MUON CAPTURE BY POLARIZED NUCLEI WITH SPIN $^1/_2$

A. P. BUKHVOSTOV and N. P. POPOV

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 29, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 1842-1852 (May, 1964)

A formula is derived for the angular distribution of the recoil nuclei from muon capture by light polarized spin-$^1/_2$ nuclei. The depolarization of the muons and target nuclei, due to the interaction responsible for both the fine and hyperfine structures, is calculated. The asymmetry of the angular distribution of tritium nuclei in muon capture by fully polarized He^3 nuclei reaches $\sim 10\%$, with the term $\sim \cos\theta$ predominating when the pseudoscalar form factor is small, and the term $\sim P_2(\cos\theta)$ becoming decisive when $C_P/C_A \sim 30$.

1. INTRODUCTION

It is known that an important role is played by the pseudoscalar form factor in muon capture by protons. The contribution of the form factor was investigated in experiments in which a unique interpretation of the experimental data is impossible unless the values $C_P/C_A > 20$ are excluded (C_P and C_A are the constants of the pseudoscalar and axial-vector interactions). Thus, for example, the probability of μ capture will be the same for values of C_P/C_A equal to 8 and 32. Analogous results are obtained also by examination of the angular asymmetry of the recoil nuclei when muons are captured by nuclei with zero spin, and the anisotropy of the neutrons of the direct process [1], in the study of $\gamma\nu$ correlation in nuclear μ capture [2].

The angular distribution of tritium nuclei in the capture of polarized muons by He^3 was calculated in [3]. The asymmetry coefficient depends monotonically on the pseudoscalar constant for $C_P/C_A < 36$. However, the asymmetry does not exceed $\sim 4\%$, which apparently is beyond the experimental accuracy [3]. We shall show that the angular asymmetry of the recoil nuclei when muons are captured by light spin-$^1/_2$ nuclei polarized in the muon beam direction can reach a noticeable magnitude, although strong depolarization (see Sec. 3) reduces the increase in the asymmetry coefficient due to the polarization of the nuclei to a value smaller than could be expected.

2. ANGULAR DISTRIBUTION OF RECOIL NUCLEI

The angular asymmetry of the recoil nuclei in the allowed muon capture by polarized spin-$^1/_2$ nuclei is of the form

$$W = 1 + a\,(\lambda_1^{(f)}\boldsymbol{\xi} + \lambda_2^{(f)}\boldsymbol{\zeta})\,\mathbf{n} + b\lambda_3^{(f)}\left[(\boldsymbol{\xi}\mathbf{n})(\boldsymbol{\zeta}\mathbf{n}) - \tfrac{1}{3}\boldsymbol{\xi}\boldsymbol{\zeta}\right], \quad (1)$$

where $\mathbf{n}$ —unit vector in the direction of emission of the recoil nucleus, $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ —initial-polarization vectors of the muon and of the initial nucleus, respectively 1). An estimate of $\lambda_i^{(j)}$ will be given in Sec. 3. In Eq. (1) we have

$$aW_0 = 1 + qM^{-1}(1 - C_P/C_A) + (qC_P/2MC_A)^2 + 3x^2(1 + q/M) + 2\sqrt{3}x[1 + (1 - C_P/2C_A)q/M] + (1 + \sqrt{3}x)\left[2qM^{-1}\left(y + \sqrt{\tfrac{1}{3}}\,z\right) - \tfrac{2}{5}q^2u\right];$$

$$bW_0 = 2qM^{-1}(1 + \sqrt{3}x)[1 - C_P/C_A + (1 + \mu_p - \mu_n)C_V/C_A + 2y - v + \tfrac{3}{5}qMu]; \quad (2)$$

W_0 coincides, apart from a common factor, with the probability of capture of polarized muons by polarized spin-$^1/_2$ nuclei:

$$W_0 = c + d\lambda_0^{(f)}\boldsymbol{\xi}\boldsymbol{\zeta};$$

$$c = 3 + qM^{-1}[1 - C_P/C_A - 2(1 + \mu_p - \mu_n)C_V/C_A] + (qC_P/2MC_A)^2 + 3x^2(1 + q/M) + 2qM^{-1} \times (-v + y + xz);$$

$$d = -2 + \tfrac{4}{3}qM^{-1}(1 + \mu_p - \mu_n)C_V/C_A - \tfrac{2}{3}qM^{-1} \times (1 - C_P/C_A) + \sqrt{3}x[2 - \tfrac{2}{3}qM^{-1}(1 + \mu_p - \mu_n)C_V/C_A + \tfrac{1}{3}(4 - C_P/C_A)qM^{-1}] - 2\sqrt{\tfrac{1}{3}}\,qM^{-1}x(v - y) + \tfrac{4}{3}qM^{-1}\left(-y + v + \tfrac{3}{2}\sqrt{3}z\right),$$

where q —neutrino energy, M —nucleon mass, μ_p and μ_n —anomalous magnetic moments of the

1) The term proportional to $[\xi \times \zeta]\cdot\mathbf{n}$ is missing when the time parity is conserved.

