MAGNETOACOUSTIC DIMENSIONAL EFFECTS IN A METAL PLATE

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We have investigated the dimensional effects that arise in the propagation of ultrasound through a metal plate in a magnetic field **H**. If **H** is parallel to the sample surface the oscillations associated with the geometric resonance should exhibit a cutoff when the diameter of the electron orbit becomes larger than the sample thickness. When the magnetic field is at an angle φ with respect to the surface of the plate the dimensional effect would become oscillatory, being a periodic function of the applied field. This dimensional effect is due to electrons close to the limiting point on the Fermi surface. An investigation of the angular dependence of the period and amplitude of the oscillations makes it possible to determine the local values of the Gaussian curvature and the electron mean free path at the Fermi surface. When $\varphi \neq 0$ in a thin plate the amplitude and width of the acoustic absorption are not determined by volume scattering, but by the transit time of resonance electrons from one side of the plate to the other.

1. There has been a great deal of recent interest in the investigation of high-frequency dimensional effects in a magnetic field **H**. In papers by one of the authors [1] and Azbel' [2] it has been shown that the magnetic-field dependence of the impedance should exhibit a number of interesting features when the electron orbits become comparable with the sample size in cyclotron resonance in plane-parallel metal plates.

Khaĭkin [3,4] and Gantmakher [5] have observed a cutoff of electron orbits in polycrystaline tin plates in a magnetic field parallel to the surface of the sample. Investigation of these dimensional effects gives a method for direct measurement of the extreme diameters of the Fermi surface. A new type of dimensional effect was observed by Gantmakher and one of the authors; [6] this effect is associated with the drift motion of the electrons deep in the metal and it was shown that an investigation of this effect can be used to find open trajectories, to measure the local values of the Gaussian curvature, and to determine the electron mean free path at the Fermi surface.

On the other hand it is well known that the energy spectra of metals can be determined by means of ultrasound. It is therefore of interest to consider the possibility of observing various dimensional effects in the propagation of sound through a metal plate in the presence of an external magnetic field. This problem is the subject of the present communication.

2. We compute sound absorption in a plane-parallel metal plate. In what follows we shall be interested in the limiting case of relatively low acoustic frequencies and a strong magnetic field; the temporal dispersion is unimportant but the spatial dispersion is very strong:

$$\omega \ll \nu \ll \Omega, \tag{1.a}$$

$$\lambda \ll d \ll l, \qquad \lambda \ll R \tag{1.b}$$

(λ is the wave length of the sound wave, l is the mean free path, $R = v/\Omega$ is the electron Larmor radius). The dimensional effects are most pronounced in this range of values of the parameters.

In the limiting case (1.a), (1.b) we need only consider the deformation interaction of the electrons with the lattice, neglecting the alternating electric fields arising in the metal in the propagation of the sound wave. In this case the equation describing the elastic oscillations in the metal is [7]

$$\rho \ddot{u}_i = \lambda_{iklm} \frac{\partial^2 u_m}{\partial x_k \partial x_l} + \frac{\partial}{\partial x_k} \int d\tau_{\mathbf{p}}\, \delta(\varepsilon - \zeta)\, \chi(\mathbf{p}, \mathbf{r}, t)\, \Lambda_{ik}(\mathbf{p}). \tag{2}$$

Here $\mathbf{u}(\mathbf{r}, t)$ is the displacement vector associated with the sound wave, ρ is the density of the metal, $\hat{\lambda}$ is the elasticity tensor, $d\tau_p = 2h^{-3}d^3p$, h is Planck's constant, d^3p is the volume element in momentum space, $\Lambda_{ik}(\mathbf{p})$ is the tensor giving the potential deformation, which vanishes after averaging over the Fermi surface $\varepsilon(\mathbf{p}) = \zeta$, $\chi\delta(\varepsilon - \zeta)$ is the nonequilibrium part of the electron distribution function and the dots denote partial differentiation with respect to time; repeated subscripts mean summation from one to three. The last term on the right side of Eq. (2)

represents the force exerted by electron gas on the element of volume of the elastic medium as a consequence of the interaction of the electrons with the lattice.

The linearized kinetic equation for the function χ is [7]

$$\dot{\chi} + \mathbf{v}\nabla\chi + \Omega\, \partial\chi/\partial\tau + \nu\chi = \Lambda_{ik}(\mathbf{p})\, \dot{u}_{ik}. \qquad (3)$$

Here $\mathbf{v} = \partial\mathcal{E}(\mathbf{p})/\partial\mathbf{p}$ is the electron velocity, $\Omega = |e|H/mc$ is the cyclotron frequency, e is the electron charge, c is the velocity of light, $m = \partial S(\mathcal{E}, p_z)/2\pi\partial\mathcal{E}$ is the effective mass of the electron, ν is the frequency of collisions of an electron with scatterers, τ is the dimensionless time (phase) associated with the motion of the electron along the orbit in the magnetic field, u_{ik} is the deformation tensor, $\delta\mathcal{E} = \Lambda_{ik}u_{ik}$ is the energy due to the interaction between the electron and the sound wave.

