PHASE SHIFT ANALYSIS OF NUCLEON-NUCLEON SCATTERING AT 630 MeV

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A phase-shift analysis of the data on np and pp scattering was carried out simultaneously. Three sets of phase shifts have with approximately equal probability with respect to the χ^2 criterion were found.

At present there are several known papers on phase shift analysis of pp scattering at 660 MeV, performed under the assumption that the resonance model for meson production processes is valid[1,2]. In the present work an attempt is made, using the results of [1,2] and the known data on np-scattering at 630 MeV, to reconstitute the amplitude of scattering of nucleons by nucleons in states with total isotopic spin t = 0.

When starting the simultaneous phase shift analysis at 630 MeV, the authors understood that at the relatively scanty experimental information at their disposal they could hardly reconstitute the nucleon-nucleon scattering amplitude to the same degree of non-ambiguity as was done in the energy region up to meson-production threshold. Nonetheless, it is hoped that the results obtained will prove useful, primarily for the planning of future experiments.

The phase shift analysis of the data on pp scattering at 660 MeV[1,2] yielded two noticeably different solutions. Both solutions were used for a search with random initial phase-shift values at t = 0. During the first stage the search was made for $l_{\max} = 4$, i.e., the amplitude was taken in the one-meson approximation starting with $l = 5$. The phase shifts for t = 1 were taken either from [1] or from [2]. It was correspondingly assumed that meson production goes essentially from states with total isotopic spin t = 1, namely the states $^3P_{0,1,2}$ and $^1D_2$ when the results of [1] are used and $^3P_{0,1,2}$, $^1D_2$, and $^3F_{2,3}$ when the results of [2] are used.[1)] The phase shifts of the waves for the states t = 0 were determined by minimizing the sum of the squares of the weighted deviations of the calculated curves from the experimental np-scattering data (Table I).

The large value of the $\chi^2/\overline{\chi^2}$ ratio and the noticeable errors in phase shifts at t = 1, which were disregarded during the first stage, made it necessary to refine all the obtained results by varying the phase shifts of all the waves and coupling constants f^2. Simultaneously an attempt was made to determine the imaginary parts of the phase shifts of the $^3S_1$ and $^3D_1$ waves and the mixing parameter ϵ_1. This attempt, however, was not successful, since the indicated parameters were determined with an error exceeding 100 per cent ($\delta(^3S_1) = (25 \pm 125)^\circ$, $\delta(^3D_1) = (18 \pm 118)^\circ$, $\epsilon_1 = (-35 \pm 134)^\circ$). The results of a search for the solutions are given in Table II.

Table I. Data used for the phase shift analysis

Experimental quantity	Energy at which the measurements were made, MeV	Number of points	Reference
σ_{pp}	660	12	[3, 4]
P_{pp}	635	14	[5]
D_{pp}	635	5	[6]
R_{pp}	635	5	[7]
C_{nn}^{pp}	660	3	[8]
C_{kp}^{pp}	660	1	[9]
σ_t^{pp}	660	1	[10]
σ_{np}	630	19	[11, 12]
P_{np}	635	8	[13]
σ_t^{np}	580	1	[14]
$\frac{\sigma_{nd}(0^\circ)}{\sigma_{np}(0^\circ)}$	630	1	Authors' data
A_{pp}	608	5	[15]

Remarks: 1—the following notation is used: σ—differential cross section; P—polarization; D, R, A—triple-scattering parameters: σ_t—total interaction cross section; σ_{nd}—differential cross section for exchange nd scattering. Within the framework of the impulse approximation, the ratio σ_{nd}/σ_{np} is equivalent in some sense to the parameters of the triple np scattering[16].

2. The quantity A_{pp} was used only for refinement in the last stage.

1)Unlike in[2], where inelastic collisions were taken into account with the aid of absorption coefficients averaged for a given l, in the present paper the imaginary additions to the phase shifts of the indicated states were regarded as independent parameters.