proton and neutron, respectively. The quantities x, y, z, u, and v are the ratios of the nuclear matrix elements, which are determined in the appendix:

$$x = \frac{C_V}{C_A}\frac{\int 1\varphi_\mu}{\int \sigma\varphi_\mu}, \qquad y = \frac{\int ir(\sigma p)\varphi_\mu}{\int \sigma\varphi_\mu},$$

$$z = \frac{C_V}{C_A}\frac{\int irp\varphi_\mu}{\int \sigma\varphi_\mu}, \qquad u = \frac{\int r(\sigma r)\varphi_\mu - {}^1/_3\int r^2\sigma\dot{\varphi}_\mu}{\int \sigma\varphi_\mu},$$

$$v = \frac{C_V}{C_A}\frac{\int [rp]\varphi_\mu}{\int \sigma\varphi_\mu}. \qquad (3)$$

In the derivation of (2) we have left out terms proportional to $(q/M)^2$, with the exception of those containing $(C_P/C_A)^2$.

In the case of muon capture by polarized protons, the coefficients a and b have an especially simple form:

$$aW_0 = (G_V + G_A - G_P)^2, \qquad bW_0 = -4G_P(G_V + G_A),$$

$$4W_0 = (3 + \lambda_0^{(f)}\xi\zeta)W_t + (1 - \lambda_0^{(f)}\xi\zeta)W_s, \qquad (4)$$

where W_s and W_t coincide, apart from a common factor, with the capture probabilities from the singlet and triplet states of the mesic atom, respectively; these are given, for example, in Primakoff's review [4]:

$$W_s = (G_V - 3G_A + G_P)^2,$$
$$W_t = (G_V + G_A)^2 - \tfrac{2}{3}G_P(G_V + G_A) + G_P^2;$$
$$G_V = C_V(1 + q/2M),$$
$$G_A = C_A - C_V(1 + \mu_p - \mu_n)\, q/2M,$$
$$G_P = [C_P - C_A - C_V(1 + \mu_p - \mu_n)]\, q/2M.$$

We note that in the absence of initial polarization of the target protons ($\zeta = 0$) formulas (1) and (4) coincide with the corresponding expressions in [4].

As can be seen, the angular asymmetry of the produced neutrons (the coefficients a and b) depends strongly on the value of the pseudoscalar. However, a study of the capture of a muon from the triplet state of the mesic atom of hydrogen entails experimental difficulties. In the case of nuclear μ capture the angular asymmetry of the recoil nuclei, as can be seen from (2), depends on several ratios of the nuclear matrix elements. The use of nuclear models in the calculation of these ratios introduces, naturally, an inaccuracy in the values of the asymmetry coefficients. However, for the transition $\mu^- + He^3 \to H^3 + \nu$, the ratios of the nuclear matrix elements can be obtained with high accuracy [5,3]: $x = -(1/3)^{1/2} C_V/C_A$, $zC_A/C_V = -y = 1/2$, and $u = v = 0$. Substituting these values in (2) we see that the expressions for a and b in the case of μ capture in polarized He^3 are obtained from the corresponding expressions (4) by making the substitutions $G_V \to C_V$, $G_A \to -G_A$, and $G_P \to -G_P - qC_A/2M$. In this case the coefficient a coincides with the corresponding expression in [3] if we neglect the term $\sim \xi\zeta$ in W_0.

3. INVESTIGATION OF DEPOLARIZATION IN THE PRESENCE OF INITIAL POLARIZATION OF THE NUCLEUS

In the depolarization of muons captured by light nuclei, an important role is played by both the fine and hyperfine structure of the μ-mesic atom. This is connected with the fact that the fine and hyperfine splitting of the levels of such mesic atoms is much larger than or comparable with their width. Indeed, the fine splitting of all the levels of the mesic atoms is approximately 100 times larger than the radiation width, and the ratio of the hyperfine splitting to the fine splitting

$$\frac{\Delta_c}{\Delta_T} = \frac{2\mu}{Z}\frac{m}{M}\frac{l(l+1)(2j+1)}{j(j+1)(2l+1)}$$

(m and M —masses of the muon and nucleon, respectively; Z —charge of nucleus; μ —its magnetic moment in nuclear magnetons) at small Z (for example, for He^3) amounts to approximately $1/10$.

To estimate the total level width it would be necessary to include not only the radiative transitions but also the Auger transitions, the probabilities of which are relatively large at small Z, particularly for the highly excited levels. However, if the number of the electrons in the atoms is small, then the muon can execute only a small number of Auger transitions to the highest levels, after which radiative transitions should take place. We shall therefore assume that below a certain level the total width is equal to the radiation width and is small compared with the fine and hyperfine splitting, and above this level there is no depolarization, since the opposite condition holds true. The analysis of the depolarization for a nucleus with spin $I = 1/2$ can be carried out in the same manner as in [6], the only difference being that there the nucleus was assumed to be unpolarized and a finite level width was taken into account.

