}{2}G_{d\to 2N}T'\left(\frac{2\mathbf{k}_0-\mathbf{k}}{2}, \mathbf{q}\right)\gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}) - \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times A(\mathbf{k}_0, \alpha_d; \mathbf{p}-\mathbf{k}, f_1', f_2') - \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times B\left(\mathbf{k}_0, \alpha_d; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right). \tag{23}$$

In an analogous manner one can derive the system of integral equations for the amplitudes of the given group of reactions when in the final state there is one deuteron and two nucleons ($A'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{f}_2)$ and $B'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q})$) or when nucleons bound into nuclei are formed in the reaction ($A''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_d)$ and $B''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_t)$). These equations are:

$$A'(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_2) = -im\, G_{d-2N}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{B(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}', f_2)}{\gamma_5(\mathbf{p}, \mathbf{k})},$$

$$B'(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}) = i\frac{3}{2}mG_{d-2N}T\left(\frac{2\mathbf{k}_0-\mathbf{k}}{2}; \mathbf{q}\right)\gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}) - \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T'((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times B\left(\mathbf{k}_0, \alpha_d; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right) - \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T'((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} A(\mathbf{k}_0, \alpha_d; \mathbf{p}-\mathbf{k}, f_1', f_2'), \tag{24}$$

where

$$\gamma_5(\mathbf{p}, \mathbf{k}) = k^2/4 - \alpha_d^2 - p^2/2 - (\mathbf{k}-\mathbf{p})^2/2;$$

$$A''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_d) = -im\, G_{d-2N}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{B'(\mathbf{k}_0, \alpha_d; \mathbf{p}, \mathbf{q}')}{\gamma_5(\mathbf{p}, \mathbf{k})},$$

$$B''(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_t) = \frac{m^2}{2\pi}G_{d-2N}G_{T-d+N}\gamma_1^{-1}(\mathbf{k}_0, \mathbf{k}) + \frac{im^2}{8\pi^3}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{G_{T-3N}(f')}{a(f')} B\left(\mathbf{k}_0, \alpha_d; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right) + \frac{im^2}{8\pi^3}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{G_{T-3N}(f_1')}{a(f_1')} A(\mathbf{k}_0, \alpha_d; \mathbf{p}, f_1', f_2'). \tag{25}$$

We go over now to the derivation of integral equations for the amplitudes of the second group of reactions, where the initial state contains a triton and a nucleon.

As in the preceding case we give the details of the derivation of integral equations for the amplitudes $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{f}_1, \mathbf{f}_2)$ and $D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, \mathbf{f}_3)$ of the reactions in which the final state consists of four interacting nucleons which are not bound into nuclei.

The value of the integral term for the amplitude $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{f}_1, \mathbf{f}_2)$ can be obtained from an analysis of the contribution from diagram c_1 in Fig. 1, which is determined by an integral over the momentum p and the energy ϵ of the intermediate particle and over the momentum $\mathbf{f}'$ of relative motion of the pair of virtual nucleons (the double vertical line in diagram c_1), which are formed in the decay of the triton into three free nucleons.

After performing the integration over ϵ the contribution from diagram c_1 can be written in the form

$$G_1(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2) = 4\pi a(f_1)\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{D_0(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', f_2)}{\gamma_4(\mathbf{p}, \mathbf{k})}. \tag{26}$$

Obviously, substituting in Eq. (26) $D(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', \mathbf{f}_2)$ for $D_0(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', \mathbf{f}_2)$ one can obtain the integral term in the equation for the amplitude $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{f}_1, \mathbf{f}_2)$ in the form

$$C_{\text{int}}(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2) = 4\pi a(f_1)\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{D(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', f_2)}{\gamma_4(\mathbf{p}, \mathbf{k})}. \tag{27}$$

