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GAMMA AND X-RADIATION CONNECTED WITH GALACTIC AND METAGALACTIC COSMIC RAYS

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We calculate the intensity of γ-rays produced in various interaction processes of cosmic rays (including their electron component) with the gas and thermal emission in interstellar and intergalactic space. The major contribution to the γ-ray intensity is made by the scatter of relativistic electrons on thermal photons, although the γ-rays from π^0-meson decay and electron bremsstrahlung may also be important. A comparison of the calculations with the observations allows us to consider that the intensity of the cosmic ray electron component in the Metagalaxy is at least one and a half to two orders less than in the Galaxy. In addition, we discuss the high-energy electron range which could produce magnetic bremsstrahlung x-radiation and cause the atmospheric showers with an anomalously small number of mesons.

THE study of cosmic gamma and x-radiation, i.e., the development of gamma and x-ray astronomy, is at present an extremely interesting and at the same time quite real problem. Cosmic x-radiation may arise in the atmospheres of stars,[1] in the interaction of sub-cosmic rays (particles with an energy of $E \lesssim 1$ to 3×10^8 eV) with the interstellar medium[2,3] and thermal emission, and lastly by the action of cosmic rays. As for the γ-rays with an energy $E_\gamma \gtrsim 5 \times 10^7$ eV (it is this range we have in mind below), they can in practice be generated only as the result of collisions of cosmic-ray particles in interstellar and intergalactic space. Thus a study of cosmic γ-radiation, and under certain conditions of x-radiation, may be an irreplaceable source of information on the cosmic rays and gas in the Universe. The corresponding analysis is promising particularly since we already have[4] a whole series of data on cosmic rays beyond the Earth, whilst there is essentially no information on sub-cosmic particles.

From the point of view of the generation of γ rays with $E_\gamma \gtrsim 5 \times 10^7$ eV the following processes are of interest:

1. The decay of π^0 mesons generated in cosmic ray collisions in the interstellar gas.
2. Bremsstrahlung of the relativistic electrons (and positrons) making up the electron component of cosmic rays. Here we may include the emission during the annihilation of positrons and during the generation of the actual electron component by $\pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$ decay.
3. Scatter of the electron component particles on thermal photons (the "inverse" Compton effect).

Processes 2 and 3 also make a contribution to the x-radiation but the major rôle is played by the soft part of the cosmic ray spectrum where these rays meet sub-cosmic particles whose spectrum is not known. Therefore if we are talking about the cosmic rays themselves (to be more precise, their electron component) only the following process is responsible for the x-radiation.

4. Magnetic bremsstrahlung x-radiation.

The question of the gamma and x-radiation connected with cosmic rays has already been discussed in a whole series of papers (see [4-11] and the references therein). However, the results are strongly dependent on the values of the parameters being used, and, largely for this reason, disagree by several orders in the various estimates. Moreover, processes 1 to 4 are well known and the focal point of the problem is in the field of actual calculations and estimates based on a definite choice of cosmic ray spectrum, thermal emission energy density, etc. The purpose of this paper is to carry out and discuss such calculations using the data that at present appear the most reliable and, partially, have been accepted or obtained by us recently.[4,8,12,13] These data could not, of course, be taken into consideration in the other papers.

1. INITIAL DATA

In the $10 \lesssim E \lesssim 10^6$ GeV energy range the intensity of all the cosmic rays can be expressed in the form (where the energy E is measured in GeV)

$$I_{cr}(E) = 1.5 \cdot E^{-2.6} \text{particles/cm}^2 \cdot \text{sec} \cdot \text{sr} \cdot \text{GeV}. \quad (1)$$

This spectrum taken in section 18 from Nikol'skii's data[97] agrees with the spectrum given by Brook et al.[15] 1) For E < 10 GeV spectrum (19.1) leads to an intensity that is somewhat too high. We shall therefore also give the approximate value we are using for the total cosmic ray intensity on Earth, which relates in practice to the particles with a total energy $E \gtrsim 2$ GeV.

$$I_{cr}(E > E_{min}) = \int_{E_{min}}^{\infty} I_{cr}(E)\,dE$$

$$= 0.23 \text{ particles/cm}^2 \cdot \text{sr} \cdot \text{sec}. \quad (2)$$

In accordance with contemporary ideas[4] we shall consider that the cosmic rays with spectrum (19.1) fill the whole Galaxy evenly including the halo, i.e., a quasi-spherical volume with a radius of 5×10^{22} cm.

The relativistic electron intensity in the Galaxy can be determined from the data on the intensity of the general Galactic radio emission. If we take the value of $\gamma_0 = 1.7$ for the electron source spectral index, then in the diffusion model with a particle departure time from the Galactic volume of $T_d = 1.5 \times 10^8$ years and a magnetic field strength of $H = 3 \times 10^{-6}$ oersteds the power necessary for the electron sources in the Galaxy is[13]

$$Q_e(E)\,dE = 8.3 \cdot 10^{43} \left(\frac{E}{mc^2}\right)^{-1.7} \frac{dE}{mc^2}$$

$$= 4.1 \cdot 10^{41} E^{-1.7} dE \text{ electrons/sec}, \quad (3)$$

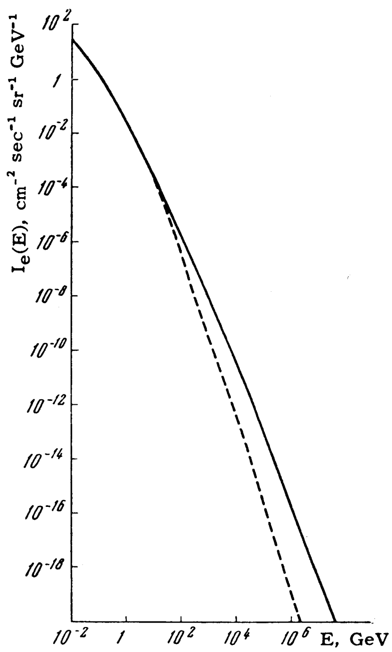
where E in the last expression is measured in GeV. The electron intensity in the vicinity of the solar system caused by these sources is shown by the solid line in the figure. In calculating this intensity it was assumed that the sources are concentrated in the Galactic disk (for further details we refer to a recent paper[13]).

