QUANTUM OSCILLATIONS OF THE FIELD EMISSION CURRENT FROM METALS IN A MAGNETIC FIELD

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Oscillations of the field-emission current from a metal in a magnetic field perpendicular to the sample surface are investigated theoretically. The oscillations are shown to have two causes: 1) oscillations of the number of electronic states in a magnetic field, and 2) oscillations of the chemical potential of the metal. The second type of oscillations usually has a considerable amplitude; the first type is appreciable for metals containing only small electron groups (and in a few other cases). The properties of emission from a complex cathode of the metal—thin dielectric film—thin metallic film type are considered.

THE study of the quantum oscillations of various physical quantities (magnetic susceptibility—the deHaas-vanAlphen effect, electric conductivity—the Shubnikov-deHaas effect etc.) is at present, along with the study of cyclotron resonance and magneto-acoustic oscillations, one of the most effective means of investigating the Fermi surfaces of metals. All oscillatory phenomena enable us to determine the same spectral characteristics (extremal cross sections of the Fermi surface and, less accurately, the effective masses); however, the investigation of some particular effect may be the most convenient in specific instances. In an earlier publication[1] (see also[2]) it was shown that by studying oscillations of the tunnel current between two metals separated by a thin dielectric film one determines not only the extremal cross sections but also the effective masses more accurately than ordinarily (without utilizing the temperature dependence of the effect). It was also shown that oscillations of the tunnel current depend greatly on oscillations of the chemical potential of metals.

Oscillations of the field emission current from a metal, which are considered in the present work, are also essentially related to the dependence of the chemical potential (and therefore of the work function) on the magnetic field.

1. Electrons can be extracted from metals by a strong electric field **F**; this is known as field emission. The current is the result of the tunneling of electrons through the potential barrier (Fig. 1). The current density vector is parallel to **F** and has the magnitude (at low temperatures)

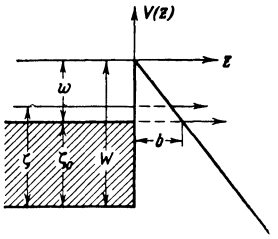

FIG. 1

$$j_0(\zeta_0) = -\frac{2e}{h^3}\int\limits_{(v_z>0)} v_z D(\mathbf{p}) f(E(\mathbf{p}))\, d^3\mathbf{p} = -\frac{2e}{h^3}\int\limits_{E_{min}}^{\zeta_0} \Phi(\varepsilon)\, d\varepsilon. \quad (1)$$

Here $E(\mathbf{p})$ is the electron dispersion law; $v = dE/dp$; D is the transmission coefficient of the barrier; f is the Fermi distribution function; ζ_0 is the chemical potential;

$$\Phi(\varepsilon) = \int\limits_{\Sigma(\varepsilon)} D(\varepsilon, p_x, p_y)\, dp_x dp_y;$$

$\Sigma(\epsilon)$ is the projection of the part of the surface $E(\mathbf{p}) = \epsilon$ where $v_z > 0$, on the $p_x p_y$ plane; the z axis is parallel to **F** and is therefore perpendicular to the surface of the sample.

Our present calculations were performed using an arbitrary function $D(\mathbf{p})$; the free electron model was used to derive estimates.[1)]

For the simplest type of potential barrier

[1)]Our next communication will be devoted to determining a transmission coefficient for an arbitrary dispersion law. Suitable assumptions are made in cases in which a complicated dispersion law leads to qualitatively different results.

(Fig. 1) described by

$$V(z) = \begin{cases} -W & \text{inside the metal } (z<0) \\ -eFz & \text{outside the metal } (z>0) \end{cases} \quad (2)$$