The boundary conditions for the function χ are determined by diffusion scattering of electrons at the surface of the sample and the periodicity condition on χ with respect to τ (period 2π). If the z-axis is taken along the external normal to the metal surface $z = 0$ the solution of Eq. (3) for monochromatic waves propagating along z is [8]

$$\chi = \frac{1}{\Omega}\int\limits_{\mu(\tau,z)}^{\tau} d\tau' \Lambda_{ik}(\tau')\, \dot{u}_{ik}\Big(z + \frac{1}{\Omega}\int\limits_{\tau}^{\tau'} v_z(\tau'')\, d\tau''\Big)$$

$$\times \exp\Big[\frac{\nu - i\omega}{\Omega}(\tau' - \tau)\Big], \qquad (4)$$

where $\mu(z, \tau)$ represents the root immediately preceding τ of one of the equations

$$z + \frac{1}{\Omega}\int\limits_{\tau}^{\mu} v_z(\tau')\, d\tau' = 0,\ d \qquad (5)$$

($\mu \leq \tau$, d is the plate thickness). Actually $\mu(z, \tau)$ depends on $z - \Omega^{-1}\int v_z d\tau'$. If Eq. (5) does not have solutions we must write $\mu = -\infty$.

Multiplying Eq. (2) by $\dot{u}_i$ and averaging over the volume we find

$$\Big\langle \frac{1}{2}\rho\frac{d\dot{u}_i^2}{dt}\Big\rangle = -\int d\tau_{\mathbf{p}}\delta(\varepsilon - \zeta)\,\langle\chi\Lambda_{ik}\dot{u}_{ik}\rangle. \qquad (6)$$

(Here we have neglected surface integrals. It can be shown that these give unimportant corrections.) The angle brackets denote averages over the sample thickness. Thus, the averaged absorption coefficient α is

$$\alpha \equiv \frac{1}{W}\Big\langle\frac{1}{2}\rho\frac{d\dot{u}_i^2}{dt}\Big\rangle = \frac{1}{W}\operatorname{Re}\int d\tau_{\mathbf{p}}\delta(\varepsilon - \zeta)\,\langle\chi\Lambda_{ik}\dot{u}_{ik}^*\rangle, \qquad (7)$$

where $W = \frac{1}{2}\rho\,|\dot{u}|^2 s$ is the flux density of acoustic energy and s is the sound velocity. The mean change in the acoustic velocity Δs is determined by the imaginary part of the integral in Eq. (7).

The absorption (as well as the velocity dispersion) of the sound wave due to the electrons is a relatively small effect and is proportional to the small parameter m/M (M is of the order of the ion mass). Hence in finding α we can use successive approximations; as the zeroth approximation in Eq. (7) we use the unperturbed plane wave $u_i^0 \exp(i\mathbf{k}\cdot\mathbf{r})$.

The effect of finite plate thickness on the propagation of ultrasound is due to the collisions of electrons with the boundaries ($\mu(\tau, z) \neq -\infty$). However, it is impossible to solve Eq. (5) in general form and to find $\mu(\tau, z)$ in explicit form. Hence we investigate certain particular cases.

3. We first consider the case in which the magnetic field $\mathbf{H} = (H, 0, 0)$ is parallel to the surface of the sample. In a strong field the maximum dimension of the electron orbit D_{max} in the z direction is smaller than d (the x axis is parallel to $\mathbf{H}$). In this case an electron with a given projection p_x on the Fermi surface can reach only one of the plate boundaries, where it experiences diffusion scattering. It is evident that the contribution of these "colliding" electrons in absorption (and dispersion) of the sound wave is at least $\Omega/|\nu - i\omega| \gg 1$ times smaller than the contribution of electrons that move inside the plate without colliding with the boundaries. (An exact calculation verifies this conclusion.) Hence, in computing the mean absorption coefficient we need only consider electrons that do not collide with the surface; for these $\mu(\tau, z) = -\infty$.