In the 0.5 to 10 GeV energy range the spectrum shown in the figure can be approximated by the expression

$$I_e(E) = 2 \cdot 10^{-2} E^{-2} \text{ electrons/cm}^2 \cdot \text{sec} \cdot \text{sr} \cdot \text{GeV} \quad (4)$$

For the $0.05 < E < 0.5$ GeV energy range this formula gives a value for the intensity that is not more than one and a half or two times too high.

1)Within the limits of accuracy of the available data and calculations we may take the value of 2.7 for the exponential index in (1) instead of the 2.6 indicated.[16,17]



It must be stressed that spectra (3) and (4) lead on Earth to the intensity $I_e(E > 1 \text{ GeV}) = 2 \times 10^{-2}$ electrons/cm^2 · sec · sr, which is too high.[13] Therefore the values of (3) and (4) must probably be reduced by a factor of 3 to 4, which is quite possible with a sensible choice of parameters (we are thinking in the first place of increasing the field strength by a factor of 1.5 to 2). Nevertheless we give and use spectrum (3) and (4) since it was for this that we made our calculations.[509]

At higher energies (E > 10 GeV) the electron intensity on Earth obtained by the method indicated (on the assumption that the source spectrum (3) is valid for the whole energy range) is

$$I_e(E)\,dE = 8.9 \cdot 10^5 \frac{dE}{mc^2} \left(\frac{E}{mc^2}\right)^{-2}$$

$$\times \int_{E/mc^2}^{\infty} d\varepsilon\, \varepsilon^{-1.7} \left[1 + 8.3 \cdot 10^7 \left(\frac{mc^2}{E} - \frac{1}{\varepsilon}\right)\right]^{-1}. \quad (5)$$

At $E > 10^5$ GeV with sufficient accuracy

$$I_e(E)\,dE = 1.3 \cdot 10^6 \left(\frac{E}{mc^2}\right)^{-2.7} \frac{dE}{mc^2} = 3{,}2 E^{-2.7} dE, \quad (6)$$

where in the last expression the energy E must be taken in GeV. Because of the rapid loss of energy in magnetic bremsstrahlung the electrons with $E > 10^5$ GeV should obviously be concentrated in the region of the sources, i.e., within the Galactic disk[8]. It must be remembered, however, that there is no direct reason for continuing spectrum (3) into the E > 10 GeV energy range since the radio astronomy data relate to electrons with $E \lesssim 10$ GeV. The spectrum (5)—(6) is therefore given here only because it will be used for the preliminary estimates later on.

Table I

	L, cm	N(L), cm^{-2}	M(L), g/cm^2	w_{ph}, $\frac{eV}{cm^3}$
Galaxy				
in direction of center	$7\cdot 10^{22}$	$3\cdot 10^{22}$	$6\cdot 10^{-2}$	0,2
in direction of anticenter	$1.5\cdot 10^{22}$	$6\cdot 10^{21}$	$1.2\cdot 10^{-2}$	
in direction of pole (including contribution from the halo)	—	$3\cdot 10^{20}$	$6\cdot 10^{-4}$	
averaged directionally (including contribution from the halo)	—	$8\cdot 10^{20}$	$1.6\cdot 10^{-3}$	0.8
halo (in direction of pole)	$3.5\cdot 10^{22}$	$1.5\cdot 10^{20}$	$3\cdot 10^{-4}$	
Metagalaxy	$R_{ph} = 5\cdot 10^{27}$	$5\cdot 10^{22}$	0.1	$2\cdot 10^{-3}$

In accordance with the available data we shall below take the values given in Table I for L, N(L), M(L), where L is the distance along the line of sight in the Galaxy, N(L) is the number of gas atoms on the line of sight and $M(L) = 2 \times 10^{-24} \times N(L)$ is the mass of this gas in g/cm^2 (for the interstellar medium which contains about 90% H and about 10% He the mean mass of the atoms is 2×10^{-24} g). It should be borne in mind that the table gives rounded-off values and even for the disk N(L) may be twice as much. The concentration $n = 10^{-2}$ cm^{-3} taken for the halo is probably almost an order too high. This fact, however, is of little significance from the point of view of calculating N(L) even in the direction of the pole. In addition, Table I gives also values for the thermal emission energy density w_{ph} and data for the Metagalaxy which are also by way of a guide (see section 8 of [4] and below).

2. GALACTIC GAMMA RADIATION

The intensity of the γ-rays arising as the result of the interaction of cosmic rays with the interstellar medium is

$$I_\gamma(E_\gamma) = N(L)\int_{E_\gamma}^{\infty} \sigma(E_\gamma, E)\, I(E)\, dE, \qquad (7)$$

where $\sigma(E_\gamma, E)$ is the differential cross section for the generation of a γ-quantum with an energy E_γ from a particle with an energy E and I(E) is the intensity of the isotropic (according to the assumption) cosmic rays $I_{cr}(E)$ (process 1) or of their electron component $I_e(E)$ (processes 2 and 3; in the case of process 3 we have $N_{ph}(L)$—the number of photons on the line of sight—instead of N(L)). For the γ-rays from π^0-meson decay (process 1) we have (see [8])

$$I_{\gamma,\pi^0}(> E_\gamma) = \int_{E_\gamma}^{\infty} I(E_\gamma)\, dE_\gamma = 5.6\cdot 10^{-28} E_\gamma^{-1,8} N(L)\ (\mathrm{cm}^2\cdot\mathrm{sec}\cdot\mathrm{sr})^{-1}, \qquad (8)$$

where E_γ is measured in GeV. This formula is valid for $E_\gamma \gtrsim 1$ to 3 GeV. It is possible that it is more correct to use a value of 1.6 to 1.7 for the exponential index in (8) instead of 1.8. This fact, however, does not in essence alter the conclusions of [8]. In the low-energy range an exponential spectrum can no longer be used. In accordance with the threshold $E_\gamma = 5 \times 10^7$ eV indicated in the preceding section we give here the numerical value (see p. 27 of [4]);

$$I_{\gamma,\pi^0}(E_\gamma > 5\cdot 10^7 \text{ eV}) = 8\cdot 10^{-27} N(L). \qquad (9)$$

For the electron bremsstrahlung we can with sufficient accuracy put

$$N(L)\,\sigma(E_\gamma, E) = \frac{M(L)}{66}\frac{1}{E_\gamma}, \qquad E_\gamma \leqslant E, \qquad (10)$$

where for the interstellar medium the emission unit is taken[18] as 66 g/cm^2. In accordance with (7) and (10)

$$I_{\gamma,\mathrm{brems}}(E_\gamma) = 1{,}5\cdot 10^{-2} M(L)\, I_e(> E_\gamma)/E_\gamma, \qquad (11)$$

or for the spectrum (4)

$$I_{\gamma,\mathrm{brems}}(E_\gamma) = 3\cdot 10^{-4} M(L)\, E_\gamma^{-2},$$

$$I_{\gamma,\mathrm{brems}}(> E_\gamma) = 3\cdot 10^{-4} M(L)\, E_\gamma^{-1}. \qquad (12)$$

The values of $I_{\gamma,\pi^0}(>E_\gamma)$ and $I_{\gamma,\mathrm{brems}}(>E_\gamma)$ are compared in Table II for $E_\gamma = 1$ GeV and $E_\gamma = 4 \times 10^7$ eV.

