QUANTUM OSCILLATIONS OF THE QUASISTATIC CONDUCTIVITY OF BISMUTH IN A MAGNETIC FIELD

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Submitted to JETP editor, January 23, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 2035-2041 (June, 1964)

Quantum oscillations of the Shubnikov-de Haas type were investigated in bismuth single crystals at 1.6°K in 5 Mc fields up to 12.5 kG. The anisotropy of the extremal cross sections of the hole and electron Fermi surfaces was investigated in detail. A study of the hole oscillations showed the dependence of their period on the magnetic field intensity and a sharp reduction of the oscillation amplitude for certain directions of the magnetic field.

PRELIMINARY results of a study of the quantum oscillations of the quasistatic conductivity of bismuth in a magnetic field, investigated by means of a sensitive modulation method, were reported earlier.[1] The present paper gives a detailed description of a further study of the anisotropy of the extremal cross sections and of certain features of the oscillations both for the electron and hole parts of the Fermi surface of bismuth.

EXPERIMENTAL DETAILS AND SAMPLES

The method of observing the quantum oscillations and the experimental technique were described in the author's earlier papers.[1,2] All the measurements were carried out at a sample temperature of 1.6°K in 5 Mc fields up to 12.5 kG.

The bismuth single crystals, prepared from material whose resistivity ratio was $\rho_{300°K}/\rho_{4.2°K} \approx 200$, were kindly supplied by M. S. Khaĭkin and V. S. Édel'man. All the samples were in the form of disks 17.8 mm in diameter and 0.5—2 mm thick. The following notation is used for the axes in a bismuth crystal: the trigonal axis - c_3, the binary axis - c_2, and the axis perpendicular to the binary and trigonal axes - c_1.

Samples having the following orientations were investigated: 1) the c_3 axis perpendicular to the flat surface of the sample, i.e., to the (c_1, c_2) plane; 2) the c_2 axis perpendicular to the flat surface of the sample, i.e., to the (c_1, c_3) plane; 3) the c_1 axis perpendicular to the flat surface of the sample, i.e., to the (c_2, c_3) plane.

The magnetic field orientation was varied by rotating the cryostat, which was adjusted so that the field was always parallel to the plane of the sample.

As reported earlier,[2] the relative amplitude of the oscillations corresponding to the different portions of the Fermi surface changes with the direction of the linear high-frequency currents in the sample (the magnetic field orientation is fixed). We can thus obtain curves which are easy to interpret. The same approach was followed in the present work.

EXPERIMENTAL RESULTS

a) Anisotropy of the Extremal Cross Sections of the Fermi Surface of Bismuth

Holes. The complete pattern of the anisotropy of the extremal cross sections was obtained for the hole surface of bismuth in the whole range of nonequivalent magnetic field directions in the planes (c_1, c_3) and (c_2, c_3) (Figs. 1 and 2). In the (c_1, c_2) plane the extremal cross sections were measured near the directions of c_1 and c_2 (Fig. 3). The experimental points for the hole surface obtained for the (c_1, c_3) and (c_2, c_3) planes are plotted in the same diagram (Fig. 1). Both systems of points fit an ellipse with semiaxis ratio 3.12 within an experimental error of 5% for the magnetic field directions near the basal plane, and 1% in the trigonal axis region. Thus, within the experimental error, the hole surface of bismuth is an ellipsoid of revolution.

Electrons. The anisotropy of the extremal cross sections for the electron part of the Fermi surface (Figs. 1-3) is in agreement with the three-ellipsoid Jones-Shoenberg model. The accuracy of measurements near the minimum cross section was 1 and 3—5% for those extremal cross sections up to which the oscillations could be observed.