Table II. Results of search for solutions with random initial conditions

Type of solution by phase shifts at t = 1	l_{max}	$\overline{\chi^2}$	Number of searches from random points	Number of solutions	χ^2	χ^2 after refinement using the measurements of A
$\delta_{t=1}$ from[1] *	4	44 **	70	1	80	95
				2	74	83
				3	76	83
				4	82	92
$\delta_{t=1}$ from[2]	4	42 **	50	1	59	70
				2	44	53
				3	49	61
$\delta_{t=1}$ from[2]	5	37 **	—	1		46
				2		38
				3		41

*Solutions described in detail in [17].
**Without account of data on A (5 points).

Table III. Phase shifts for $l_{max} = 4$ (in degrees)*

	Set 1	Set 2	Set 3
f^2	0.082±0.005	0.065±0.009	0.071±0.007
	Real part of phase shifts		
$^1S_0$	−27.84±4.02	−25.85±3.87	−28.05±4.17
$^3S_1$	− 4.99±4.63	3.83±6.37	−5.88±7.93
$^3P_0$	−43.10±10.0	−48.03±8.14	−50.86±7.23
$^1P_1$	− 3.25± 5.0	−54.13±7.60	40.19±5.74
$^3P_1$	−36.74±4.00	−31.76±3.04	−35.80±3.96
$^3P_2$	17.04±1.50	15.41±1.64	15.95±1.25
ε_1	4.45±8.34	8.09±6.39	9.54±3.16
$^3D_1$	−35.70±6.73	26.78±6.09	20.09±3.82
$^1D_2$	10.27±2.69	7.60±2.37	6.05±2.33
$^3D_2$	8.06±5.33	22.16±4.50	−16.28±4.85
$^3D_3$	− 6.49±1.76	9.37±3.08	11.20±1.61
ε_2	− 2.77±1.43	−0.05±2.31	−3.91±2.31
$^3F_2$	− 6.11±1.36	−1.41±1.64	−4.79±2.17
$^1F_3$	−17.39±2.55	4.78±2.78	−2.42±2.22
$^3F_3$	− 0.90±1.37	−2.11±2.53	0.35±2.23
$^3F_4$	2.00±0.72	2.77±0.75	1.96±0.64
ε_3	− 2.88±2.58	3.73±5.25	6.85±3.02
$^3G_3$	− 1.84±3.02	−1.48±2.42	6.83±1.35
$^1G_4$	6.97±0.79	6.51±0.81	6.60±0.71
$^3G_4$	24.58±2.16	−0.60±2.80	18.42±2.40
$^3G_5$	0.36±1.14	−1.50±1.34	0.64±1.23
	Imaginary part of phase shifts		
$^3P_0$	−2.10±3.94	−2.29±5.15	5.00±4.89
$^3P_1$	−3.51±2.04	−9.13±2.06	−3.84±3.04
$^3P_2$	3.05±2.14	11.42±2.70	6.55±3.09
$^1D_2$	14.43±4.94	8.82±5.00	6.63±4.18
$^3F_2$	3.45±2.35	4.28±1.71	4.38±1.65
$^3F_3$	10.19±3.05	14.60±3.19	12.74±3.40
χ^2	70.2	53.47	61.0

*Parametrization of Stapp et al.[19]