Equation (1) gives[3]

$$j_0(\zeta_0) = \begin{cases} -\pi e m_0 h^{-3} (eFa_0)^2 D_{max}, & eFa_0 \ll \zeta_0 \\ -2\pi e m_0 h^{-3} \zeta_0^2 D_{max}, & \zeta_0 \ll eFa_0 \end{cases}, \quad (3)$$

where $D_{max} \sim \exp(-4w/3eFa_0)$, $a_0 = h/\sqrt{2m_0 w}$ is a distance of the order of the lattice constant, and $w = W - \zeta_0$ is the work function. This expression is valid for $w \gg eFa_0$ (corresponding to fields $F \ll 10^7 - 10^8$ V/cm) or $b \gg a_0$ (b is the minimum width of the potential barrier), which condition we shall assume. In view of this last condition, the deviation of the real potential from (2) does not affect the correctness (with logarithmic accuracy) of (3). The main (exponential) factor of (3) is also similar in the case of an arbitrary dispersion law, because it is determined by the electron wave function inside the barrier, i.e., outside the metal.

Since the transmission coefficient depends very sharply on p, an essential contribution to emission comes from electrons within a small region of p-space where D is close to its maximum value. For free electrons this region is close to the surface of the Fermi sphere, where p_z is maximal.

According to (3) measurable currents appear when w/eFa_0 is of the order of a few tenths, i.e., when $eFa_0 \sim 10^{-2} - 10^{-1}$ eV, $F \sim 10^6 - 10^7$ V/cm.

2. When a monocrystalline sample is placed in a magnetic field the emission current is modified by the quantization of electron energy. We consider the case of a magnetic field perpendicular to the metal surface ($\mathbf{H} \parallel Oz$). In the quasiclassical approximation (large n) quantized levels of orbital motion $\epsilon_n(p_z)$ are determined from the familiar relation[4,5]

$$S(\varepsilon, p_z) = ehHc^{-1}(n + 1/2), \quad (4)$$

where $S(\epsilon, p_z)$ is the area of the section through the surface $E(\mathbf{p}) = \epsilon$ by the plane $p_z = \text{const}$; we assume that the orbits bounding these cross sections are closed and non-selfintersecting.

In this approximation, to calculate the current corresponding to an electron state (n, p_z) we must average the expression for the current over the orbit in the absence of a magnetic field, using the classical law of orbital motion[6]

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c}[\mathbf{vH}], \quad \text{i.e.} \quad \frac{dl}{dt} = \frac{e}{c} v_\perp H. \quad (5)^*$$

This gives for the density of the field emission current in a magnetic field

$$j = -\frac{e^2 H}{h^2 c} \sum_\sigma \int \sum_n \langle v_z D \rangle_{np_z} f(\varepsilon_n(p_z) + \sigma\mu_0 H)\, dp_z. \quad (6)$$

Here $\sigma = \pm 1/2$ is the projection of electron spin and $\mu_0 = e\hbar/m_0 c$; summation is over all orbits containing segments with $v_z > 0$ (Fig. 2) from which an electron can pass outside when it approaches the metal surface; the symbol $\langle\ \rangle_{np_z}$ denotes the aforementioned averaging over an orbit.

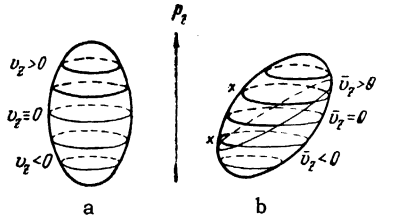

FIG. 2. Orbital segments with $v_z > 0$ are denoted by thick lines. Crosses denote a pair of symmetric orbits.

Using the central symmetry of constant-energy surfaces, the contributions of symmetric orbits (Fig. 2b) can be combined, thus transforming (6) into

$$j = -\frac{e^2 H}{h^2 c} \sum_\sigma \int_{(\bar{v}_z>0)} \sum \overline{(|v_z|D)_{np_z}} \times f(\varepsilon_n(p_z) + \sigma\mu_0 H)\, dp_z \quad (7)$$

(in this and the following equations D(p) must be understood to mean D(p sign v_z). Here the summation includes only orbits on which $v_z > 0$; the bar denotes time averaging in accordance with the orbital motion law (5):

$$\overline{|v_z|D} = \oint |v_z| D \frac{dl}{v_\perp} \Big/ \oint \frac{dl}{v_\perp}. \quad (8)$$

Equations (7) and (8) are, of course, valid when the sign of v_z does not change along the orbit (Fig. 2a).