The final-state density matrix $\rho^{(f)}$ is expressed in terms of the initial-state density matrix ρ in the following fashion:

$$\rho^{(f)} = \mathop{S}_{k} w_k \rho^{(f_k)}, \qquad (5)$$

$$\rho^{(f_k)}_{\nu\nu'} = N_k \sum_{\mu\mu'} H_{\nu\mu}H^+_{\mu'\nu'} \sum_{\varepsilon\varepsilon'} H_{\mu\varepsilon}H^+_{\varepsilon'\mu'} \ldots \sum_{\alpha\alpha'} H_{\beta\alpha}H^+_{\alpha'\beta'}\rho_{\alpha\alpha'}, \qquad (6)$$

where $H_{\beta\alpha}$ etc. —matrix element of the transition from the state α into the state β; N_k —normalization factor, determined from the condition $\mathrm{Sp}\,\rho^{(f_k)} = 1$; w_k —probability of k-th cascade; S denotes the sum over all the possible cascades. By cascade is meant a definite sequence of levels through which the mesic atom goes through. The levels are characterized, in addition to the principal quantum number, also by the values of l, j, and F (l —orbital angular momentum, $\mathbf{j} = \mathbf{l} + \mathbf{s}$; $\mathbf{F} = \mathbf{j} + \mathbf{I}$; $\mathbf{s}$ and $\mathbf{I}$ —spins of the muon and the nucleus). The sublevels α and α' of one level differ in the values of the projection of the total angular momentum **F**.

Let us find first the matrix ρ in the right-hand side of (6). It can be assumed that prior to the start of the depolarization the density matrix has a form of a direct product of the density matrices of the nucleus and of the muon:

$$\rho_0 = (1 + 2\zeta\mathbf{I})(1 + 2\xi\mathbf{s})/4(2l + 1), \tag{7}$$

where ζ and ξ are the polarization vectors of the nucleus and muon, respectively ($I = 1/2$). The factor $(2l + 1)/4$ corresponds to the requirement $\mathrm{Sp}\,\rho_0 = 1$.

The matrix ρ differs from ρ_0 in the absence of elements relating the states with different j and F. (Nondiagonal matrix elements of ρ_0 do not make any contribution to the final density matrix, owing to the incoherence of the emitted quanta.) Therefore

$$\rho = D_F(D_j(\rho_0)), \tag{8}$$

where the operators D_j and D_F separate the part of the matrix which is diagonal in j and F.

The separation of the elements of the matrix ρ_0 which are diagonal in j entails no difficulty, if it is recognized that

$$D_j(\mathbf{s}) = \frac{j(j+1) - l(l+1) + 3/4}{2j(j+1)}\mathbf{j} = \frac{1}{2l+1}P_+\mathbf{j} - \frac{1}{2l+1}P_-\mathbf{j}, \tag{9}$$

where P_+ and P_- are the operators of projection on the states with $j = l + 1/2$ and $j = l - 1/2$. From this we get, taking (7) into consideration:

$$D_j(\rho_0) = \frac{1}{4(2l+1)}(1 + 2\zeta\mathbf{I})\left(1 + \frac{2}{2l+1}\xi\mathbf{j}P_+ - \frac{2}{2l+1}\xi\mathbf{j}P_-\right). \tag{10}$$

To perform the operation D_F we make use of the formulas

$$D_F(\mathbf{I}) = \frac{F(F+1) - j(j+1) + 3/4}{2F(F+1)}\mathbf{F},$$

$$D_F(\mathbf{j}) = \frac{F(F+1) + j(j+1) - 3/4}{2F(F+1)}\mathbf{F}, \tag{11}$$

$$D_F\left[j_ij_k + j_kj_i - \frac{2}{3}j(j+1)\delta_{ik}\right] = \mathcal{F}_{ik}\begin{Bmatrix} F & F & 2 \\ j & j & 1/2 \end{Bmatrix}$$
$$\times\left[\frac{j(j+1)(2j+1)(2j+3)(2j-1)(2F+1)}{F(F+1)(2F-1)(2F+3)}\right]^{1/2}(-1)^{F+j+1/2}$$
$$= \mathcal{F}_{ik}\begin{cases} \dfrac{2j-1}{2j+1} & F = j + 1/2, \\ \dfrac{2j+3}{2j+1} & F = j - 1/2, \end{cases} \tag{12}$$

where

$$\mathcal{F}_{ik} = F_iF_k + F_kF_i - \tfrac{2}{3}F(F+1)\delta_{ik}.$$

We note, in addition, that

$$j_iI_k - j_kI_i = -i\varepsilon_{ikm}[(\mathbf{jI})j_m - j_m(\mathbf{jI})], \tag{13}$$

hence $D_F(j_iI_k - j_kI_i) = 0$, since **jI** is diagonal in F, and

$$j_iI_k + j_kI_i = \tfrac{1}{2}(F_iF_k + F_kF_i) - \tfrac{1}{2}(j_ij_k + j_kj_i) - \tfrac{1}{4}\delta_{ik}. \tag{14}$$