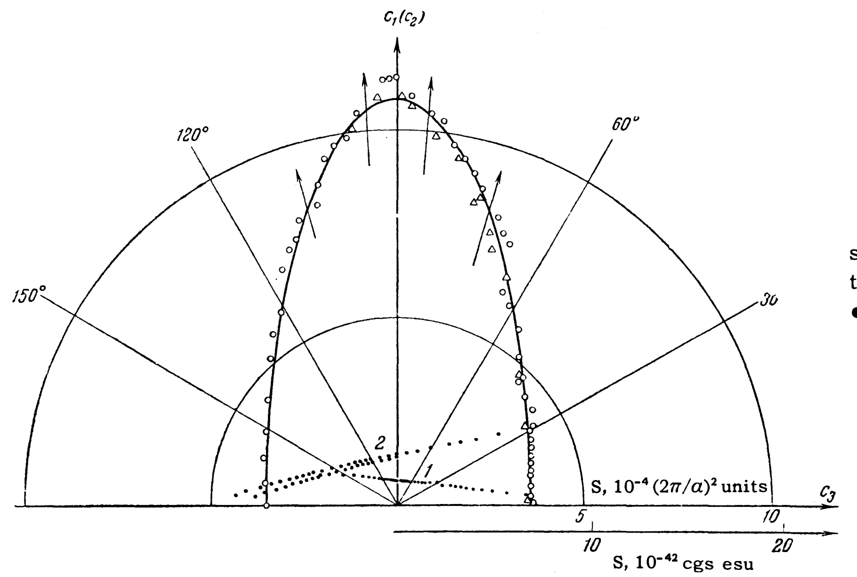

FIG. 1. Anisotropy of the extremal cross sections of the Fermi surface of bismuth obtained for the (c_1, c_3) plane: o – holes, • – electrons, Δ – holes in the (c_2, c_3) plane.

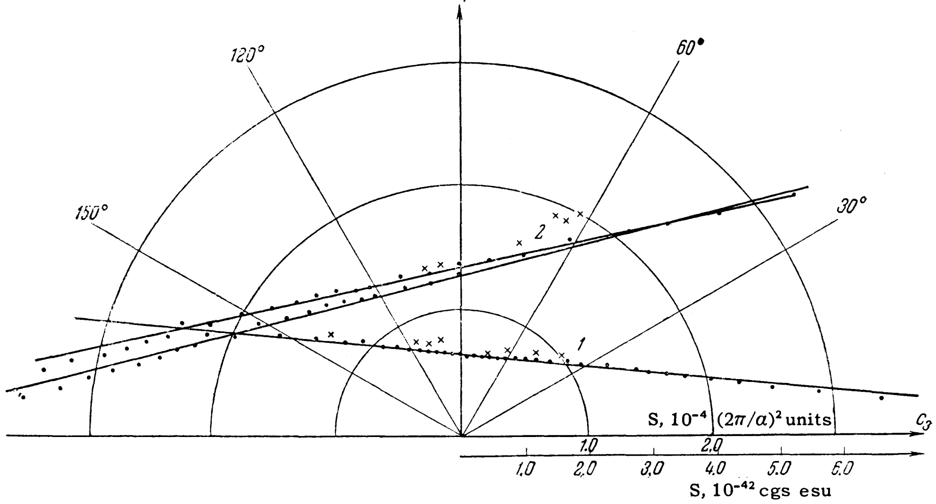

FIG. 2. Anisotropy of the extremal cross sections of the electron Fermi surface of bismuth obtained for the (c_1, c_3) plane; it shows part of Fig. 1 on a larger scale. Crosses represent the results of Brandt et al.[5]

The limits of observation in the (c_1, c_3) plane were set not only by the sensitivity of the apparatus but also by the ability to distinguish less intense oscillations against the background of strong ones.

Over a wide range of angles the experimental points for all the planes fit straight lines within the limits referred to above (Figs. 1–3). This means that the electron surfaces are markedly elongated in one direction and in their middle sections differ little from cylinders. The slope of the straight line for branch 1 in Figs. 1 and 2 gives the angle made by the axis of such a cylinder with the basal plane; this angle is 6° ± 30′. For branch 2 in Figs. 1 and 2, the angle of the slope, as expected, is twice as large as that for branch 1. The splitting of branch 2 in Figs. 1 and 2 has occurred because of a ≈ 1° error in the sample orientation. The distinctness of the splitting of branch 2 is a measure of the accuracy of the results.