The integral terms in the equation for the amplitude $D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, \mathbf{f}_3)$ are obtained from an analysis of the contributions of the diagrams d_1^1 and d_1^2 in Fig. 1. Carrying out a computation of the contributions of these diagrams, followed by a substitution in these expressions of $C(\mathbf{k}_0, \alpha_t; \mathbf{k}-\mathbf{p}, \mathbf{f}_1, \mathbf{f}_2)$ and $D(\mathbf{k}_0, \alpha_t; \mathbf{p}, (\mathbf{p}-2\mathbf{k})/2, \mathbf{f}')$ for $C_0(\mathbf{k}_0, \alpha_t; \mathbf{k}-\mathbf{p}, \mathbf{f}_1, \mathbf{f}_2)$ and $D_0(\mathbf{k}_0, \alpha_t; \mathbf{p}, (\mathbf{p}-2\mathbf{k})/2, \mathbf{f}')$, respectively, we obtain the integral terms in the form

$$D^{(1)}_{\text{int}}(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times C(\mathbf{k}_0, \alpha_t; \mathbf{p}-\mathbf{k}, f_1', f_2'); \tag{28}$$

$$D^{(2)}_{\text{int}}(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3) = -\frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times D\left(\mathbf{k}_0, \alpha_t; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right). \tag{29}$$

In the expression (28) $\mathbf{f}_1'$ is the value of the relative momentum of two mutually scattering nucleons (a virtual nucleon with momentum $\mathbf{k}_0 - \mathbf{p}$ and a real one with momentum $-\mathbf{k}_0$) and $\mathbf{f}_2'$ is the value of the relative momentum of two scattering nucleons formed in the decay of the triton. In the expression (29) $\mathbf{f}'$ is the value of the relative momentum of two scattering nucleons emitted from the block of three-nucleon scattering.

Thus taking into account (28) and (29), one can write down the following system of integral equations for the amplitudes $C(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{f}_1, \mathbf{f}_2)$ and $D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, \mathbf{f}_3)$ for the processes in which the initial state consists of a triton and a nucleon

$$C(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2) = 2im\, G_{T\to 3N}(f_1)\, a(f_2)\, \gamma_2^{-1}(\mathbf{k}_0, \mathbf{k}) + 4\pi a(f_1)\int\frac{d\mathbf{p}}{(2\pi)^3} D(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', f_3)\,\gamma_4^{-1}(\mathbf{p}, \mathbf{k});$$

$$\begin{aligned} D(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3) &= -\frac{3im}{4\pi^3}\int G_{T\to 3N}(\xi')\, T''\left(\frac{2\mathbf{k}_0-\mathbf{k}}{2}; \mathbf{q}\right) a^{-1}(\xi')\,\xi'^2 d\xi' \\ &- \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times C(\mathbf{k}_0, \alpha_t; \mathbf{p}-\mathbf{k}, f_1', f_2') \\ &- \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T''((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times D\left(\mathbf{k}_0, \alpha_t; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right). \end{aligned} \tag{30}$$

It is easy to show that the corresponding systems of equations for the amplitudes $C'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{f}_2)$, $D'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q})$, $C''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_d)$ and $D''(\mathbf{k}_0, \alpha_t, \mathbf{k}, \alpha_t)$ (cf. Fig. 2b and c) have the forms

$$C'(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2) = 2im\, G_{T\to N+d}\, a(f_2)\,\gamma_2^{-1}(\mathbf{k}_0, \mathbf{k}) - im\, G_{d\to 2N}\int\frac{d\mathbf{p}}{(2\pi)^3} D(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}', f_3)\,\gamma_5^{-1}(\mathbf{p}, \mathbf{k}), \tag{31}$$

$$\begin{aligned} D'(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}) &= -\frac{3im}{16\pi^3}\int G_{T\to 3N}(\xi')\, T'\left(\frac{2\mathbf{k}_0-\mathbf{k}}{2}; \mathbf{q}\right) a^{-1}(\xi')\,\xi'^2 d\xi' \\ &- \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{T'((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f_1')} \times C(\mathbf{k}_0, \alpha_t; \mathbf{p}-\mathbf{k}, f_1', f_2') \\ &- \frac{3m}{8\pi^2}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{T'((\mathbf{k}-2\mathbf{p})/2; \mathbf{q})}{a(f')} \times D\left(\mathbf{k}_0, \alpha_t; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right); \end{aligned}$$

$$C''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_d) = \frac{m^2}{2\pi} G_{d\to 2N}\, G_{T\to N+d}\,\gamma_3^{-1}(\mathbf{k}_0, \mathbf{k}) - im\, G_{d\to 2N}\int\frac{d\mathbf{p}}{(2\pi)^3} D'(\mathbf{k}_0, \alpha_t; \mathbf{p}, \mathbf{q}')\,\gamma_5^{-1}(\mathbf{p}, \mathbf{k}),$$

