ON THE CHARACTER OF LOW-ENERGY INTERACTIONS OF PIONS FROM THE REACTIONS $p + d \to He^3 + 2\pi$ *AND* $\pi + N \to N + 2\pi$

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THE information currently available on the characteristics of low-energy pion interactions is extremely contradictory. Abashian, Booth, and Crow [1] investigated the energy spectrum of He^3 in the reaction $p + d \to He^3 + 2\pi$ and found a maximum in the spectrum near the highest possible energy values. This phenomenon was attributed to the large pion scattering length in the zero isotopic spin state ($a_0 \approx 2.5$). Evidence of the presence of strong pion interaction at low energy was also found by Richter et al [2] in their work on the $\gamma + p \to p + 2\pi$ reaction and by Button et al [3] in work on the $p + \bar{p} = 6\pi$ reaction. On the other hand, no pion resonance interactions were detected in a study of the $\pi + N \to N + 2\pi$ reaction by Batusov et al [4]. It was found, to the contrary, that the probability of production of two pions with low kinetic energy is comparatively small. With increase in the pion relative kinetic energy the production probability increases noticeably. Nor was any evidence of the existence of a low energy resonance in two-pion systems found in several other studies [5-8] of the $\gamma + N \to N + 2\pi$ reaction. An investigation of the spectrum of pions in τ decay indicated that the scattering length a_0 cannot be large; the experimental data agree with the theoretical curves only for $a_0 \lesssim 1$.[8]

We shall show here that the difference between the behavior of the energy distributions with respect to $\sqrt{s}$ ($\sqrt{s}$ being the total energy of the produced pions in their c.m.s.) at $\sqrt{s} \sim 2$ in the reactions $p + d \to He^3 + 2\pi$ and $\pi + N \to N + 2\pi$ can be explained by the presence near $s = 4$ of a logarithmic singularity in the production amplitudes. This logarithmic singularity was discovered by Aitchison [9] (Fig. 1a). This diagram shows that the intermediate state of the reaction $\pi + N \to N + 2\pi$ consists of a pion and the isobar $(3/2, 3/2)$. His analysis of this diagram is, however, not entirely correct, for no account was taken of

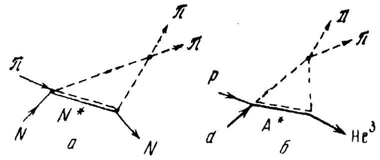

FIG. 1

the existence of a threshold singularity for the pion-nucleon scattering amplitude, which means that the amplitude pole corresponding to the isobar $(3/2, 3/2)$ lies on the second (unphysical) sheet. As a consequence there is no deformation of the integration contour in the dispersion integral over s for any W (total energy of the system).

The location of Aitchison's logarithmic singularity depends on W. It approaches the physical region near $s = 4$ only when $W \approx \mathrm{Re}\, M^* + 1$ ($M^* = 8.9 - 0.3i$ is the complex mass of the isobar in units of pion mass). If our interest is only in the energy distribution with respect to s, then the amplitude of the $\pi + N \to N + \pi + \pi$ reaction with $W \approx M^* + 1$ can be represented as follows:[10]

$$A(s, W) =$$

$$A_0\left[f + i\frac{\alpha\Gamma}{4\beta}\ln\frac{(\alpha + \sqrt{s-4})(\alpha^2 + 4 - s)}{(\alpha - \sqrt{s-4})((W-M)^2 - 4)}\right]; \tag{1}$$

where A_0 is a normalization constant, f an unknown complex constant of the order of unity, a the pion scattering length, M the nucleon mass;

$$\beta\alpha = \tfrac{1}{2}(M^2 + W^2) - M^{*2} - 1,$$

$$\beta = \tfrac{1}{4}\left[\left[(W+M)^2 - 4\right]\left[(W-M)^2 - 4\right]\right]^{1/2},$$

$$\Gamma = -\,\mathrm{Im}\, M^{*2}. \tag{1a}$$

Since f embodies, among others, Breit-Wigner poles with respect to other pairing energies, in general its magnitude at a fixed W can depend strongly on s. f can be considered effectively constant only if formula (1) is used subsequently to determine the energy distribution with respect to s. Since $a \lesssim 1$ for $W \approx M^* + 1$, the logarithm in (1) is approximately zero at $s \sim (W - M)^2$. In this case the probability of two-pion production is determined by the constant f only. With decreasing s, the logarithm changes slowly, provided s stays large. The logarithm changes rapidly and becomes significantly different from zero only for values of s in the neighborhood of 4. The absolute value of the expression in the square brackets can then become, depending upon the sign of f, either larger or smaller than the absolute value of f. Thus the proximity of the logarithmic singularity to the physical region can lead to two dif-