Since the accepted electron intensity (4) should, as has been indicated, be reduced by a factor of 3 and in addition for $E_\gamma \gtrsim 5 \times 10^7$ eV the spectrum (12) is still 1.5 to 2 times too high it may be considered that the gamma bremsstrahlung is comparable with but nevertheless 2 to 3 times weaker than the γ-rays due to π^0-meson decay.

It is easy to show that the γ-radiation arising in the generation of electrons and positrons ($\pi \to \mu \to e$ decay) and the annihilation of positrons is considerably weaker than the bremsstrahlung (at $E_\gamma \gtrsim 5 \times 10^7$ eV). In fact during the generation of a particle with an energy

Table II.
Gamma Radiation Intensity I_γ ($> E_\gamma$) (in $cm^{-2} \cdot sec^{-1} \cdot sr^{-1}$)

Process	Galactic radiation				Metagalaxy	E_γ
	towards center	towards anti-center	towards pole	averaged directionally		
(a) $\pi^0 \to \gamma + \gamma$	$1.7 \cdot 10^{-5}$	$3.4 \cdot 10^{-6}$	$1.7 \cdot 10^{-7}$	$4.5 \cdot 10^{-7}$	$2.8 \cdot 10^{-5} \xi_{cr}$	1 BeV
(b) Bremsstrahlung	$1.8 \cdot 10^{-5}$	$3.6 \cdot 10^{-6}$	$1.8 \cdot 10^{-7}$	$4.8 \cdot 10^{-7}$	$3 \cdot 10^{-5} \xi_e$	
(c) Inverse Compton effect	10^{-5}	$3 \cdot 10^{-6}$	$2 \cdot 10^{-5}$	$2 \cdot 10^{-5}$	$10^{-2} \xi_e$	
(a) $\pi^0 \to \gamma + \gamma$	$2.4 \cdot 10^{-4}$	$5 \cdot 10^{-5}$	$2.4 \cdot 10^{-6}$	$6.6 \cdot 10^{-6}$	$4 \cdot 10^{-4} \xi_{cr}$	$5 \cdot 10^7$ eV
(b) Bremsstrahlung	$3.6 \cdot 10^{-4}$	$7.2 \cdot 10^{-5}$	$3.6 \cdot 10^{-6}$	$9.6 \cdot 10^{-6}$	$6 \cdot 10^{-4} \xi_e$	
(c) Inverse Compton effect	$5 \cdot 10^{-5}$	$\sim 10^{-5}$	10^{-4}	10^{-4}	$4 \cdot 10^{-2} \xi_e$	

$$E = mc^2/\sqrt{1 - v^2/c^2} \gg mc^2$$

an energy is emitted ($\alpha = e^2/\hbar c$)

$$dW_\gamma = \frac{\alpha}{\pi}\left(\frac{c}{v}\ln\frac{c+v}{c-v} - 2\right) dE_\gamma \approx \frac{2\alpha}{\pi}\left(\ln\frac{2E}{mc^2} - 1\right) dE_\gamma .$$

This classical expression (cf.[19], p. 220) for the estimate is also valid at $E_\gamma \le E$. The number of photons is dW_γ/E_γ and the γ-ray intensity is

$$I_{\gamma,\,\mathrm{gen}}(E_\gamma) = \frac{L}{4\pi}\int_{E_\gamma}^{\infty} \frac{dW_\gamma}{dE_\gamma}\frac{q_e(E)\,dE}{E_\gamma}$$

$$\approx 4 \cdot 10^{-4}\left(\ln\frac{2\bar{E}_\gamma}{mc^2} - 1\right)\frac{Q_e(E > E_\gamma)}{E_\gamma},$$

where $Q_e(E > E_\gamma)$ is the total number of particles generated per second on the line of sight (the path L). We have shown elsewhere[13] that on a path of M(L) g/cm^2 the total number of electrons generated is

$$Q(E > 0) = \frac{4\pi M(L)}{180} \cdot 0.44 \approx 3 \cdot 10^{-2} M(L).$$

Hence, for example, $I_{\gamma,\mathrm{gen}}(E_\gamma = 5 \times 10^{-2}$ GeV$) \simeq 1 \times 10^{-3}$ M(L), whilst $I_{\gamma,\mathrm{brems}}(E_\gamma = 5 \times 10^{-2}$ GeV$) \simeq 0.12$ M(L).

The annihilation cross section is

$$\sigma_{\mathrm{an}} = \pi r_e^2 \frac{mc^2}{E}\left(\ln\frac{2E}{mc^2} - 1\right),$$

$E \gg mc^2$, one photon acquiring an energy $E_\gamma \simeq E_+$. Therefore

$$I_{\gamma,\,\mathrm{an}}(E_\gamma) = N(L)\,\sigma_{\mathrm{an}} I_{e+}(E_\gamma)$$

$$\approx 0.1\, M(L)\frac{mc^2}{E_\gamma}\left\{\ln\frac{2E_\gamma}{mc^2} - 1\right\} I_{e+}(E_\gamma).$$

Here N(L) is the number of electrons, but for the interstellar medium we can use the value given above for the number of atoms so N(L) is replaced by $M(L)/2 \times 10^{-24}$. For spectrum (4)

$$I_{e+}(E) \lesssim \tfrac{1}{2} I_e(E) = 10^{-2} E^{-2}$$

and

$$I_{\gamma,\mathrm{an}}(E_\gamma) \lesssim 10^{-3}\left\{\ln\frac{2E_\gamma}{mc^2} - 1\right\} M(L)\frac{mc^2}{E_\gamma} E_\gamma^{-2}.$$

Even for $E_\gamma = 5 \times 10^{-2}$ GeV this value is 7 times less than $I_{\gamma,\mathrm{brems}}$ (see (12)).