Table IV. Phase shifts for $l_{max} = 5$ (in degrees)*

	Set 1	Set 2	Set 3
f^2	0.060±0.007	0.066±0.009	0.065±0.008
	Real part of phase shifts		
$^1S_0$	−26.18±3.90	−25.82±4.25	−30.17±10.3
$^3S_1$	−3.06±5.69	−5.45±14.7	−1.51±10.8
$^3P_0$	−41.16±17.2	−50.46±14.2	−45.16±16.4
$^1P_1$	−24.45±7.32	−38.21±16.1	53.02±7.33
$^3P_1$	−35.47±5.32	−35.21±4.14	−36.99±5.76
$^3P_2$	19.41±2.79	16.36±2.63	17.97±3.31
ε_1	7.32±9.24	14.05±10.7	−0.41±4.65
$^3D_1$	−36.01±9.03	23.63±9.92	13.83±4.14
$^1D_2$	10.64±3.03	8.57±3.08	3.91±4.00
$^3D_2$	14.56±7.14	26.43±9.84	−15.88±4.89
$^3D_3$	−4.64±2.48	5.81±5.24	11.96±2.27
ε_2	−3.14±2.00	−1.20±3.02	−4.33±2.32
$^3F_2$	−6.46±2.07	−2.25±2.19	−6.62±2.59
$^1F_3$	−17.91±3.51	6.61±3.95	2.68±3.62
$^3F_3$	−1.55±1.54	−1.09±2.66	2.83±3.08
$^3F_4$	1.94±1.11	2.54±0.75	1.44±0.99
ε_3	4.15±3.54	11.34±5.53	6.60±3.09
$^3G_3$	3.36±3.48	0.32±3.14	6.70±1.89
$^1G_4$	6.40±0.89	6.55±1.19	4.39±1.83
$^3G_4$	19.00±3.89	3.92±3.25	13.25±2.79
$^3G_5$	2.00±1.78	1.39±2.43	2.19±2.20
ε_4	−5.75±0.90	4.65±1.15	−5.50±1.14
$^3H_4$	0.69±0.74	0.27±0.64	0.39±0.98
$^1H_5$	2.67±2.17	3.09±2.40	−6.65±1.82
$^3H_5$	−3.39±1.02	−3.37±1.27	−0.81±1.17
$^3H_6$	0.86±0.47	0.80±0.41	1.05±0.59
	Imaginary part of phase shifts		
$^3P_0$	1.75±6.11	4.08±7.32	1.24±6.14
$^3P_1$	−2.58±3.11	−6.02±3.47	−3.37±5.20
$^3P_2$	1.82±2.34	6.69±3.80	3.89±6.11
$^1D_2$	13.19±8.06	7.35±5.12	7.61±7.38
$^3F_2$	3.80±3.56	3.40±2.46	4.41±2.61
$^3F_3$	9.12±4.05	13.21±4.05	12.03±5.69
χ^2	46	38	41

*Parametrization of Stapp et al.[19]

The completion of the measurement of the parameter A of triple pp scattering[15] made it possible to refine further the solutions obtained. It was observed as a result that the phase-shift sets obtained on the basis of the solution from [1] is characterized by values χ^2 which have less than one per cent probability for the given number of degrees freedom ($\overline{\chi^2} = 49$). In this case the largest contribution to χ^2 for all four solutions of this type is made by the points $R_{pp}(72°)$ and $C_{nn}^{pp}(54°)$; it amounts to a total of approximately 20 units for each solution. It must be noted that this circumstance was observed also in [18] where