In averaging α over the plate thickness the limits of integration over z are given by

$$z_1 < z < z_2,$$

where the points

$$z_1 = \frac{c}{|e|H}\,(p_y(\tau) - p_{y\,min}(\zeta, p_x)),$$

$$z_2 = \frac{c}{|e|H}(p_y(\tau) - p_{y\,max}) + d,$$

(the maximum and minimum of p_y are taken only as functions of τ) are the boundaries of the regions in phase space that contain electrons that collide and electrons that do not collide with the surfaces of the plate. Inasmuch as the distribution function for these electrons χ does not depend on z the average in Eq. (7) results in a factor $(z_2 - z_1)/d = 1 - D(\zeta, p_x)/d$ under the integral sign over p where $D(\zeta, p_x) = (c/|e|H)(p_{y\,max} - p_{y\,min})$ is the diameter of the electron orbit (with a given energy and projected momentum p_x) in the z-direction.

In a weaker magnetic field the maximum (in p_x) diameter D_{max} is greater than d. Orbits of electrons with $D(p_x) > d$ are not contained within the plate, that is to say, collisions are experienced in each orbit. Hence, in integrating over p_x we consider only those electrons for which $D(p_x) \leq d$. Thus, the general expression for the mean absorption coefficient α in this case is

$$\alpha = \mathrm{Re}\,\frac{2}{h^3 W} \int\limits_{D(p_x)\leqslant d} dp_x\, m\left(1 - \frac{D(\zeta, p_x)}{d}\right) \int\limits_0^{2\pi} d\tau\, \Lambda \dot{u}^* \chi. \quad (8)$$

When $d \to \infty$ this expression goes over to the familiar relation for the absorption coefficient in an infinite metal (cf. for example [9,10]).

In computing the integrals over τ in Eq. (8) we use the method of stationary phase ($kR \gg 1$). The usual calculations for the simplest case of a closed convex surface lead to the expression

$$\alpha = \frac{2|e|H}{h^3 Wck}\,\mathrm{Re} \int\limits_{D(p_x)\leqslant d} \frac{dp_x\,[1 - D(p_x)/d]\, m}{\nu - i\omega}$$

$$\times \left\{ \sum_{\alpha=1}^{2} \frac{|g_\alpha^2|}{|p''_{y\alpha}|} + \frac{2 g_1 g_2^*}{|p''_{y1} p''_{y2}|^{1/2}} \sin kD(p_x) \right\}; \quad (9)$$

$$g_\alpha = \Lambda_{ik}(\zeta, p_x, \tau_\alpha)\, \dot{u}_{ik}, \qquad p''_{y\alpha} = \partial^2 p_y(\zeta, p_x, \tau_\alpha)/\partial\tau^2,$$

where the $\tau_\alpha(\zeta, p_x)$ are the points of stationary phase, i.e., solutions of the equation $kv_z(\zeta, p_x, \tau) = \omega$.

The first term in the curly brackets in Eq. (9) gives the monotonic (nonoscillating) part of the acoustic absorption due to electrons in the magnetic field; the second term describes the magnetoacoustic oscillations of α as a function of H (geometric resonance). The monotonic part of the absorption α_M increases linearly with H when $d > D_{max}$ and becomes proportional to H^2 when $D_{max} \gg d$. When $d = D_{max}$ the derivative $d\alpha_M/dH$ exhibits a discontinuity at which $d^2\alpha_M/dH^2$ becomes infinite in this approximation. (Actually, at this point $|d^2\alpha_M/dH^2| \sim \Omega/\nu - i\omega$.)

Oscillations of the absorption for the geometric resonance occur only when $d > D_{max}$ (more precisely when $d - D_{max} \gg \lambda$) and the amplitude of the oscillations in $(1 - D_{max}/d)^{-1}$ is smaller than in an infinite metal.

In weak fields, in which case $d < D_{max}$, the amplitude of the geometric resonance in the plate is reduced sharply (by a factor $\Omega/|\nu - i\omega|$ compared with the case $d > D_{max}$) since it is determined by electrons in the vicinity of the central cross-section of the Fermi surface. When $d < D_{max}$ the resonance orbits are not contained within the plate and part of the electron orbit is cutoff; this is analogous to the effect in cyclotron resonance [1-4] and in the anomalous skin effect at low frequencies.[5]

If the Fermi surface is complicated and there are several extremum dimensions the oscillation of the geometric resonance with a given period will vanish depending on the extent to which the appropriate orbit remains inside the plate.