$$\begin{aligned} D''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_t) &= -\frac{m^2}{16\pi^4}\int G^2_{T\to 3N}(\xi')\, a^{-1}(\xi')\,\xi'^2 d\xi' \\ &+ \frac{im}{8\pi^3}\int\frac{d\mathbf{p}}{(2\pi)^3} f_2'^2 df_2' \frac{G_{T\to 3N}(f_1')}{a(f_1')} \times C(\mathbf{k}_0, \alpha_t; \mathbf{p}-\mathbf{k}, f_1', f_2') \\ &+ \frac{im^2}{8\pi^3}\int\frac{d\mathbf{p}}{(2\pi)^3} f'^2 df' \frac{G_{T\to 3N}(f')}{a(f')} \times D\left(\mathbf{k}_0, \alpha_t; \mathbf{p}, \frac{\mathbf{p}-2\mathbf{k}}{2}, f'\right). \end{aligned} \tag{32}$$

5. CONCLUSION

In the preceding sections integral equations have been obtained for the interaction amplitudes of four identical particles without spin for the case $kr_0 < 1$. It is important to note that since the particles are identical it is necessary to symmetrize the amplitudes with respect to all particles that participate in the reaction.

For comparison with experimental data on nucleon scattering on tritons or on He^3, or on deuteron-deuteron scattering, and also in order to obtain the wave function of the bound state of four nucleons, it is necessary to write down the equations which were obtained for real particles, taking into account the spin and isospin variables.

As has been shown before [2,3], within the framework of the diagram method spin and isospin can be taken into account by associating with each interaction block of two or three nucleons an operator, so that the contribution of each diagram will also be an operator. The value of this contribution will be given by the matrix element of the given operator between the initial and final functions of the spin and isospin variables of the reaction under consideration.

The integral equations for the four nucleon interaction amplitudes which have been obtained can be simplified if they are represented as a system of equations for partial wave amplitudes corresponding to a given angular momentum. In this case the integral equations for the partial wave amplitudes $A_l(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_1, f_2)$, $A'_l(\mathbf{k}_0, \alpha_d; \mathbf{k}, f_2)$ and $A''_l(\mathbf{k}_0, \alpha_d; \mathbf{k}, \alpha_d)$ and also the amplitudes $C_l(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_1, f_2)$, $C'_l(\mathbf{k}_0, \alpha_t; \mathbf{k}, f_2)$ and $C''_l(\mathbf{k}_0, \alpha_t; \mathbf{k}_0, \alpha_d)$ will depend on two variables and the equations for the amplitudes $B_l(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q}, f_3)$, $B'_l(\mathbf{k}_0, \alpha_d; \mathbf{k}, \mathbf{q})$ and $B''_l(\mathbf{k}_0, \alpha_d, \mathbf{k}, \alpha_t)$ and the amplitudes $D_l(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q}, f_3)$, $D'_l(\mathbf{k}_0, \alpha_t; \mathbf{k}, \mathbf{q})$ and $D''(\mathbf{k}_0, \alpha_t; \mathbf{k}, \alpha_t)$ will depend on three variables.

Since the kernels of the integral equations for four nucleon interaction amplitudes have the same structure as the kernel of the equation for three-nucleon interaction amplitudes, the indicated simplification of the equations can be carried out by means of the method proposed by Skornyakov and Ter-Martirosyan for the solution of the equations for the three nucleon interaction amplitude[5].

In the future we propose to obtain integral equations for four-nucleon interaction for concrete cases of reactions.

It must be pointed out that the diagram method can also be applied to the investigation of the problem of five-nucleon interactions. As in the four-nucleon case, for five nucleons one obtains four mutually coupled integral equations, the concrete form of which is being presently investigated by us.

By means of the proposed method one can obtain integral equations for the interaction amplitudes of a larger number of nucleons, which will open the possibility to construct wave functions for light nuclei and thus to create a model-free theory of light nuclei.

In conclusion the authors consider it a pleasant obligation to express their sincere gratitude to K. A. Ter-Martirosyan for proposing the problem and discussing this work.

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