Let us now examine process 3 (the inverse Compton effect). The importance of allowing for this when calculating the γ-ray intensity was stressed recently.[9]

Obviously

$$I_\gamma(E_\gamma) = N_{\mathrm{ph}}\int_{E_\gamma}^{\infty} \bar{\sigma}(E_\gamma, E)\, I_e(E)\, dE, \qquad (13)$$

where $N_{\mathrm{ph}}\bar{\sigma}(E_\gamma, E) = L\int n_{\mathrm{ph}}(\epsilon)\,\sigma(E_\gamma, \epsilon, E)\,d\epsilon$, $n_{\mathrm{ph}}(\epsilon)$ is the concentration of thermal photons in the energy range ϵ, $\epsilon + d\epsilon$, $N_{\mathrm{ph}} = L\int n_{\mathrm{ph}}(\epsilon)\,d\epsilon = Ln_{\mathrm{ph}}$ is the number of photons along the line of sight and $\sigma(E_\gamma, \epsilon, E)$ is the effective cross section for the generation of a γ-photon with an energy E_γ in the scatter of a photon with an energy ϵ on an electron with a total energy E; here both the thermal and the electron emission are considered to be distributed isotropically.

We shall first calculate $\sigma(E_\gamma, E)$ by an approximate method similar to that used by Felton and Morrison.[9] The energy loss of an electron with an energy E due to scatter in unit time is (for $E_\gamma \gg \epsilon$ and $E \ll (mc^2/\epsilon)\,mc^2$)

$$-\frac{dE}{dt} = cn_{\mathrm{ph}}\int E_\gamma \sigma(E_\gamma, E)\, dE_\gamma = \frac{4}{3} c w_{\mathrm{ph}} \sigma_T \left(\frac{E}{mc^2}\right)^2$$

$$= 2.67 \cdot 10^{-14}\, w_{\mathrm{ph}} (E/mc^2)^2 \text{ eV/sec}. \qquad (14)$$

Here

$$w_{\mathrm{ph}} = \int \varepsilon\, n_{\mathrm{ph}}(\varepsilon)\, d\varepsilon = n_{\mathrm{ph}}\bar{\varepsilon}, \qquad \bar{\varepsilon} = 2.7\, kT,$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = 6.65 \cdot 10^{-25} \text{cm}^2$$

and in the last expression the energy density w_{ph} is measured in eV/cm^3; formula (14) follows from

Feenberg and Primakoff's paper[20] and is also derived in the Appendix (an approximate expression was obtained[4] which is three quarters the value of (14)). To obtain formula (14) we may put [2)]

$$\sigma(E_\gamma, E) = \sigma_T\delta\left(E_\gamma - \tfrac{4}{3}\bar{\varepsilon}\,(E/mc^2)^2\right). \quad (15)$$

If we substitute (15) in (13) we obtain

$$I_\gamma(E_\gamma) = \frac{\sqrt{3}}{4}\frac{N_{ph}\sigma_T mc^2}{\sqrt{\bar{\varepsilon}E_\gamma}} I_e\left(mc^2\sqrt{\frac{3E_\gamma}{4\bar{\varepsilon}}}\right)$$

$$= \frac{N_{ph}\sigma_T}{2}(mc^2)^{1-\gamma}\left(\frac{4}{3}\bar{\varepsilon}\right)^{(\gamma-1)/2} K_e E_\gamma^{-(\gamma+1)/2}, \quad (16)$$

where in the last expression we have $I_e(E) = K_e E^{-\gamma}$ and $N_{ph} = n_{ph}L$.

We show at the end of this paper that formula (16) is obtained in a precise calculation, the factor $f(\gamma)$ appearing in addition. In this case, for example $f(2) \simeq 0.8$ and $f(3) \simeq 1.0$. By putting also $\bar{\epsilon} = 2.7\,kT \simeq 1.2$ eV ($T = 5000°$) and measuring E_γ in eV, w_{ph} in eV/cm^3 and K_e in $(\text{cm}^2\cdot\text{sec}\cdot\text{sr})^{-1}\,(\text{GeV})^{\gamma-1}$ we have

$$I_\gamma(E_\gamma) = 2.8\cdot 10^{-25}(2.5\cdot 10^3)^{\gamma-1} f(\gamma)\, Lw_{ph}\, K_e E_\gamma^{-(\gamma+1)/2}$$

photons/cm^2 · sec · sterad · eV

$$I_\gamma(>E_\gamma) = 5.6\cdot 10^{-25}(2.5\cdot 10^3)^{\gamma-1}\frac{f(\gamma)Lw_{ph}}{\gamma-1}K_e E_\gamma^{-(\gamma-1)/2}$$

photons/cm^2 · sec · sterad (17)

The loss formula (14) is valid only if $4\bar{\epsilon}E/mc^2 < mc^2$, i.e., for the energies

$$E < 5\cdot 10^{10}\ \text{eV}. \quad (18)$$

Obviously formulae (16) to (17) can be used only in region (18). If $E > 5\times 10^{10}$ eV, formulae (16) to (17) produce a value for the intensity I_γ that is too high. In the region $E \gg 5\times 10^{10}$ eV we put approximately

$$\sigma = \pi r_e^2\frac{(mc^2)^2}{\bar{\varepsilon}E}\ln\frac{2\bar{\varepsilon}E}{(mc^2)^2}$$

and consider that the energy of the scattered photon is equal to the initial energy of the electron E. Then if we proceed in the approximate way similar to that used for deriving formula (16)

$$I_\gamma(E_\gamma) = N_{ph}\pi r_e^2\frac{(mc^2)^2}{\bar{\varepsilon}}\int_{E_\gamma}^{\infty}\frac{1}{E}\ln\frac{2\bar{\varepsilon}E}{(mc^2)^2}I_e(E)\,\delta(E_\gamma - E)\,dE$$

$$= 2.5\cdot 10^{-25}\frac{(mc^2)^2}{\bar{\varepsilon}^2}Lw_{ph}E_\gamma^{-1}I_e(E_\gamma)\ln\frac{2\bar{\varepsilon}E_\gamma}{(mc^2)^2}. \quad (19)$$

[2)]In accordance with (15) the total cross section is $\int\sigma(E_\gamma, E)\,dE_\gamma = \sigma_T$, as should be the case at $E \ll (mc^2/\epsilon)mc^2$. From this and from (14) it is clear that the mean energy of a γ-photon is $\frac{3}{4}\epsilon(E/mc^2)^2$ and this is also taken into consideration in (15).