We introduce the operators P_{++}, P_{+-}, P_{-+}, and P_{--} of projection on states with different j and F (the first index denotes $j = l \pm 1/2$, the second denotes $F = j \pm 1/2$. Then the formulas (11)—(14) yield

$$D_F(\mathbf{I}P_+) = \frac{1}{2(l+1)}\mathbf{F}(P_{++} - P_{+-}),$$
$$D_F(\mathbf{I}P_-) = \frac{1}{2l}\mathbf{F}(P_{++} - P_{--}),$$
$$D_F(\mathbf{j}P_+) = \mathbf{F}\left(\frac{2l+1}{2(l+1)}P_{++} + \frac{2l+3}{2(l+1)}P_{+-}\right),$$
$$D_F(\mathbf{j}P_-) = \mathbf{F}\left(\frac{2l-1}{2l}P_{-+} + \frac{2l+1}{2l}P_{--}\right),$$
$$D_F[(\zeta\mathbf{I})(\xi\mathbf{j})P_+] = \left(\frac{1}{4(l+1)}\zeta_i\xi_k\mathcal{F}_{ik} + \frac{2l+1}{12}\zeta\xi\right)P_{++}$$
$$- \left(\frac{1}{4(l+1)}\zeta_i\xi_k\mathcal{F}_{ik} + \frac{2l+3}{12}\zeta\xi\right)P_{+-},$$
$$D_F[(\zeta\mathbf{I})(\xi\mathbf{j})P_-] = \left(\frac{1}{4l}\zeta_i\xi_k\mathcal{F}_{ik} + \frac{2l-1}{12}\zeta\xi\right)P_{-+}$$
$$-\left(\frac{1}{4l}\zeta_i\xi_k\mathcal{F}_{ik} + \frac{2l+1}{12}\xi\zeta\right)P_{--}. \tag{15}$$

Substituting these expressions in (8) and (10) we get

$$\rho = \sum_{jF}\frac{1}{2F+1}\left[\frac{2F+1}{4(2l+1)} + \frac{1}{4}\lambda_0\zeta\xi + \frac{3}{F+1}(\lambda_1\xi + \lambda_2\zeta)\mathbf{F}\right.$$
$$\left. + \frac{15\lambda_3}{(F+1)(2F+3)}\zeta_i\xi_k\mathcal{F}_{ik}\right]P_{jF}. \tag{16}$$

The summation here is understood over four states with $j = l \pm 1/2$, $F = j \pm 1/2$; P_{jF} —operators of projection on these states. The values of λ_i in different states are listed in Table I.

Table I

State (j, F)	λ_0	λ_1	λ_2	λ_3
$(l+\frac{1}{2}, l+1)$	$\frac{2l+3}{3(2l+1)}$	$\frac{(l+2)(2l+3)}{12(l+1)(2l+1)}$	$\frac{(l+2)(2l+3)}{12(l+1)(2l+1)}$	$\frac{(l+2)(2l+3)(2l+5)}{60(l+1)(2l+1)^2}$
$(l+\frac{1}{2}, l)$	$-\frac{2l+3}{3(2l+1)}$	$\frac{2l+3}{12(2l+1)}$	$-\frac{1}{12}$	$-\frac{2l+3}{60(2l+1)}$
$(l-\frac{1}{2}, l)$	$-\frac{2l-1}{3(2l+1)}$	$-\frac{(l+1)(2l-1)}{12l(2l+1)}$	$\frac{l+1}{12l}$	$-\frac{(l+1)(2l+3)}{60l(2l+1)}$
$(l-\frac{1}{2}, l-1)$	$\frac{2l-1}{3(2l+1)}$	$-\frac{2l-1}{12(2l+1)}$	$-\frac{2l-1}{12(2l+1)}$	$\frac{2l-1}{60(2l+1)}$

We now turn to (6). The summation over $\alpha\alpha'$ is carried out in the same manner as in [6]. For the matrix

$$\rho^{(1)} = \sum_{\alpha\alpha'} H_{\beta\alpha} H^{+}_{\alpha'\beta'} \rho_{\alpha\alpha'}$$

we obtain the expression (16) with replacement of l, j, and F by l_1, j_1, and F_1, corresponding to the new level. The coefficients $\lambda_i^{(1)}$ in the matrix ρ_1 are expressed in terms of the initial coefficient λ_i by means of the formula

$$(\lambda_i^{(1)})_{j_1F_1} = \sum_{jF} w_{jF}^{j_1F_1} (\beta_i)_F^{F_1} (\lambda_i)_{jF}, \qquad (17)$$

where $w_{jF}^{j_1F_1}$ — probability of transition from the state (jF) into the state (j_1F_1), and the quantities $\beta_1 = \beta_2$ have the same form as the expression for $a_1^{(1)}/b_1$ in formula (18) of [6], in which we replace j by F and j_1 by F_1; β_3 coincides with $a_5^{(1)}/b_5$ of the same formula following the same substitution; $\beta_0 = 1$.

We neglect transitions in which l increases by unity, the probability of which is small when $l \neq 0$; then for a specified l there are ten possible types of transitions, depending on the initial and final states; their probability w, and also the values of the coefficient β_i for such transitions, are listed in Table II. If we make all the remaining summations in (6) and (5), we obtain for the final state, in which $l = 0$:

Table II

Initial state (j, F)	Final state (j_1F_1)	w	$\beta_1=\beta_2$	β_3
$(l+\frac{1}{2}, l+1)$	$(l-\frac{1}{2}, l)$	1	1	1
$(l+\frac{1}{2}, l)$	$(l-\frac{1}{2}, l)$	$\frac{1}{l(2l+1)}$	$1-\frac{1}{l(l+1)}$	$1-\frac{3}{l(l+1)}$
	$(l-\frac{1}{2}, l-1)$	$\frac{(l+1)(2l-1)}{l(2l+1)}$	1	1
$(l-\frac{1}{2}, l)$	$(l-\frac{1}{2}, l)$	$\frac{l+1}{l^2(2l+1)}$	$1-\frac{1}{l(l+1)}$	$1-\frac{3}{l(l+1)}$
	$(l-\frac{1}{2}, l-1)$	$\frac{1}{l^2(4l^2-1)}$	1	1
	$(l-\frac{3}{2}, l-1)$	$\frac{(l-1)(2l+1)}{l(2l-1)}$	1	1
$(l-\frac{1}{2}, l-1)$	$(l-\frac{1}{2}, l)$	$\frac{1}{l^2(2l-1)^2}$	$1-l^{-2}$	$\frac{(l^2-1)(4l^2-9)}{l^2(4l^2-1)}$
	$(l-\frac{1}{2}, l-1)$	$\frac{(l-1)(2l+1)}{l^2(2l-1)^2}$	$1-\frac{1}{l(l-1)}$	$1-\frac{3}{l(l-1)}$
	$(l-\frac{3}{2}, l-1)$	$\frac{2l+1}{l(2l-1)^2}$	$1-\frac{1}{l(l-1)}$	$1-\frac{3}{l(l-1)}$
	$(l-\frac{3}{2}, l-2)$	$\frac{(2l+1)(2l-3)}{(2l-1)^2}$	1	1

$\beta_0=1$

$$\rho^{(f)} = \tfrac{1}{3}\,[\tfrac{3}{4} + \tfrac{1}{4}\lambda_0^{(f)}\xi\zeta + \tfrac{3}{2}(\lambda_1^{(f)}\xi + \lambda_2^{(f)}\zeta)\,\mathbf{F} + \tfrac{3}{2}\lambda_3^{(f)}\xi_i\zeta_k\mathcal{F}_{ik}]\,P_t + (\tfrac{1}{4} - \tfrac{1}{4}\lambda_0^{(f)}\xi\zeta)\,P_s, \qquad (18)$$

$$\lambda_i^{(f)} = \sum_{jF} (\beta_i)_{jF}\,(\lambda_i)_{jF}, \qquad (19)$$

$$(\beta_i)_{jF} = \sum_k w_k\,(\beta_i)_k. \qquad (20)$$

In (19) the summation is over four possible states with different j and F (for specified l), while in (20) the summation is over all the cascades that starts from the initial state with given j and F. The quantity $(\beta_i)_k$ for the k-th cascade in (20) is a product of the values of β_i for the individual transitions in this cascade.

Table II and formulas (19) and (20) enable us to calculate the coefficients $\lambda_i^{(f)}$ in the final state. To this end it is necessary to consider the possible cascades. If in the initial state $j = l + 1/2$ and $F = l + 1$, then only transitions with successive decrease in j and F are possible, for which, as can be seen from Table II, all $\beta_i = 1$. For the case of an initial state with $j = l + 1/2$ and $F = l$, let us consider the cascade

$$(l + \tfrac{1}{2},\, l) \to (l - \tfrac{1}{2},\; l - 1) \to \ldots \to (k + \tfrac{1}{2},\; k) \to (k - \tfrac{1}{2},\, k) \to \ldots,$$

in which a transition without a change in F occurs when $F = k$. The probability of such a cascade is

$$w_k = \frac{l+1}{2l+1}\,\frac{1}{k\,(k+1)},$$

and the coefficients $(\beta_i)_k$ take the form

$$(\beta_1)_k = 1 - 1/k\,(k+1), \qquad (\beta_3)_k = 1 - 3/k\,(k+1).$$

Consequently, from formula (20) we have for this case

$$(\beta_0)_{+-} = \sum_{k=1}^{l} \frac{l+1}{2l+1}\,\frac{1}{k\,(k+1)} = \frac{l}{2l+1},$$

$$(\beta_1)_{+-} = (\beta_2)_{+-} = \sum_{k=1}^{l} \frac{l+1}{2l+1}\left[\frac{1}{k\,(k+1)} - \frac{1}{k^2(k+1)^2}\right] \approx \frac{l+1}{2l+1}\left(\frac{l}{l+1} - \alpha_1\right),$$

$$(\beta_3)_{+-} = \sum_{k=1}^{l} \frac{l+1}{2l+1}\left[\frac{1}{k\,(k+1)} - \frac{3}{k^2(k+1)^2}\right] \approx \frac{l+1}{2l+1}\left(\frac{l}{l+1} - 3\alpha_1\right),$$

$$\alpha_1 = \sum_{k=1}^{\infty} \frac{1}{k^2(k+1)^2} = \frac{\pi^2}{3} - 3 \approx 0.290. \qquad (21)$$

Assume, further, that in the initial state $j = l - 1/2$ and $F = l$. Let us consider a group of cascades in which j and F both decrease until l reaches a value k, after which a transition of one of the following types occurs:

$$(k - \tfrac{1}{2},\, k) \to (k - \tfrac{1}{2},\, k)$$
$$\text{or}\quad (k - \tfrac{1}{2},\, k) \to (k - \tfrac{1}{2},\, k - 1).$$

In the first case, the probability of which is

$$(2l+1)\,(k+1)/lk\,(2k+1)^2,$$

we have

$$(\beta_1)_k = 1 - 1/k\,(k+1), \quad (\beta_2)_k = 1 - 3/k\,(k+1),$$

in the second case, the probability of which is

$$(2l+1)/lk\,(2k+1)\,(4k^2-1),$$

the values of β_i are given by formulas (21), in which we must put $l = k - 1$.