It is easily seen that [2)]

$$\bar{v}_z|_{np_z} = \left(-\frac{\partial S}{\partial p_z} \Big/ \frac{\partial S}{\partial \varepsilon}\right)_{np_z} = \frac{d\varepsilon_n}{dp_z}, \quad (9)$$

$$\overline{(|v_z|D)_{np_z}} = \left[\frac{1}{\delta p_z} \int_{\delta\Sigma} D(\varepsilon, p_x, p_y)\, dp_x dp_y \Big/ \left|\frac{\partial S}{\partial \varepsilon}\right|\right]_{np_z}, \quad (10)$$

where the region $\delta\Sigma$ is the projection on the $p_x p_y$ plane of the portion of the surface $E(\mathbf{p}) = \epsilon$ lying between the plane p_z of a quantized orbit and the

*$[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

[2)]We note that (9) was obtained in[7] when quasiclassical wave functions were used in calculating the quantum-mechanical mean velocity v_z of an electron in a magnetic field.

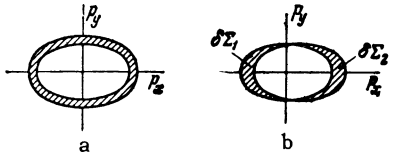

FIG. 3. The shaded region $\delta\Sigma$

infinitesimally close plane $p_z + \delta p_z$ (Fig. 3).

Transforming in the integral (7) to the variable $\epsilon_n(p_z)$ by means of (9) and using (10), we obtain

$$j = -\frac{e^2H}{h^2c}\sum_\sigma\int\sum_n D(\varepsilon, n) f(\varepsilon + \sigma\mu_0 H)\, d\varepsilon,$$

where

$$D(\varepsilon, n) = \left[\frac{1}{|\delta S|}\int_{\delta\Sigma} D(\varepsilon, p_x, p_y)\, dp_x dp_y\right]_n \qquad \left(\delta S = \frac{\partial S}{\partial p_z}\delta p_z\right) \tag{11}$$

plays the part of the effective transmission coefficient. Further calculations follow the customary procedure for distinguishing the oscillating parts of the physical quantities,[4] and yield

$$j = \frac{1}{2}\sum_\sigma j_0(\zeta + \sigma\mu_0 H) + j_1^{osc} + j'; \tag{12}$$

where ζ is the chemical potential of the metal in a magnetic field and contains a small added oscillating term:[4] $\zeta = \zeta_0 + \zeta^{osc}$; j_1^{osc} is an oscillating term; j' is a term that is monotonically dependent on H and vanishes when H = 0 (this term will not interest us).

Equation (12) shows that the current oscillations in a magnetic field have two causes:

1) The oscillatory variation of the number of electron states (the term j_1^{osc}). These are ordinary de Haas-van Alphen oscillations.

2) Oscillations of the chemical potential [the first term of Eq. (12)]. This effect results from the variation of the barrier minimum, and therefore of the transmission coefficient D_{max}, as ζ varies (Fig. 1). For $\zeta_0 \ll eFa_0$ the principal role is played by the dependence of the current on ζ, as determined from the pre-exponential factor [see Eq. (3)].

3. The oscillating part j_1^{osc} of the current is the sum of terms associated with each orbit on the Fermi surface that bounds an extremal area $S_m(\zeta)$ (assuming henceforth $\zeta = \zeta_0$). The expressions determining each oscillating term will differ in the following cases: a) $v_z = 0$ on an extremal orbit (Fig. 2a); b) the sign of v_z is reversed on an extremal orbit (Fig. 2b). In both cases $v_z = 0$ on an extremal orbit [see Eq. (9)].