For the spectrum (4) we have from (17)

$$I_\gamma(>E_\gamma) = 2.2\cdot 10^{-23}Lw_{ph}E_\gamma^{-1/2}. \quad (20)$$

The mean energy of a γ-quantum from an electron with an energy E is

$$E_\gamma = \frac{4}{3}\bar{\varepsilon}\left(\frac{E}{mc^2}\right)^2 = 1.6\left(\frac{E}{mc^2}\right)^2\text{eV}. \quad (21)$$

Since the spectrum (4) relates to the range $0.5 \lesssim E \lesssim 10$ GeV, the spectrum (20) is applicable for the range 1 MeV $\lesssim E_\gamma \lesssim$ 600 MeV. However, even for $E_\gamma \lesssim$ 2 to 3 GeV the value of $I_\gamma(>E_\gamma)$ is only comparatively too high because of the inaccuracy of spectrum (20).

Table II gives the values of $I_\gamma(>E_\gamma)$ in accordance with (20) using the data in Table I. Rounding off is carried out here and in particular the mean value in all directions is considered to be the same as in the directions of the pole (the error here does not exceed a factor of two). The characteristic feature of the γ-radiation from the scatter of electrons on thermal photons is the approximate isotropy of the intensity, whilst the remaining processes (interaction with the gas) lead to sharp anisotropy. This, of course, was clear right from the start (see Table I). According to Kraushaar and Clark[5] the mean directional value is $I_\gamma(E_\gamma > 5\times 10^7\ \text{eV}) = (3.7\ \text{to}\ 11)\times 10^{-4}$ photons/cm^2 · sec · sterad or on the average

$$I_\gamma(E_\gamma > 5\cdot 10^7\ \text{eV})$$

$$= 5.5\cdot 10^{-4}\ \text{photons/cm}^2\cdot\text{sec}\cdot\text{sterad} \quad (22)$$

It is still not clear whether this value is the upper limit or the actual flux of γ-rays from the cosmos.[3)] In the latter case if the intensity is quasi-isotropic (according to Kraushaar and Clark[5] this is apparently the case) the corresponding γ-rays can be considered Galactic only by connecting them with process 3—scatter on thermal photons. Even in this case, however, there is a discrepancy of an order or even more since the electron spectrum (4) is, as has already been indicated, too high. It is therefore necessary to estimate the intensity of the γ-radiation from the Metagalaxy.

3. METAGALACTIC EMISSION

The density of the gas in Metagalactic space is unknown but the most probable values (for more details see, for instance, [4]) are

[3)]According to recent data,[28] $I_\gamma(E_\gamma > 4\times 10^7\ \text{eV}) \leq (3.3 \pm 1.3)\times 10^{-4}$.

$$n = 10^{-5}\,\text{cm}^{-3},\ \rho = 2\cdot 10^{-29}\,\text{g/cm}^3,$$
$$N(L) = 5\cdot 10^{22}\,\text{cm}^{-2},\ M(L) = 0.1\,\text{g/cm}^2, \quad (23)$$

where the path L is the photometric radius of the Metagalaxy $R_{ph} = 5 \times 10^{27}$ cm. The possibility that the gas concentration in the Metagalaxy is $n \ll 10^{-5}$ cm^{-3} cannot be considered to be excluded but the opposite inequality is impossible for reasons based on relativistic cosmology. The selection of the value for the mean thermal photon energy density is also very important. In published papers wide use is made of the value $w_{ph} = \text{eV/cm}^3$ but we think that this is two orders too high.[4)] From observational data and estimates we have the density

$$w_{ph} \approx 2\cdot 10^{-3}\,\text{eV/cm}^3. \quad (24)$$

This corresponds, if we are speaking of energy density, to thermal emission with a temperature $T = 0.8°$K. The density (24) is a lower limit, but we know no actual reason for increasing w_{ph} although this has not yet been excluded.

We have no direct information on the cosmic rays in Metagalactic space. Therefore for the Metagalactic gamma and X-radiation, which should be highly isotropic (we are not now thinking of discrete sources) it is more correct to set the problem as follows. Let us say that in the whole of Metagalactic space in a definite energy range

$$I_{cr}(E) = K_{cr}\,E^{-\gamma}, \qquad I_e(E) = K_e E^{-\gamma_e}. \quad (25)$$

It is then easy to find the intensity of the γ-rays for all the processes 1 to 3 under discussion. The results, if we are dealing with the integral spectrum $I_\gamma(>E_\gamma)$, depend comparatively little on the choice of the indices γ and γ_e in (25).

In this connection we shall limit ourselves to two examples. We shall consider that the cosmic ray spectrum in the Metagalaxy $I_{cr,Mg}$ differs from the Galactic spectrum (see, e.g., (1)) only by the factor ξ_{cr}. Then for the γ-rays from the decay of π^0-mesons we can use formulae (8) and (9). The corresponding values are given in Table II. Obviously the value (22) can be obtained only if $\xi_{cr} \sim 1$, which seems to us to be highly improbable.[4,12] We shall now consider that the Metagalactic electrons have a Galactic spectrum (4) multiplied by ξ_e. Then, by using the above data (see Table I), we come to the intensities given in Table II.

If $\xi_e \sim 10^{-2}$, the flux $I_\gamma(E_\gamma > 5 \times 10^7\ \text{eV})$ reaches the value (22). We think that this result is very important. Even if we reduce the Galactic value (4) by a factor of 3 the electron intensity in the Metagalaxy should be 30 times less than in the Galaxy. This, in any case, does not lead to contradictions of an energetic nature and is possible (for further detail see [12] also section 13 of [4]).

As a check we shall assume that in the Metagalaxy

$$I_e(E) = K_e E^{-2.6}. \quad (26)$$

Then from formula (17) and the experimental value (22) we find

$$K_e = 8.3\cdot 10^{-4}\,\text{BeV}^{1.6}\,\text{cm}^{-2}\cdot\text{sr}^{-1}\cdot\text{sec}^{-1}$$
$$= 6.0\cdot 10^{-8}\,\text{cm}^{-2}\cdot\text{sr}^{-2}\cdot\text{sec}^{-1}\cdot\text{erg}^{1.6}. \quad (27)$$

This leads to an electron intensity $I_{e,Mg}(E > 1\ \text{GeV}) = 5.2 \times 10^{-4}\,(\text{cm}^2\cdot\text{sec}\cdot\text{sr})^{-1}$, i.e., about 40 times less than the electron intensity in the Galaxy according to (2).