It should be noted that all the results in Figs. 1–3 are in good agreement with the results of measurements, by various methods, of the extremal cross sections of the Fermi surface of bismuth published in the last few years,[3-5] but they are more accurate. For comparison, Fig. 2 shows the corresponding experimental points taken from the paper of Brandt et al.[5]

In spite of this, we cannot with certainty conclude from our results that the electron Fermi surfaces are indeed ellipsoids. According to the Abrikosov-Fal'kovskiĭ theory,[6] the greatest departures from the ellipsoidal shape should be observed, if at all, near the vertices of the electron "ellipsoids". In the present work, we have been unable to observe oscillations sufficiently far for this purpose and the accuracy of the published

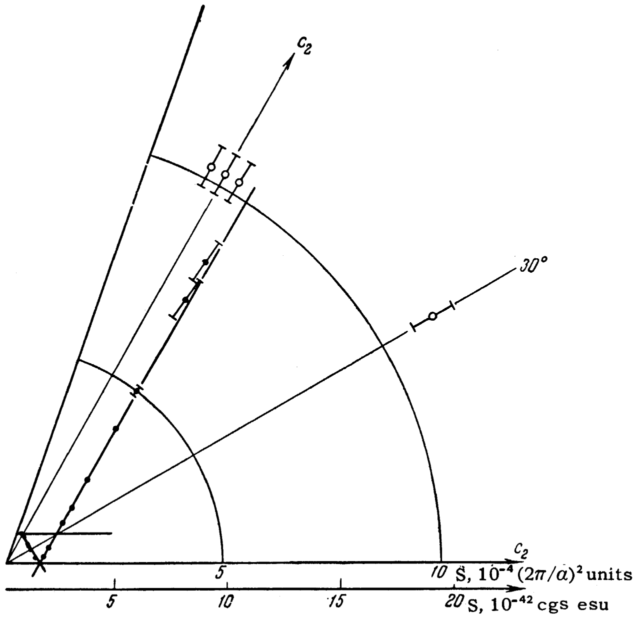

FIG. 3. Anisotropy of the extremal cross sections of the Fermi surface of bismuth in the (c_1, c_2) plane: o – holes, • – electrons. The errors of measurements are shown for the large cross sections. For the remaining points, the error is represented by the dimension of the point.

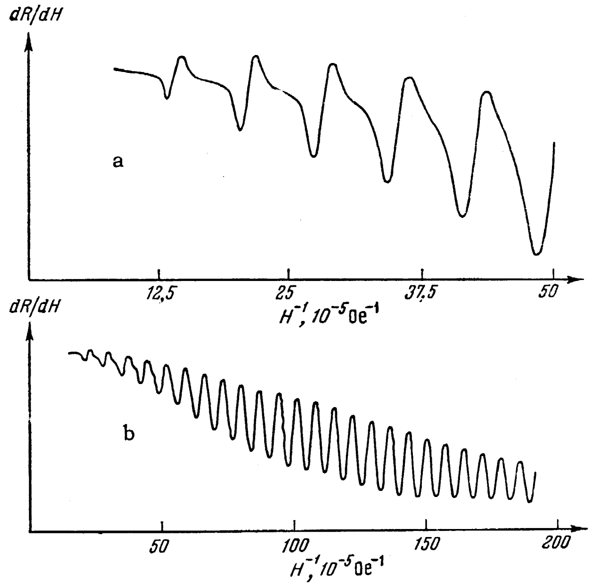

FIG. 4. Change of the form of the oscillations on increase of the magnetic field intensity. Both curves have been recorded for the same direction of the magnetic field (**H** ∥ c_2). In recording the upper curve, the scale has been extended along the axis of the reciprocal of the magnetic field and the amplification has been increased.

results [3,5] is insufficient to speak of any deviations of the electron surface from the ellipsoidal shape.