In case a) the effective transmission coefficient on an extremal orbit, $D(\zeta, n_m) = D$, is the mean value of the true transmission coefficient in the absence of a magnetic field for the region $\delta\Sigma$ [see Eq. (11)], since the area of $\delta\Sigma = |\delta S|$ (Fig. 3a). The oscillating term associated with this orbit is

$$j_1^{osc} = \pm\frac{2em}{\pi\hbar^3}(\mu H)^2 D\sum_{p=1}^{\infty}\frac{(-1)^p}{p^2}\Psi(p\lambda) \times\cos\left(\pi p\frac{m}{m_0}\right)\cos\left(p\frac{cS_m(\zeta)}{e\hbar H}\right); \tag{13}$$

$$\mu = \frac{e\hbar}{|m|c}, \qquad m = \frac{1}{2\pi}\frac{dS_m}{d\zeta},$$

$$\Psi(u) = \frac{u}{\sinh u}, \qquad \lambda = 2\pi^2\frac{kT}{\mu H};$$

the upper sign will henceforth pertain to S_m = max and the lower sign to S_m = min. Equation (13) and the subsequent results are correct when the condition for quasiclassical quantization, $\mu H \ll \zeta$, is satisfied.

In case b) the effective transmission coefficient (11) on an extremal orbit becomes infinite, since

$$|\delta S| = |\text{area } \delta\Sigma_1 - \text{area}|\delta\Sigma_2| = 0,$$

$$\text{area } \delta\Sigma = \text{area } \delta\Sigma_1 + \text{area } \delta\Sigma_2 \neq 0$$

(Fig. 3b). The oscillating current component associated with this orbit is

$$j_1^{osc} = -\text{sign } m\sqrt{\frac{2}{\pi}}\frac{em^{1/2}}{\hbar^3}\frac{\oint|\operatorname{tg}\varphi|\,dl}{|\partial^2 S/\partial p_z^2|_m^{1/2}}(\mu H)^{3/2} D\sum_{p=1}^{\infty}\frac{(-1)^p}{p^{3/2}} \times\Psi(p\lambda)\cos\left(\pi p\frac{m}{m_0}\right)\cos\left(p\frac{cS_m(\zeta)}{e\hbar H}\mp\frac{\pi}{4}\right), \tag{14}$$

where D is the mean value of D(p) for the region $\delta\Sigma$ as in (13), φ is the angle between v and the xy plane. If $|\tan\varphi| \sim 1$ on the average along the orbit the ratio of this expression to (13) is of the order $(\zeta/\mu H)^{1/2} \gg 1$.

Upon comparing (13) and (14) with (3), we see that for sufficiently low temperatures ($\lambda \lesssim 1$) the relative smallness of oscillations depends mainly on the parameters $\mu H/eFa_0$ (or $\mu H/\zeta_0$) and D/D_{max}. In fields $H \lesssim 10^4$ Oe the first of these parameters is not too small only for groups with a small effective mass ($m/m_0 \sim 10^{-3} - 10^{-2}$). The ratio of the transmission coefficient on an extremal orbit to its maximum value (in the free electron model) is

$$\frac{D}{D_{max}} \sim \exp\left\{-\frac{4}{3}\left[\frac{(w+\zeta)^{3/2}}{w^{1/2}} - w\right]\frac{1}{eFa_0}\right\}.$$

Ordinarily $\zeta \sim 1 - 10$ eV and this ratio is exponentially small; D/D_{max} can be of the order of unity only when the chemical potential of the elec-

trons is unusually small ($\zeta \lesssim eFa_0$). Thus an appreciable magnitude of the considered oscillations can be expected when the metal contains only small groups (compare with the conclusion in [1]).[3)]

4. The first term of (12) can be put into the form $j_0(\zeta_0) + j_2^{osc}$, where $j_0(\zeta_0)$ is the current in the absence of a magnetic field (1), and

$$j_2^{osc} = -\frac{2e}{h^3}\Phi(\zeta_0)\,\zeta^{osc} \qquad (15)$$

A comparison with (1) gives[4)]

$$\frac{j_2^{osc}}{j_0} \sim \begin{cases} \zeta^{osc}/eFa_0, & eFa_0 \ll \zeta_0 \\ \zeta^{osc}/\zeta_0, & \zeta_0 \ll eFa_0 \end{cases}, \qquad (16)$$

since with decreasing ϵ, $\Phi(\epsilon)$ decreases exponentially at a distance of the order eFa_0.