Let us also estimate the possible γ-ray intensity from the whole combination of galaxies $I_{\gamma,gal}$. Our Galaxy emits in the γ-band (as the result of the inverse Compton effect) about as much as in the radio band, i.e., its power is $L_\gamma \sim 3 \times 10^{38}$ erg/sec. The γ-ray flux will be too high only if we consider all the galaxies to be just as powerful γ-emitters (the majority of galaxies emit less than ours and the contribution from radio galaxies, particularly bearing in mind that the thermal emission density in them is not high,[5)] is probably considerably less than the contribution from normal galaxies[4]). Furthermore $J_{\gamma,gal} = N_g L_\gamma R_{ph}/4\pi$, where $N_g \sim 5 \times 10^{-75}\,\text{cm}^{-3}$ is the concentration of the galaxies and L_γ is their mean power. Hence

$$J_{\gamma,gal} \lesssim 6\cdot 10^{-10}\,\text{erg/cm}^2\cdot\text{sec}\cdot\text{sr}$$

or

$$I_{\gamma,gal}(E_\gamma > 5\cdot 10^7\,\text{eV}) \lesssim J_{\gamma,gal}/E_\gamma \lesssim 10^{-5}$$
$$\text{photons/cm}^2\cdot\text{sec}\cdot\text{sr}$$

This estimate is not, of course, particularly accurate but it may nevertheless be taken that the con-

4)Here it is assumed that the emission intensity of all the galaxies is equivalent to half the intensity of a galaxy of 10th photographic stellar magnitude per square degree. If we take for the Sun $m_{pq} = -26.26$ we obtain $w_{ph} = 1.8 \times 10^{-3}$ eV/cm³.

5)We are not thinking now of galaxies of the "super-star" type. If the optical emission of these galaxies (3C273-B etc.) is of the nature of magnetic bremsstrahlung the γ-ray flux (due to the inverse Compton effect) corresponding to it may be very large.[475] In view of the small number of these sources their contribution can be picked out in measurements even with measurements with a comparatively poor angular resolution.

tribution of $I_{\gamma,gal}$ is essentially less than value (22). We would also mention that the value (24) for w_{ph} is a lower limit and it may turn out that it has to be increased by several factors. In such an increase, of course, the coefficient ξ_e is increased by the same factor.

Therefore to explain the experiments of Kraushaar and Clark[5] (see (22)) it is sufficient to consider that in Metagalactic space there are electrons (with $E \gtrsim 1$ GeV) whose concentration is 1.5 to 2 orders less than the Galactic concentration. This value, however, as follows from what is said below, is still rather high and should be looked upon as an upper limit (the nature of Kraushaar and Clark's measurements[5] points to the same thing).

The Metagalactic electrons with an energy E produce radio emission largely at a frequency $\nu = 4.6 \times 10^{-6} H_\perp (E\ \text{eV})^2$. Even in a field $H \sim 10^{-7}$ Oe at a frequency $\nu = 400$ Mc/s it is large electrons with $E \sim 3 \times 10^{10}$ eV that are emitters. At these and greater energies there is every reason to use a spectrum with $\gamma > 2$. Therefore, just as earlier in this book, we shall take $\gamma = 2.6$ and shall make the effective temperature of the Metagalactic radio emission at a frequency of 400 Mc/s less than 10°. Then from the formula[4]

$$\frac{4\pi}{c} K_e H_{\text{Mg}}^{(\gamma+1)/2} = \frac{8.9 \cdot 10^{22}\, T_{\text{eff}}}{a(\gamma) L} \left(\frac{\nu}{6.26 \cdot 10^{18}}\right)^{(\gamma+3)/2}, \quad a(2.6) = 0.083$$

we can obtain for the spectrum (26)—(27)

$$H_{\text{Mg}} < \left(\frac{6 \cdot 10^{-32}}{4\pi K_e/c}\right)^{0.555} = 10^{-8}\,\text{Oe}. \qquad (28)$$

The field $H_{Mg} \sim 10^{-8}$ Oe is still less than the equilibrium field $H_{Mg} = \sqrt{4\pi\rho u^2} \gtrsim 10^{-7}$ Oe (here $\rho u^2/2 \gtrsim 10^{-15}$ erg/cm³ is the kinetic energy density of the Metagalactic gas). The value of K_e, however, can probably be reduced by several factors so there is no need to think of any difficulties here. In addition, we should mention that according to an earlier estimate[22] which is based on unconfirmed measurements of the polarization of source 3C295, the field is $H_{Mg} < (3 - 10) \times 10^{-8}$ Oe.

4. HIGH-ENERGY ELECTRONS AND MAGNETIC BREMSSTRAHLUNG X-RADIATION

The very important question arises of the nature of the electron spectrum at high energies since the spectrum (4) relates only to the region $1 < E < 10$ GeV and the extrapolated spectrum (6) is only by way of illustration. If the spectrum (6) were valid up to energies of $E \sim 10^{15}$ eV the electron intensity in this region of the spectrum would be about half the intensity of the proton-nuclear component. Moreover, if these electrons do exist they would produce the extensive showers which are anomalously poor in μ-mesons and are generally ascribed to γ-rays.[6,23,24] The number of these showers is not more than 10^{-3} to 10^{-4} of the number of all the showers with the same primary particle energy.

Of course, electrons are equivalent to γ-rays as the sources of these showers.[6)] The above-mentioned data[6,23,24] may therefore be looked upon as an indication of the upper limit of the electron intensity. A similar result is obtained from the data on X-radiation.[7] If we continue spectrum (6) up to an energy $E \sim 10^{15}$ eV we should obtain an intensity

$$I_B(E > 1.7\ \text{keV}) \approx 1.7 \cdot 10^3\ \text{quanta/cm}^2 \cdot \text{sec} \cdot \text{sr}$$

in the direction of the center of the Galaxy.