4. If the magnetic field is inclined with respect to the plate oscillations of the dimensional effect periodic in the applied field will arise by virtue of electron drift from one surface to the other. Physically this oscillatory behavior is related to the "focusing" of electrons by the external magnetic field.[6] The amplitude is determined by the relatively small circle around the reference point on the Fermi surface corresponding to a given direction of **H**. At the reference point the electron velocity is parallel to **H**. Hence, for these electrons the function $\mu(\tau, z)$ can be found easily if the electron gyration is neglected and v_z is replaced by the average value $\bar{v}_z = (2\pi)^{-1}\int v_z d\tau$. As a result we have

$$\mu(\tau, z) = \begin{cases} \tau - \Omega z/\bar{v}_z, & \bar{v}_z > 0 \\ \tau + \Omega(d - z)/\bar{v}_z, & \bar{v}_z < 0 \end{cases}. \quad (10)$$

Using Eq. (8) we can write the expression for the average absorption coefficient in the form

$$\alpha = \frac{2}{h^3 Wd}\,\mathrm{Re}\int dp_H \frac{m}{\Omega} \int\limits_0^{2\pi} d\tau\, g^*(\tau, p_H) \int\limits_0^d dz \quad (11)$$

$$\times \int\limits_{\tau - \Omega z/|\bar{v}_z|}^{\tau} d\tau'\, g(\tau', p_H) \exp\left[\frac{\nu - i\omega + ik\bar{v}_z}{\Omega}(\tau' - \tau)\right].$$

We expand the function $g(\tau, p_H)$ in a Fourier series in τ:

$$g(\tau, p_H) = \sum_{-\infty}^{\infty} g_n(p_H)\, e^{in\tau}.$$

The results of the calculation of the integrals over τ, τ' and z are

$$\alpha = \frac{4\pi}{h^3 W}\,\mathrm{Re}\int m\,dp_H \sum_{n=-\infty}^{\infty} \frac{|g_n(p_H)|^2}{\tilde{\nu}}$$

$$\times \left\{1 - \frac{|\bar{v}_z|}{d} \frac{1 - \exp(-d\tilde{\nu}/|\bar{v}_z|)}{\tilde{\nu}}\right\}, \quad (12)$$

$$\tilde{\nu} = \nu - i\omega + ik\bar{v}_z + in\Omega. \quad (13)$$

The first term in the curly brackets represents the absorption of sound by electrons in the infinite metal α_∞ investigated by Gurevich.[9] It gives a smooth dependence of the absorption coefficient on H. Actually, when $\nu \to 0$

$$\mathrm{Re}\,\tilde{\nu}^{-1} \to \pi\delta(k\bar{v}_z - n\Omega - \omega). \quad (14)$$

In a strong field, in which $k\bar{v}_{z\,max} \leq \Omega$, the argument of the δ-function can vanish only when $n = 0$ and only one term remains in the summation over n. As H is reduced the summation over n will start to include more and more terms $|n| = 1, 2, \ldots$ etc. Near the limiting or reference point (pole)

$$g_n(p_H) = a_n|(p_H - p_{H\,ext})/p_{H\,ext}|^{1/2},$$

the appearance of new terms with $|n| \neq 0$ leads to a discontinuity in the derivative of the absorption coefficient as a function of magnetic field ($d^{|n|}\alpha_\infty/dH^{|n|}$). The absorption coefficient itself changes smoothly. The second nonoscillating term in Eq. (12) is evidently $kd \gg 1$ times smaller than the first and can be neglected. The oscillation in absorption due to the finite plate thickness is described by the last term in Eq. (12), which contains an exponential.

If

$$|\bar{v}_z|/\Omega \ll d \tag{15}$$

the basic contribution to the integration over p_H comes from the vicinity of the reference point

$$|\Delta p_H|/|p_{H\,ext}| \sim \bar{v}_{z.\mathrm{ref}}\ \Omega d \ll 1.$$

Expanding the exponential in powers of p_H close to the reference point and keeping the linear term we find

$$\Delta\alpha_{\mathrm{osc}} = \frac{16\pi\sin\varphi\,|a_1|^2}{h^3Wk^2d^3\,|p_Hv_H|}\exp\left(-\frac{\nu d}{|v_H|\sin\varphi}\right) \times\left[\frac{d}{dp_H}\left(\frac{\Omega}{|v_H|}\right)\right]^{-2}\cos kd\cos\left(\frac{\Omega d}{v_H\sin\varphi}\right). \tag{16}$$

The values of all functions are taken at the reference point. In Eq. (16) we retain only the basic oscillatory terms with $|n| = 1$.