The oscillating part of the chemical potential is [8]

$$\zeta^{osc} = \frac{2\sqrt{2\pi}}{dU/d\zeta}\left(\frac{e\hbar H}{c}\right)^{3/2}\sum\left|\frac{\partial^2 S}{\partial p_z^2}\right|_m^{-1/2}\sum_{p=1}^{\infty}\frac{(-1)^{p+1}}{p^{3/2}} \times \Psi(p\lambda)\cos\left(\pi p\frac{m}{m_0}\right)\cos\left(p\frac{cS_m(\zeta)}{e\hbar H}\mp\frac{\pi}{4}\right),$$

where $U(\zeta)$ is the volume of p-space occupied by electrons; the first summation is performed over all extremal orbits on the Fermi surface. The order of magnitude of the amplitude of ζ^{osc} is determined by the largest group; for $\lambda \lesssim 1$ we have

$$\zeta_{max}^{osc} \sim \tilde{\mu}H\,(\tilde{\mu}H/\zeta)^{1/2};$$

here $\tilde{\mu}$ is the magneton of this group averaged over the orbits and magnetic field directions; ζ is computed from the bottom of an electron group, or from the top of a hole group. Consequently in the case of a large group ($\zeta \sim 1 - 10$ eV, $m \sim m_0$) we have $\zeta_{max}^{osc} \sim 10^{-7} - 10^{-6}$ eV (for $H \sim 10^4$ Oe), and the relative amplitude of the current oscillations (16) is $j_{2max}^{osc}/j_0 \sim 10^{-6} - 10^{-4}$. When only small groups are present, j_{2max}^{osc}/j_0 is limited only by the condition $\mu H \ll \zeta$ and can be of the order 10^{-2}.

5. In conclusion we consider the properties of field emission from a complex cathode consisting of two metals separated by a thin dielectric film through which electron tunneling can occur.

When a negative bias of the order of one to a few volts is applied to metal 1, the potential barrier for tunneling electrons will be considerably lower than for electrons of metal 2 (Fig. 4). Let metal 2 be a film whose thickness is no more than the order of the mean free path of these "hot" electrons. A considerable field current then exists even in a relatively weak field F that cannot induce appreciable emission from metal 2 (in the absence of a potential difference between the metals).[5)]

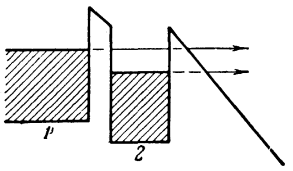

FIG. 4

When a magnetic field is applied the field emission current observed under the foregoing conditions exhibits various oscillations associated with both metals (if they are monocrystalline).

The experimental investigation of field emission current oscillations is, of course, an extremely complicated problem. A flat emitter requires extremely higher accelerating voltages than the conventional[10] sharp points or thin wires. High-voltage pulses could possibly be used.

In the case of the usual geometry[10] most of the emission comes from a very small region of a monocrystalline cathode. The necessary conditions for observing oscillations can then also be realized by properly orienting the magnetic field relative to the single crystal surface.

Note added in proof (February 18, 1964). We note that the oscillating term defined by (14) is due to electrons performing an incomplete revolution between collisions with the metal surface. However, in the case of diffuse reflection a contribution to the oscillations comes only from electrons completing many revolutions before colliding with the surface; these electrons must come from the interior of the metal. As a result, even in case b) the oscillating term j_1^{osc} has the form (13) except that D will be defined somewhat differently than in case a).

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[3)] A complicated electron dispersion law leads to the conclusion that the relative size of j_1^{osc} can also be quite large in certain other cases (for example, when in addition to small groups, large groups are present but are situated farther than the small groups from the center of the first Brillouin zone.)

[4)] Except for some special cases when $\Phi(\varepsilon)$ reaches a maximum for $\varepsilon < \zeta_0$.

[5)] If the potential difference exceeds the work function of metal 2, tunneling electrons can leave metal 2 even in the absence of an external electric field. Such "tunnel emitters" were suggested and realized by Mead.[9]

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Translated by I. Emin
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