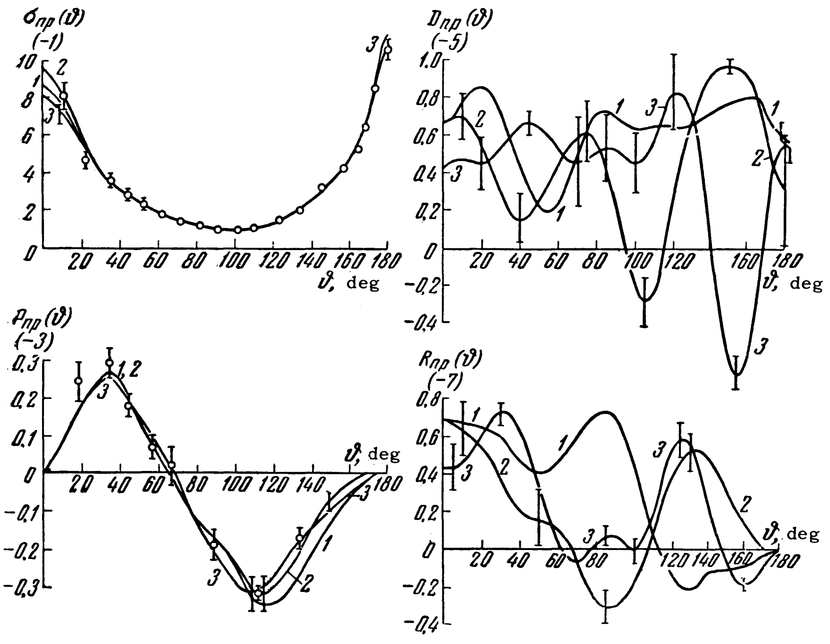

FIG. 1

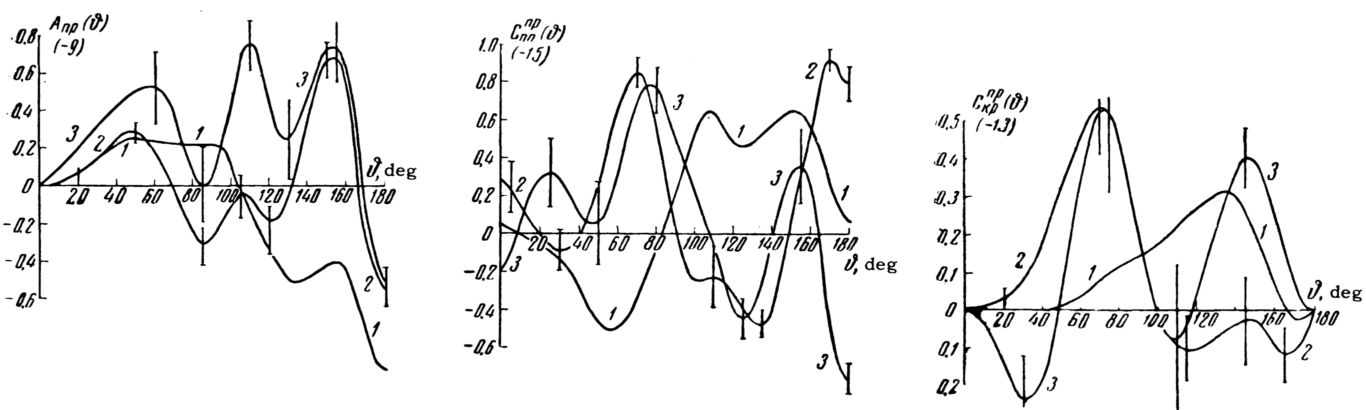

FIG. 2

the phase shift analysis data on pp scattering, obtained by the measuring the parameter A, were refined.

Thus, the solutions obtained on the basis of the set of phase shifts at t = 1 from [2] describe the existing experimental data much better. It must be noted, to be sure, that the imaginary parts of the phase shifts of the $^3P_0$ and $^3P_1$ waves are negative in all three solutions of this type, and consequently the solutions go somewhat into the unphysical region relative to these parameters.

The solutions that lie lowest relative to χ^2 were checked for stability against an increase in the number of experimentally determined parameters. To this end, these solutions were refined for $l_{max} = 5$. The number of parameters determined from the experimental data was increased from 28 to 33. From a comparison of the quantities listed in Tables III and IV we can see that the solutions were sufficiently stable, and the mean values of the phase shifts changed in most cases by less than one standard deviation. The value of χ^2 was reduced by at least 15 units. The solutions go over much less into the unphysical region relative to the imaginary parts of the phase shifts of ^3P waves. All this points apparently to the need for using the one-meson approximation at 630 MeV, starting with $l = 6$ and higher. However, a noticeable increase in the experimental information is necessary for a reliable determination of the parameters of the H waves.