Thus, in the case of an initial state $(l - 1/2,\, l)$ we get

$$(\beta_0)_{-+} = \sum_{k=1}^{l} \frac{2l+1}{l}\,\frac{k+1}{k\,(2k+1)^2} + \sum_{k=2}^{l} \frac{2l+1}{l}\,\frac{k-1}{k\,(4k^2-1)^2} = \frac{2l+1}{l}\sum_{k=1}^{l}\left[\frac{1}{4k^2-1} - \frac{1}{(4k^2-1)^2}\right] \approx 1 - \frac{2l+1}{l}\,\alpha_2,$$

$$\alpha_2 = \sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2} = \frac{1}{2}\left(\frac{\pi^2}{8} - 1\right) \approx \frac{1}{9},$$

$$(\beta_1)_{-+} = (\beta_2)_{-+} = \sum_{k=1}^{l} \frac{2l+1}{l}\,\frac{k+1}{k\,(2k+1)^2}\left(1 - \frac{1}{k\,(k+1)}\right) + \sum_{k=2}^{l} \frac{2l+1}{l}\,\frac{1}{(4k^2-1)^2}\left(\frac{k-1}{k} - \alpha_1\right) = (\beta_0)_{-+} - \frac{2l+1}{l} \times \left(\sum_{k=1}^{l} \frac{1}{k^2(2k+1)^2} + \sum_{k=2}^{l} \frac{\alpha_1}{(4k^2-1)^2}\right) \approx 1 - \frac{2l+1}{l}\,(\alpha_2 + \alpha_3),$$

$$(\beta_3)_{-+} = \sum_{k=1}^{l} \frac{2l+1}{l}\,\frac{k+1}{k\,(2k+1)^2}\left(1 - \frac{3}{k\,(k+1)}\right) + \sum_{k=2}^{l} \frac{2l+1}{l}\,\frac{1}{(4k^2-1)^2}\left(\frac{k-1}{k} - 3\alpha_1\right) = (\beta_0)_{-+} - \frac{2l+1}{l}\left(\sum_{k=1}^{l} \frac{3}{k^2\,(2k+1)^2} + \sum_{k=2}^{l} \frac{3\alpha_1}{(4k^2-1)^2}\right) \approx 1 - \frac{2l+1}{l}\,(\alpha_2 + 3\alpha_3),$$

$$\alpha_3 = \sum_{k=1}^{\infty} \frac{1}{k^2\,(2k+1)^2} + \sum_{k=2}^{\infty} \frac{\alpha_1}{(4k^2-1)^2} \approx 0.126. \qquad (22)$$

Let us consider the last possibility for the initial state $j = l - 1/2$, $F = l - 1$. As in the preceding analysis, we assume that up to $l = k$ there occur transitions with decreasing j and F, after which one of the following three transitions takes place:

1) $(k - \tfrac{1}{2},\; k - 1) \to (k - \tfrac{1}{2},\, k)$,

2) $(k - \tfrac{1}{2},\, k - 1) \to (k - \tfrac{1}{2},\, k - 1)$,

$$3)\ (k-\tfrac{1}{2},\ k-1)\to(k-\tfrac{3}{2},\ k-1).$$

The probabilities of such chains are, respectively,

$$\frac{2l+1}{2l-1}\frac{1}{k^2(4k^2-1)},\qquad \frac{2l+1}{2l-1}\frac{k-1}{k^2(2k-1)},\qquad \frac{2l+1}{2l-1}\frac{1}{k(2k-1)}.$$

The coefficients β have in the first case the form

$$\beta_0=1,\qquad (\beta_1)_k=1-\frac{1}{k^2},\qquad (\beta_3)_k=\frac{(k^2-1)(4k^2-9)}{k^2(4k^2-1)},$$

and in the second and third cases they are given by formulas (21) and (22), in which we must replace l by $k-1$ and then multiply by $1-1/k(k-1)$ for β_1 and by $1-3/k(k-1)$ for β_3.

Leaving out the intermediate steps, we obtain for the initial state $(l-{}^1/_2,\ l-1)$:

$$(\beta_0)_{--}=\sum_{k=1}^{l}\frac{2l+1}{2l-1}\frac{1}{k^2(4k^2-1)}+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{(k-1)^2}{k^2(2k-1)^2}$$
$$+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{1}{k(2k-1)}\left(1-\frac{2k-1}{k-1}\alpha_2\right)$$
$$\approx\frac{2l+1}{2l-1}\left(\frac{1}{3}+\frac{l-1}{2l+1}-\alpha_2\frac{l-1}{l}\right),$$
$$(\beta_1)_{--}=(\beta_2)_{--}=\sum_{k=1}^{l}\frac{2l+1}{2l-1}\frac{1}{k^2(4k^2-1)}\left(1-\frac{1}{k^2}\right)$$
$$+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{k-1}{k(2k-1)^2}\left(1-\frac{1}{k(k-1)}\right)\left(\frac{k-1}{k}-\alpha_1\right)$$
$$+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{1}{k(2k-1)}\left(1-\frac{1}{k(k-1)}\right)$$
$$\times\left[1-\frac{2k-1}{k-1}(\alpha_2+\alpha_3)\right]\approx\frac{2l+1}{2l-1}\left[\frac{l-1}{2l+1}\right.$$
$$\left.\times\left(1-\frac{\alpha_1}{3}\right)-(\alpha_2+\alpha_3)\frac{l-1}{l}-\alpha_4\right],$$
$$\alpha_4\approx0.022,$$

$$(\beta_3)_{--}=\sum_{k=1}^{l}\frac{2l+1}{2l-1}\frac{(k^2-1)(4k^2-9)}{k^4(4k^2-1)^2}$$
$$+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{k-1}{k(2k-1)^2}\left(1-\frac{3}{k(k-1)}\right)$$
$$\times\left(\frac{k-1}{k}-3\alpha_1\right)+\sum_{k=2}^{l}\frac{2l+1}{2l-1}\frac{1}{k(2k-1)}$$
$$\times\left(1-\frac{3}{k(k-1)}\right)\left[1-\frac{2k-1}{k-1}(\alpha_2+3\alpha_3)\right]$$
$$\approx\frac{2l+1}{2l-1}\left[\frac{l-1}{2l+1}(1-\alpha_1)-\frac{l-1}{l}(\alpha_2+3\alpha_3)+\alpha_5\right],$$
$$\alpha_5\approx0.180. \tag{23}$$

To obtain the final expression for the $\lambda_i^{(f)}$ it is necessary to use formula (19) with $(\lambda_i)_{jF}$ from Table I and $(\beta_i)_{jF}$ from (21)–(23). We then get

$$3\lambda_0^{(f)}=\frac{1}{3}+\frac{4l+3}{(2l+1)^2}+\alpha_2,$$
$$12\lambda_1^{(f)}\approx-\frac{\alpha_1}{3}+(3-\alpha_1)\frac{4l+3}{(2l+1)^2}+(\alpha_2+\alpha_3)\left(3-\frac{1}{l^2}\right)+\alpha_4,$$
$$12\lambda_2^{(f)}\approx1+\frac{4(4l+3)}{(2l+1)^2}+\frac{2}{3}\alpha_1-(\alpha_2+\alpha_3)\left(1+\frac{4l+1}{l^2}\right)+\alpha_4,$$
$$60\lambda_3^{(f)}\approx\frac{4l+15}{(2l+1)^2}+3\alpha_1\left[\frac{1}{3}+\frac{4l+3}{(2l+1)^2}\right]+(\alpha_2+3\alpha_3)$$
$$\times\left(1+3\frac{2l+1}{l^2}\right)+\alpha_5. \tag{24}$$

We have neglected here terms of the form $[l(l+1)(2l+1)^2]^{-1}$.

In formulas (24) the dependence on l is quite weak for large l; putting $l = 15$, which is close to the maximum possible value, we obtain the lower limits for the $\lambda_i^{(f)}$:

$$\lambda_0^{(f)}=0.172,\qquad \lambda_1^{(f)}=0.057,$$
$$\lambda_2^{(f)}=0.097,\qquad \lambda_3^{(f)}=0.019. \tag{25}$$

With decreasing l, the values of $\lambda_i^{(f)}$ increase somewhat.

We present for comparison the results obtained when the width of the excited levels is large compared with the hyperfine splitting, which is manifest only on the K shell. If we take ξ in (18) to mean the vector of muon polarization at the instant when the muon falls on the K shell, then the coefficients $\lambda_i^{(f)}$ for this case can be obtained from Table I, by putting $l = 0$. We have

$$\lambda_0^{(f)}=1,\qquad \lambda_1^{(f)}=\lambda_2^{(f)}=\lambda_3^{(f)}=\tfrac{1}{2}.$$

It must be borne in mind that depolarization due to the fine structure causes ξ in this case to have a value of approximately $^1/_6$ [7], whereas in all the preceding analysis ξ was taken to mean the initial polarization of the muon, i.e., $\xi = 1$.

4. CONCLUSION

We present the results of the calculation for the angular distribution of tritium nuclei in the capture of muons by He^3 nuclei polarized in the direction of the muon beam. We use the values of $\lambda_i^{(f)}$ from (25) and the explicit expressions for a and b (see Sec. 2). We then find that in the case of capture in fully polarized He^3 ($\xi = 1$) the coefficient $a(\lambda_1^{(f)}+\lambda_2^{(f)})$ reaches $\sim 10\%$ in the absence of pseudoscalar interaction; in this case $b\lambda_3^{(f)}\approx 0$. When $C_P/C_A \approx 36$, the coefficient a vanishes and $|b|\lambda_3^{(f)}$ reaches approximately 6%. Thus the asymmetry proportional to $\cos\theta$ (θ — angle between the direction of emission of the recoil nucleus and the direction of the muon beam) can be increased as a result of target-nucleus polarization by approximately 2.5 compared with the asymmetry following capture by unpolarized nuclei, but account must be taken in this case of the term proportional to $P_2(\cos\theta)$. We note that in view of the small value of the coefficient $\lambda_0^{(f)}$ the population of the levels of the hyperfine structure of the mesic atom He^3 will be practically statistical. In the case of partially polarized He^3 nuclei ($\zeta = 0.5$) we have