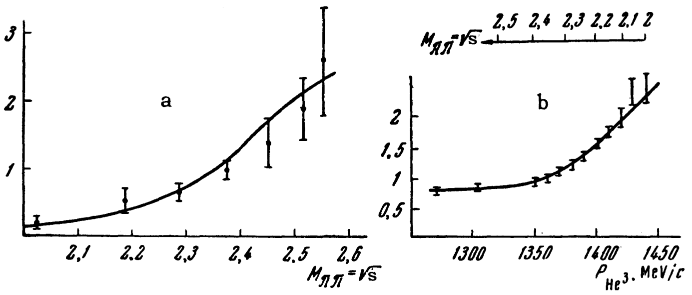

FIG. 2. Energy distribution of pions in the reactions $\pi^- + p \to n + \pi^+ + \pi^-$ (a) and $p + d \to He^3 + 2\pi$ after division by the phase space volume.

ferent effects: (1) to a fairly sharp increase in the probability of pion production when s is close to 4, or (2) to an equally sharp decrease in this probability for $s = 4$.

In the $\pi + N \to N + \pi + \pi$ reaction the logarithmic singularity lies near the physical region for incident pion energies of the order of 300—600 MeV ($W = 9.5—11$). Batusov et al [4] studied the $\pi^- + p \to n + \pi^- + \pi^+$ reaction with incident pion energy of 290 MeV ($W = 9.4$). Figure 2a shows the energy distribution with respect to $\sqrt{s}$ divided by the phase space volume. Also shown is the curve obtained from (1) for $f = 1.16$ and $a = 1$ (for this reaction a is determined by the scattering length a_0, which is evidently of the order of unity. From Fig. 2a it is clear that the second effect is realized above in the $\pi^- + p \to n + \pi^- + \pi^+$ reaction and the production probability decreases when $s = 4$. The increase in the two-pion production probability at large s may be manifest by the appearance of a maximum at the end of the spectrum in the distribution with respect to $M_{\pi\pi} = \sqrt{s}$. This effect was observed by Kirz et al [11] in analyzing the $\pi^- + p \to n + \pi^+ + \pi^-$ reaction at incident pion energies 300—700 MeV. At 700 MeV this "maximum" disappeared (at such energy the logarithmic singularity is sufficiently remote from the physical region). In reactions where the produced pions cannot have zero isotopic spin (for example, for $\pi^+ + \pi^+$ or $\pi^+ + \pi^0$ production), the drop in production probability at $s \sim 4$ should be insignificant, for in such cases the quantity a in (1) is determined only by a_2 (the scattering length of pions with $T = 2$) which is evidently small, and thus the second term in the formula is significantly less than 1 in absolute value. In fact, no drop in the production probability was detected in the reaction $\pi^+ + p \to n + \pi^+ + \pi^+$ at $s = 4$.[11]

A logarithmic singularity of the type discussed can explain the increase in the production probability of pions at $s \sim 4$ in the reaction $p + d \to He^3 + 2\pi$.[1] The pion should form an "isobar" in a nucleus composed of three nucleons. Then the amplitude of this reaction includes the diagram shown in Fig. 1b, where A^3 is an "isobar" with mass $2M + M^*$. Abashian et al,[1] studied this reaction at just those energies for which the logarithmic singularity due to the diagram approaches the physical region near $s = 4$. The reaction amplitude is then given by formula (1) (with appropriate change in mass). In Fig. 2b the energy distribution with respect to $\sqrt{s}$ divided by the phase space volume, is shown for an incident nucleon energy $E_p = 743$ MeV. Also shown is the curve obtained from (1) with $f = -0.5 + 0.24i$ and $a = 1$. With decrease in E_p the logarithmic singularity moves away for the physical region; however, it still makes a noticeable contribution to the amplitude at $s \sim 4$ for those E_p at which this reaction was investigated by Abashian et al.[1] This analysis shows that there is no reason to think that the scattering length a_0 is large. The authors learned recently that a similar explanation of the experimental data of Abashian et al. was proposed by B. N. Valuev in the fall of 1963.

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163