At the present time it is hardly possible to choose the most probable among the obtained solutions. This will be possible only after additional experiments are made. To facilitate the comparison of the analysis results with the experimental data, Figs. 1—4 show the calculated variation of the experimentally observed quantities with

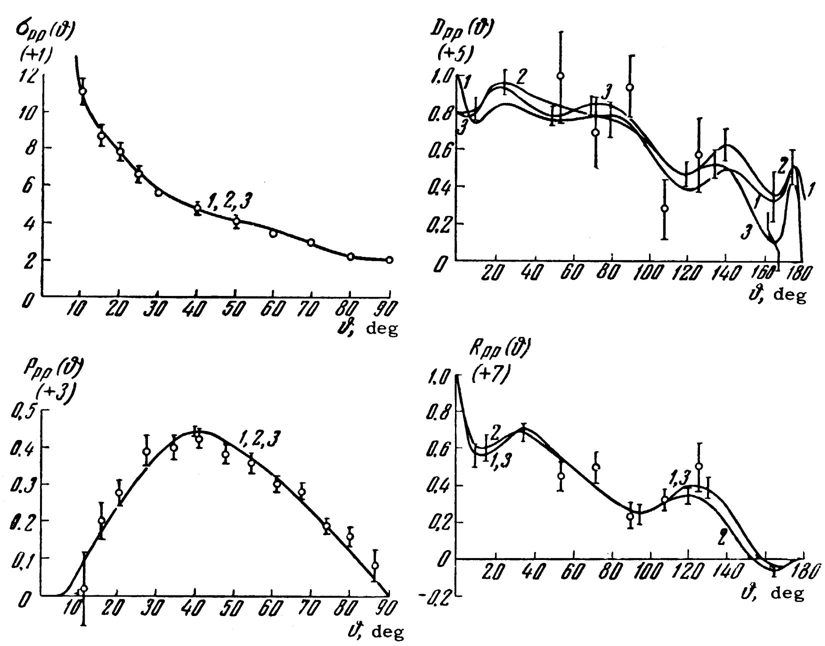

FIG. 3

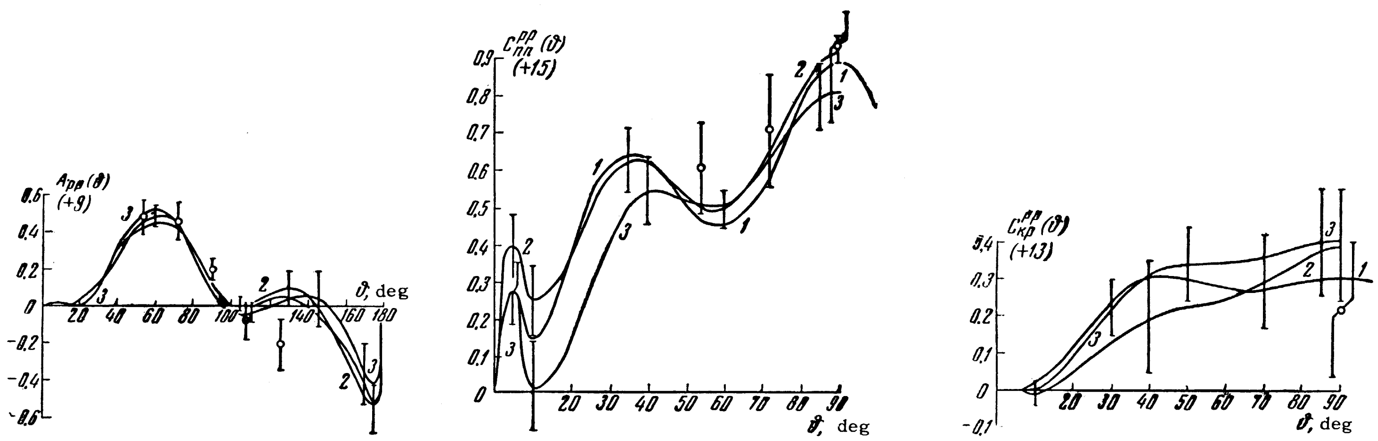

FIG. 4

the scattering angle. The full dots show the experimental data used in the present work. The error corridor of the calculated curves is indicated by means of vertical segments. The number on each curve corresponds to the number of the solution.

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