$$a\,(\lambda_1^{(f)} + \lambda_2^{(f)}) \approx 0.07 \qquad (C_P/C_A \approx 0);$$

$$b\lambda_3^{(f)} \approx -0.04 \qquad (C_P/C_A \approx 36).$$

It must be noted that the coefficient $b\lambda_3^{(f)}$ is negligibly small as $C_P/C_A \rightarrow 0$ for all nuclei with spin $1/2$, and increases with increasing C_P. At the same time, the character of the dependence of a on C_P/C_A turns out to be appreciably different for different nuclei. Thus, for the capture of the muon by a proton the coefficient a is vanishingly small when $C_P/C_A \approx 0$ and increases with increasing C_P, whereas for capture in He^3 it vanishes when $C_P/C_A = 36$, having a minimum at this value. For capture in fully polarized hydrogen with $C_P/C_A = 36$ we have $a\,(\lambda_1^{(f)} + \lambda_2^{(f)}) \approx 10\%$ and $b\lambda_3^{(f)} \approx 0.5\%$.

Thus, the appreciable depolarization of the initial nucleus and of the muon due to the hyperfine interaction leads to a small value of the asymmetry; nonetheless, the experimental observation of the asymmetry of the recoil nuclei would be useful, since the monotonic dependence of the asymmetry coefficient on C_P/C_A over a wide range of possible values of the pseudoscalar constant makes it possible to interpret uniquely the experimental results.

The authors are deeply grateful to I. M. Shmushkevich for continuous interest in the work and for valuable remarks.

APPENDIX

The nuclear matrix elements are denoted as follows:

$$\int O_i\varphi_\mu \equiv \sqrt{\frac{4\pi\,(2I_1+1)}{2I+1}} \int \Psi^*_{I_1M_1} e^{-\alpha Z m r} O_i \tau_- \Psi_{IM}\, d\mathbf{r},$$

Ψ_{IM} and $\Psi_{I_1M_1}$—wave functions of the initial and final states of the nucleus, τ_-—operator for the transformation of a proton into a neutron, and α—fine structure constant.

We present the values of O_i for different nuclear matrix elements:

$\int 1\varphi_\mu$:	$C_0 j_0(qr)$,
$\int \boldsymbol{\sigma}\varphi_\mu$:	$-C_1 j_0(qr)\,\boldsymbol{\sigma}\mathbf{Y}_{1\Lambda}^{-1}$,
$\int \mathbf{r}\mathbf{p}\varphi_\mu$:	$\frac{3}{q} C_0 j_1(qr)\,\mathbf{p}\mathbf{Y}_{00}^{1}$,
$\int [\mathbf{r}\mathbf{p}]\,\varphi_\mu$:	$\frac{\sqrt{6}}{q} iC_1 j_1(qr)\,\mathbf{p}\mathbf{Y}_{1\Lambda}^{0}$,
$\int \mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\,\varphi_\mu$:	$\frac{\sqrt{3}}{q} C_1 j_1(qr)\,Y_{1\Lambda}\boldsymbol{\sigma}\mathbf{p}$,
$\int \mathbf{r}(\mathbf{r}\boldsymbol{\sigma})\,\varphi_\mu - \frac{1}{3}\int r^2\boldsymbol{\sigma}\varphi_\mu$:	$\frac{5\sqrt{2}}{q^2} j_2(qr)\,C_1\boldsymbol{\sigma}\mathbf{Y}_{1\Lambda}^{1}$,

where

$$C_J \equiv [C_{IMJ\Lambda}^{I_1M_1}]^{-1}.$$

Here $C_{\cdots}^{\cdot\cdot}$—Clebsch-Gordan coefficient, $\mathbf{Y}_{L\Lambda}^{T}$—spherical vector, $j_l(qr)$—spherical Bessel function. When the exponential is replaced by unity and only the first terms in the expansion of the Bessel functions are retained, we obtain nuclear matrix elements in the Konopinski-Uhlenbeck notation.

[1] L. D. Blokhintsev and É. I. Dolinskiĭ, JETP **41**, 1986 (1961), Soviet Phys. JETP **14**, 1410 (1962).

[2] G. M. Bukat and N. P. Popov, JETP (in press).

[3] G. Ya. Korenman and R. A. Éramzhyan, JETP **45**, 1111 (1963), Soviet Phys. JETP **18**, 767 (1963).

[4] H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).

[5] A. Fujii and H. Primakoff, Nuovo cimento **12**, 327 (1959). C. Werntz, Nucl. Phys. **16**, 59 (1960).

[6] A. P. Bukhvostov and I. M. Shushkevich, JETP **41**, 1895 (1961), Soviet Phys. JETP **14**, 1347 (1962).

[7] I. M. Shmushkevich, Nucl. Phys. **11**, 419 (1959).

Translated by J. G. Adashko
267