b) Form and Phase of Oscillations

The change in the form of the oscillations on increase of the magnetic field intensity is depicted in Fig. 4, which shows an oscillation record for electrons when H ∥ c_2. The record gives a quantity proportional to the derivative dR/dH,[1)] where R is an equivalent resistance introduced by the sample into the oscillator circuit. The form of the curve obtained by integration of the experimental record shown in Fig. 4 changes in such a way that with increase of the oscillation number the minima become narrower and sharper and the maxima broaden and become flatter. This change of form is observed for all directions of the field on reaching the sixth or seventh oscillation (including the oscillations associated with the hole surface when H ∥ c_3). The form of the oscillations in strong fields makes it easier to determine the phase of the oscillations because the narrow peaks in the integrated curve represent the points at which the number of levels occupied by electrons changes by unity. If the values of the reciprocal of the magnetic field, corresponding to the peaks of the integrated curve, are plotted as a function of the number of the oscillation, all the points lie on a straight line which passes through the origin of coordinates; the fit is within 5% of the oscillation period. This means that the spin splitting of the Landau levels in the magnetic field is approximately equal to the separation between these levels.[7]

This change in the form is already noticeable at the sixth or seventh oscillation for holes (H ∥ c_3 or close to this direction), but the peaks of the integrated curve are separated by a half-integral number of periods from the zero value of the reciprocal of the magnetic field.

c) Dependence of the Hole Oscillation Period on the Magnetic Field

For magnetic field directions making an angle > 60° with the trigonal axis in the planes (c_1, c_3) and (c_2, c_3), it was found that the oscillations of the hole Fermi surface are not a strictly periodic function of the reciprocal of the magnetic field. The nature of the deviations from the periodicity may be illustrated clearly by plotting successive maxima and minima $(H^{-1})_n$ of the oscillations associated with the hole surface as a function of the number n, and then calculating the average slope from these points. The result of applying this treatment to two experimental records is shown in Fig. 5. The origin of the plot is arbi-

1)It is possible that the differentiation is the reason for the oscillation amplitude decrease in strong fields (Fig. 4), since $df(H^{-1})/dH = H^{-2}f'(H^{-1})$.

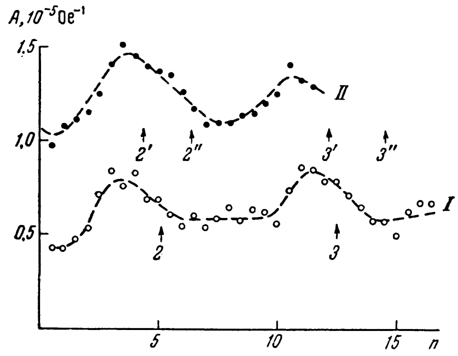

FIG. 5. Shifts of the successive maxima and minima of the hole oscillations, represented by the reciprocal magnetic field, from the positions given by the average period in the investigated range of fields: I) magnetic field in the (c_1, c_3) plane, $\psi = 59°$; II) magnetic field in the (c_2, c_3) plane, $\psi = 64.5°$ (ψ' is the angle between the magnetic field and the trigonal axis). $A = H_n^{-1} - n\Delta H^{-1}$, where ΔH^{-1} is the average period.

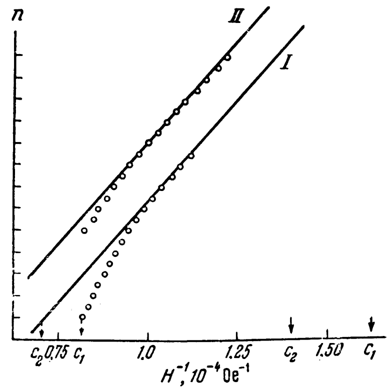

FIG. 6. The reciprocal magnetic field for the successive maxima and minima of the hole oscillations as a function of the oscillation number: I) H || c_1; II) H || c_2. The arrows show the positions of the first and second peaks of the electron oscillations with the longest period.

trary for both axes. The arrows in Fig. 5 show the relative positions of the second and third peaks of the electron oscillations (the integrated curve). The dependence of the oscillation period on the magnetic field intensity is given by the derivative of the curves which join the points in Fig. 5.