The period of the oscillation in magnetic field ΔH is given by

$$\Delta H = \frac{2\pi c}{|e|}\sin\varphi\frac{mv_H}{d} = \frac{2\pi c}{|e|}\frac{\sin\varphi}{K^{1/2}d}, \tag{17}$$

where K is the local value of the Gaussian curvature at the reference point, φ is the angle of inclination of the field, **H** with respect to the surface of the plate. The oscillations amplitude is small and is a diminishing function of magnetic field:

$$\frac{\Delta\alpha_{\mathrm{osc}}}{\alpha_\infty} \sim \frac{v_H^2\sin\varphi}{kd^3\Omega^2}\exp\left(-\frac{d}{l\sin\varphi}\right). \tag{18}$$

It is evident from Eqs. (17) and (18) that an investigation of the dependence of the period and amplitude of the dimensional effect can be used to measure directly the local values of the Gaussian curvature and the electron mean free path at all elliptic reference points of the Fermi surface.

A similar dimensional effect should be observed in a magnetic field parallel to the surface in the presence of open trajectories. In this case the oscillation period is given by

$$\Delta H = \frac{2\pi b\,c\cos\theta}{|e|d}, \tag{19}$$

where b is the period of the open trajectories (in momentum space) and θ is the angle between the direction of the opening and the normal to the surface of the plate.

5. When ultrasound is propagated at an angle with respect to the magnetic field in an infinite metal the absorption and dispersion of the acoustic velocity exhibit resonance oscillations that are periodic in the reciprocal field. [10] The position of the resonance peaks is given by $(\overline{\mathbf{k}\cdot\mathbf{v}/\Omega})_{\mathrm{ext}} \approx n$ (n is an integer which is an extremum in p_H) and the width is due to collisions of electrons with scatterers. In a thin plate in which d, the thickness, is small compared with the mean free path along the normal to the surface $l\sin\varphi$ smearing and reduction in the heights of the resonance peaks should be observed. This effect arises because the effective mean free path for resonance electrons is the quantity $d/\sin\varphi$.

The above statement can be verified rather simply. We use Eq. (10) for $\mu(z, \tau)$. The averaged absorption coefficient α is then

$$\alpha = \frac{2}{h^3W}\mathrm{Re}\int\frac{m\,dp_H}{\Omega}\int g^*(\tau)\,d\tau\int_{\tau-\Omega d/|\bar{v}_z|}^{\tau}d\tau' g(\tau', p_H) \times\exp\left\{\int_{\tau}^{\tau'}\frac{\nu - i\omega + i\mathbf{kv}}{\Omega}d\tau''\right\}\left[1 - \frac{(\tau-\tau')}{\Omega d}|\bar{v}_z|\right]. \tag{20}$$

When $d \ll |v_z|/|\nu - i\omega|$ we can neglect $\nu - i\omega$ in the exponential. If $(\Omega d/|v_z|)_{\mathrm{ext}} \gg 1$ in this case, Eq. (20) for α becomes

$$\alpha = \frac{2}{h^3W}\mathrm{Re}\int\frac{m\,dp_H}{\Omega}\sum_{j=0}^{N-1}\left(1 - \frac{j}{N}\right)\exp\left(-2\pi ij\frac{k\bar{v}_z}{\Omega}\right) \times\int_0^{2\pi}g^*(\tau)\,d\tau\int_{\tau-2\pi}^{\tau}d\tau' g(\tau', p_H)\exp\left(i\int_{\tau}^{\tau'}\frac{kv_z}{\Omega}d\tau_2\right),$$

where $N = E(\Omega d/2\pi\,|\bar{v}_z|)$, while $E(x)$ is taken for discrete values of x.

Carrying out the summation over j we have

$$\sum_{j=0}^{N-1}\left(1 - \frac{j}{N}\right)e^{2\pi ijx} = \left(1 + \frac{1}{2\pi iN}\frac{\partial}{\partial x}\right)\frac{1 - e^{-2\pi iNx}}{1 - e^{-2\pi ix}},$$

$$x = \frac{k\dot{v}_z}{\Omega}. \tag{21}$$

The real part of the functional (21) for $N \gg 1$ is a sum of smeared-out δ-functions

$$\frac{1}{2}\sum_{n=-\infty}^{\infty}\delta_N(x-n)\approx\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\frac{N^{-1}}{N^{-2}+(x-n)^2}$$
$$=\sum_{n=-\infty}^{\infty}\frac{|\bar{v}_z|/d\Omega}{(k\bar{v}_z/\Omega-n)^2+(2\pi\bar{v}_z/\Omega d)^2}. \qquad (22)$$

It follows from Eq. (22) that the role of the effective collision frequency ν_{eff} in this case is played by the quantity $2\pi\,|\,\bar{v}_z\,|/d$, i.e., the characteristic mean free path $l_{\mathrm{eff}}\sim d/\sin\varphi$ is determined by the thickness of the sample and the orientation of the field with respect to the surface of the plate.

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