At the same time experimentally $I_B \simeq 5$ quanta/cm² · sec · sterad. In addition, the electron injection power would be

$$\int\limits_{E \ll E_{max}}^{E_{max}} Q_e(E)\, E dE = 2.8 \cdot 10^{44}\, mc^2 \left(\frac{E_{max}}{mc^2}\right)^{0.3} = 1.4 \cdot 10^{41}\ \text{erg/sec}$$

It is thus clear that the intensity (6) can and must be reduced by at least three orders, thus opening up a basic possibility of explaining recent results.[6,7,10]

As an example we shall consider that the source spectrum (3) is valid only for $E < 10^{10}$ eV while for $E > 10^{10}$ eV

$$Q_e(E) = 1.6 \cdot 10^{42}\, E^{-2.3}\ \text{electrons/sec} \cdot \text{GeV} \qquad (29)$$

Then in the $E > 10^5$ GeV energy range

$$I_e(E) = 6.8 \cdot E^{-3.3}. \qquad (30)$$

Spectrum (30) is shown by a dotted line in the figure.

In accordance with (30) $I_e(E > 10^{15}\ \text{eV}) = 4.8 \times 10^{-14}$ whilst $I_{cr}(E > 10^{15}\ \text{eV}) = 2.4 \times 10^{10}$ quanta/cm² · sec · sr. At the same time the source power is

$$U_e = \int\limits_{10^{10}\ \text{eV}}^{\infty} E\, Q_e(E)\, dE = 4.3 \cdot 10^{39}\ \text{erg/sec},$$

which is only 1.5 times higher than the power

$$U_e = \int\limits_{3 \cdot 10^8}^{10^{10}} E\, Q_e(E)\, dE = 2.8 \cdot 10^{39}\ \text{erg/sec},$$

obtained[13] for the spectrum (3). The magnetic

6)The effect of the Earth's magnetic field[25] is significant only at energies $E > 1$ to 5×10^{17} eV.

bremsstrahlung x-radiation for electrons with the spectrum (3) has the intensity[8]

$$I_B(\nu) = 5.4\cdot 10^{19}\, a(\gamma)\, LK_e H^{(\gamma+1)/2}\left(\frac{1.6\cdot 10^{13}}{\nu}\right)^{(\gamma+1)/2}$$
$$= 3\cdot 10^{-19}\, L\left(\frac{4.8\cdot 10^{7}}{\nu}\right)^{2.15}, \qquad (31)$$

where in the second expression K_e is the coefficient in the spectrum $I_e(E)\,dE = K_e E^{-\gamma}\,dE$ and E is measured in GeV. In accordance with (31) for $L = 7\times 10^{22}$ cm (the direction of the center of the Galaxy)

$$I_B(1.6\cdot 10^3 < h\nu < 6.4\cdot 10^3 \text{ eV})$$
$$= 2.6 \text{ photons/cm}^2\cdot\text{sec}\cdot\text{sr} \qquad (32)$$

Since the ultra-high energy electrons will be distributed only in the region of the disk (the region of the sources) or near it the intensity of the x-radiation towards the pole is two orders less than the value (32). The magnetic bremsstrahlung intensity will also be far greater than the thermal emission intensity in the Galaxy in the far ultraviolet region. For example, with a thermal emission energy density of $w_{ph} = 0.2$ eV/cm³ (for T = 5000° K) the intensity is

$$I_\nu\left(10^3\text{Å} < \lambda = \frac{c}{\nu} < 1.5\cdot 10^3\text{ Å}\right)$$
$$\approx 0.3 \text{ photon/cm}^2\cdot\text{sec}\cdot\text{sr}$$

whilst for the magnetic bremsstrahlung even in the direction of the pole (L = 100 pc) in the same range $I_\nu \simeq 4$ photons/cm² · sec · sr.

Thus if we assume that electrons with a spectrum like (29) are generated in the Galaxy it would really be possible to obtain a flux of electrons with an energy $E > 10^{15}$ eV necessary to explain the experiments of Firkowski et al.,[6] Suga et al.,[23] and of Giacconi et al.[7] in relation to the anisotropic component of the x-radiation. This does not, of course, mean that electrons with a spectrum like (29) do exist. We wish only to stress that the assumption that these electrons exist does not contradict the available data and energy considerations. A certain difficulty connected with this assumption is that the electrons with $E \sim 10^{14}$ to 10^{15} eV lose half their energy in a time of 10^3 to 10^4 years. Their sources should therefore be operative at present, i.e., a contribution of explosions of the Galactic core is excluded. Even if it is a question of supernovae there should be large fluctuations in the spatial distribution of ultra-high energy electrons at a mean explosion frequency of 10^{-2} explosions/year. However, a decisive method of checking the hypothesis under discussion would be direct measurements of the intensity of the cosmic ray electron component at energies of $E > 10^{10}$ eV. In particular even if we reduce the source power (29) by a factor of 3 to 4 the electron intensity at an energy $E \sim 10^{11}$ to 10^{12} eV should be of the order of one percent of the intensity of all cosmic rays. There are at present clearly possible experimental ways for finding electrons with this intensity.

APPENDIX

EFFECTIVE CROSS SECTION FOR "INVERSE" COMPTON SCATTERING

Let us calculate the effective cross section $\sigma(E_\gamma, \epsilon, E)\,dE_\gamma$ (see formula (13) above and the paragraph following it) for the generation of a γ-quantum with an energy in the range dE_γ during the scatter of a thermal photon with an energy ϵ on an electron whose total energy is E. We shall consider the photon distribution to be isotropic.

We shall use the Compton cross section taken in the general invariant form (see [26], Sec. 29; we put c = 1 in formulae (A1)—(A5))·

$$\sigma_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{v})\, d\Omega_2 = 2r_e^2 \frac{E_\gamma^2}{m^2\varkappa_1^2}\left\{4\left(\frac{1}{\varkappa_1}+\frac{1}{\varkappa_2}\right)^2 - 4\left(\frac{1}{\varkappa_1}+\frac{1}{\varkappa_2}\right) - \left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}\right)\right\} d\Omega_2, \qquad (A.1)$$

where

$$\varkappa_1 = -\frac{2}{m^2}\varepsilon E\,(1 - v\cos\nu_1),$$
$$\varkappa_2 = \frac{2}{m^2}E_\gamma E\,(1 - v\cos\nu_2), \qquad (A.2)$$

$r_e = e^2/mc^2$, m and e are the mass and charge of an electron, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the original and scattered photons, Θ_1 and Θ_2 are the angles between these momenta and the velocity of the electron v, $d\Omega_1$ and $d\Omega_2$ being corresponding solid angles.