An inspection of Fig. 5 shows that the period of the hole oscillations is modulated by the long-period components of the electron oscillations. Similar modulation has been observed in studies of the Shubnikov-de Haas oscillations [8] and the de Haas-van Alphen oscillations.[5] Granier et al.[8] explained this phenomenon by the leakage of electrons from one band to another, which occurs in a magnetic field when the lower Landau levels are expelled from one or two electron regions. Increase or decrease of the hole volume is the reason for the change in the oscillation period. The equality of the numbers of electrons and holes should naturally be retained.

The greatest change in the period of the hole oscillations - by a factor greater than 1.5 - is observed for H || c_1 (Fig. 6) since for this direction of the magnetic field it is easier to approach the first peak of oscillations due to one of the electron ellipsoids. The periodicity of the electron oscillations is not affected up to the second oscillation, and, according to Kunzler et al., [7] the first oscillations are shifted very little. The strong dependence of the period of the hole oscillations on the magnetic field accompanied by the almost exact periodicity of the electron oscillations seems surprising.

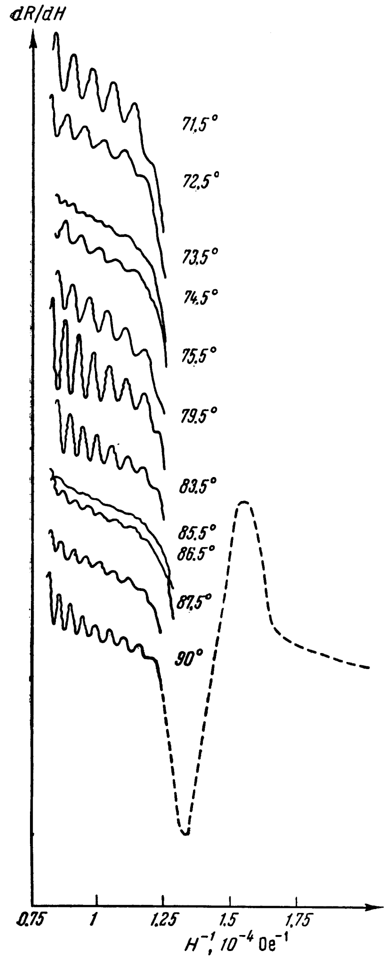

FIG. 7. Records of the hole oscillations in the (c_2, c_3) plane. The numerals to the right of these curves give the directions of the magnetic field with respect to the c_3-axis. The positions of the missing oscillations are clearly visible. The same results are obtained for the (c_1, c_3) plane. The dashed curve shows the second electron oscillation.

d) Missing Hole Oscillations for Certain Directions of the Magnetic Field.

The monotonic reduction of the hole oscillation amplitude as the angle between the field and the axis increases is interrupted at the angles of 73, 85.5, 94.5, and 107° in the (c_1, c_3) and (c_2, c_3) planes. These directions are shown in Fig. 1 by arrows. The amplitude of the oscillations drops sharply in an angular interval of $\simeq 1°$ near these directions[2)] and at 73° an apparent doubling of the period is observed. Figure 7 contains several experimental records of the oscillations in the (c_2, c_3) plane, which show clearly the nature of the effect. Singularities of the same type have been observed in oscillations of the magnetic susceptibility [5] both for holes (for directions represented by the angles $\psi = 72$ and 108° only) and electrons.

It is possible that this effect is related to the g-factor anisotropy, as mentioned in the work of Brandt et al.[5] When spin splitting becomes half the separation between the Landau levels, the first harmonic of the oscillations should disappear and the period should be doubled. However, the existence of two magnetic field directions for which the effect is observed in the case of holes indicates that an accurate calculation is needed.

2)This effect is independent of the direction of high-frequency currents in the sample.

Thus quantum oscillations in bismuth have interesting features, the study of which in stronger fields and at lower temperatures may yield additional information on the electron spectrum of this metal.

In conclusion, the author thanks M. S. Khaĭkin and V. S. Édel'man for the bismuth samples and for discussing the results, and also A. A. Abrikosov, L. A. Fal'kovskiĭ, and R. T. Mina for valuable discussions.

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Translated by A. Tybulewicz
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