The energy of the scattered photon is (Θ is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$)

$$E_\gamma = \frac{\varepsilon\,(1 - v\cos\nu_1)}{1 - v\cos\nu_2 + \varepsilon E^{-1}(1-\cos\nu)}$$
$$= \varphi\,(\varepsilon, E, \nu_1, \nu_2, \nu). \qquad (A.3)$$

The desired cross section is defined by the expression

$$\sigma\,(E_\gamma, \varepsilon, E) = \frac{1}{4\pi}\int (1 - v\cos\nu_1)\,\sigma_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{v})$$
$$\times\delta\,(E_\gamma - \varphi\,(\varepsilon, E, \nu_1, \nu_2, \nu))\, d\Omega_1 d\Omega_2. \qquad (A.4)$$

To calculate the integrals in (A.4) we shall use a convenient approximate form[20] of relation (A.3) for $\epsilon \ll E_\gamma < E$:

$$E_\gamma \approx \frac{\varepsilon(1 - v\cos\nu_1)}{1 - v\cos\nu_2 + \varepsilon E^{-1}(1 - v\cos\nu_1)\cos\nu_2} \quad (A.5)$$

and limit ourselves to the case

$$4\varepsilon E/mc^2 \ll mc^2. \quad (A.6)$$

Upon these assumptions the direct calculation of (A.4) leads to the cross section

$$\sigma(E_\gamma, \varepsilon, E) = \frac{\pi r_e^2}{4}\frac{(mc^2)^4}{\varepsilon^2 E^3}\left\{2\frac{E_\gamma}{E} - \frac{(mc^2)^2 E_\gamma^2}{\varepsilon E^3} + 4\frac{E_\gamma}{E}\ln\frac{(mc^2)^2 E_\gamma}{4\varepsilon E^2} + \frac{8\varepsilon E}{m^2c^4}\right\}, \quad (A.7)$$

where E_γ is within the limits

$$\varepsilon \leqslant E_\gamma \leqslant 4\varepsilon\,(E/mc^2)^2. \quad (A.8)$$

By virtue of (A.7) and (A.8) the total scattering cross-section is

$$\sigma_t = \int \sigma(E_\gamma, \varepsilon, E)\, dE_\gamma = \frac{8\pi}{3} r_e^2 \equiv \sigma_T \quad (A.9)$$

which is equal to the Thomson cross-section and corresponds to the approximation (A.6) under discussion. The mean energy losses of an electron are obviously

$$-\frac{dE}{dt} = c\iint \sigma(E_\gamma, \varepsilon, E) n_{ph}(\varepsilon) E_\gamma\, dE_\gamma\, d\varepsilon = c\frac{8}{3}\pi r_e^2\left(\frac{E}{mc^2}\right)^2 \frac{4}{3}\int \varepsilon n_{ph}(\varepsilon)\, d\varepsilon = c n_{ph}\sigma_T \frac{4}{3}\bar{\varepsilon}\left(\frac{E}{mc^2}\right)^2, \quad (A.10)$$

where the total number of photons per unit volume and their mean energy are

$$n_{ph} = \int n_{ph}(\varepsilon)\, d\varepsilon \text{ and } \bar{\varepsilon} = \frac{1}{n_{ph}}\int \varepsilon n_{ph}(\varepsilon)\, d\varepsilon. \quad (A.11)$$

It follows from expressions (A.9) and (A.10) that the mean energy of a scattered photon is

$$\bar{E}_\gamma = \frac{4}{3}\bar{\varepsilon}\,(E/mc^2)^2. \quad (A.12)$$

Let us now determine the intensity of the scattered γ-quanta if on the path L the electron intensity $I_e(E)$ and photon concentration $n_{ph}(\epsilon)$ are homogeneous and isotropic. Obviously

$$I_\gamma(E_\gamma) = L\int_0^\infty n_{ph}(\varepsilon)\, d\varepsilon \times \int_{E_{min}}^\infty \sigma(E_\gamma, \varepsilon, E)\, I_e(E)\, dE, \quad (A.13)$$

where the lower limit $E_{min} = mc^2(E_\gamma/4\epsilon)^{1/2}$ is determined from (A.8). Making the electron spectrum exponential

$$I_e(E) = K_e E^{-\gamma}, \quad (A.14)$$

we find

$$\int_{E_{min}}^\infty \sigma(E_\gamma, \varepsilon, E)\, I_e(E)\, dE = 2^\gamma \frac{\gamma^2 + 4\gamma + 11}{(\gamma+1)(\gamma+3)^2(\gamma+5)} 8\pi r_e^2 K_e (mc^2)^{1-\gamma} \varepsilon^{(\gamma-1)/2} E_\gamma^{-(\gamma+1)/2}. \quad (A.15)$$

For black radiation

$$n_{ph}(\varepsilon)\, d\varepsilon = \frac{n_{ph}}{2.404\,(kT)^3}\frac{\varepsilon^2\, d\varepsilon}{e^{\varepsilon/kT} - 1}, \qquad \bar{\varepsilon} = 2.7\, kT. \quad (A.16)$$

Substituting expression (A.15) and (A.16) in the expression for the intensity (A.13) and integrating with respect to $d\epsilon$ we obtain

$$I_\gamma(E_\gamma) = f(\gamma)\frac{2}{3}\sigma_T L w_{ph} (mc^2)^{1-\gamma}\left(\frac{4}{3}\bar{\varepsilon}\right)^{(\gamma-3)/2} K_e E_\gamma^{-(\gamma+1)/2}, \quad (A.17)$$

where

$$f(\gamma) = 4.74\,(1.05)^\gamma \frac{\gamma^2 + 4\gamma + 11}{(\gamma+1)(\gamma+3)^2(\gamma+5)}\Gamma\left(\frac{\gamma+5}{2}\right)\zeta\left(\frac{\gamma+5}{2}\right) \quad (A.18)$$

$\Gamma(x)$ is an Euler gamma-function and

$$\zeta(x) = \sum_{n=1}^\infty n^{-x}$$

is a Riemann function.[27]

In particular

$$f(1) = 0.84, \quad f(2) = 0.86, \quad f(3) = 0.99, \quad f(4) = 1.4. \quad (A.19)$$

It is easy to see that expression (A.17) agrees with formula (16) given earlier with an accuracy up to the factor $f(\gamma)$.

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