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SCATTERING OF POLARIZED GAMMA QUANTA BY THE FIELD OF A NUCLEUS ACCOMPANIED BY CREATION OF ELECTRON-POSITRON PAIRS

I. M. NADZHAFOV, N. A. TENYAKOVA, and A. I. MUKHTAROV

Azerbaĭdzhan State University

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The angular and energy distribution of electron-positron pairs and of scattered γ quanta in the reaction $\gamma + (Ze) \rightarrow (Ze)' + \gamma' + e_+ + e_-$ is investigated taking into account the polarization of all the particles. The degree of polarization is determined both for the pairs and for the scattered γ-quanta.

It is well known that quantum mechanical processes of high order occur at very high energies of the primary particles. Since particles of high energy can approach one another very "closely," the investigation of high order processes can give us some information both on the structure of elementary particles, and also on the applicability of electrodynamics at small distances. From this point of view the theoretical investigation of high-energy processes is of some interest. One such process is the scattering of γ quanta by the Coulomb field of a nucleus accompanied by the creation of electron-positron pairs.

Consider a γ-quantum of energy $\mathcal{E}_\gamma = c\hbar\kappa$, and of momentum $\mathbf{p}_\gamma = \hbar\boldsymbol{\kappa}$ scattered by the Coulomb field of a nucleus (Ze) and creating an electron-positron pair of energies $E_- = c\hbar K_-$, $E_+ = c\hbar K_+$ and of momenta $\mathbf{p}_- = \hbar\mathbf{k}_-$, $\mathbf{p}_+ = \hbar\mathbf{k}_+$ (where $K_\pm = (k_\pm^2 + k_0^2)^{1/2}$, $k_0 = m_0c/\hbar$). We denote the energy and the momentum of the scattered γ-quantum by $\mathcal{E}_{\gamma'} = c\hbar\kappa'$ and $\mathbf{p}_{\gamma'} = \hbar\boldsymbol{\kappa}'$.

The theoretical analysis of this process without taking the polarization of the particles into account has been carried out earlier by a number of authors [1,2]. In the paper by DeTollis et al [1] only two particular cases are investigated, but apparently the final results are incorrect (cf., below). In the paper by Nelipa [2] it is shown that the evaluation of the square of the matrix element can be reduced to the evaluation of seven traces, but nothing definite can be said on the basis of these results about the energy or the angular dependence of the cross section for the process.

The effective differential cross section for the process of the scattering of γ quanta by the field of nucleus accompanied by the creation of electron-positron pairs can be represented in the first Born approximation of the following form:

$$d\sigma = 4\,(2\pi)^{-4}\alpha^2 r_0^2 Z^2 \frac{\varkappa' k_+ k_- k_0^2\, dK_+ dK_-}{\varkappa q^4 K_+ K_-} |M|^2\, d\Omega_{\gamma'} d\Omega_- d\Omega_+. \quad (1)$$

Here

$$M = b^*(k_-, s_-, 1)\,(A + i\boldsymbol{\sigma}\mathbf{B} + \rho_1\boldsymbol{\sigma}\mathbf{C} + i\rho_1 D)\, b(-k_+, s_+, -1)$$

is the matrix element for the transition (cf. [3]), $b(k, s, 1)$ is the spinor amplitude, $\mathbf{q} = \boldsymbol{\kappa} - \boldsymbol{\kappa}' - \mathbf{k}_- - \mathbf{k}_+$ is the recoil momentum of the nucleus (we neglect the kinetic energy of recoil of the nucleus), $d\Omega_{\gamma'}$, $d\Omega_-$, $d\Omega_+$ are the solid angles into which the scattered γ quantum, the electron and the positron are respectively emitted, σ_i, ρ_i are the Dirac matrices, A, B, C, and D are certain functions of the energies and the momenta of the particles.

It is well known that at high energies the spin of fermions is oriented in the direction of their momentum. In the case under consideration we can restrict ourselves without loss of generality to taking into account only the longitudinal polarization of the electron and the positron (here it is done in accordance with the method proposed by Sokolov [3]). Then the square of the matrix element can be written in the following form:

$$|M|^2 = \frac{1}{4}[f_0 + ll'f_1 + s_+s_-(f_2 + ll'f_3) + s_+(lf_4 + l'f_5) + s_-(lf_6 + l'f_7)], \quad (2)$$

where s_-, s_+ and l, l' define the component of the spin of the electron, the positron and the two photons (l'—of the scattered photon) along their direction of motion: $s_- = 1$ ($s_+ = 1$) corresponds to the spin of the electron (or of the positron) being directed parallel to its momentum, $s_- = -1$ ($s_+ = -1$) corresponds to the spin of the electron (or of the positron) being directed oppositely to its momentum, while for $l = 1$ the photon is right circularly polarized (the photon spin is directed along the momentum), and for $l = -1$ the photon is left circularly polarized (the photon spin is directed oppositely to its momentum); f_i ($i = 0, 1 \ldots, 7$) are functions of the energies and of the angles. The expressions for these functions are in the general case very awkward, and it is not possible to reproduce them here completely. Therefore, we shall give their values only in the special cases considered below.

It is well known that at high energies scattering occurs principally into a very small solid angle. Therefore, it is sensible to investigate the following two particular cases which have been investigated prior to our work [1].

1. The scattered photon is moving in the direction of motion of the primary photon, while the pair is emitted at an angle θ, i.e., $\kappa \parallel \kappa'$, $\mathbf{k}_+ = \mathbf{k}_-$, $\kappa \mathbf{k}_\pm = \kappa k \cos\theta$. In this case the functions f_i ($i = 0, 1, \ldots, 7$) are simplified considerably:

$$\begin{aligned} f_0 = f_1 &= (1 - \beta\cos\theta)^{-3}\,[(1-\beta)^3 + (3+\beta^2)\,\beta y], \\ f_2 = f_3 &= (1-\beta\cos\theta)^{-4}\,[2\,(1-\beta)\,(2+\beta^2-\beta^3)\,y \\ &\quad -(2+\beta^2+\beta^4)\,y^2 - (1-\beta)^4], \\ f_i = 0, \quad i &= 4, 5, 6, 7, \quad \beta = v/c = k/K, \quad y = 1-\cos\theta. \end{aligned} \tag{3}$$

On expressing the energies of the particles in units of m_0c^2, we obtain for the effective cross section in the case under consideration

$$\begin{aligned} &d\sigma/dK_-dK_+d\Omega_{\gamma'}d\Omega_-d\Omega_+ \\ &= (2\pi)^{-4}\,\alpha^2 r_0^2 Z^2_{\_}(1+ll')\,\varkappa'\beta^2(f_0 + s_-s_+f_2)/\varkappa q^4. \end{aligned} \tag{4}$$

Moreover, the square of the recoil momentum of the nucleus is equal to

$$q^2 = 4K^2\,[2\,(1-\beta\cos\theta) + \beta^2 - 1].$$

It follows from expression (4) that both the incident and the scattered γ quanta must have the same polarization, i.e., either both must be right circularly polarized ($l = l' = 1$) or both must be left circularly polarized ($l = l' = -1$). But if $l' = -l$, the effective cross section for the process vanishes.

For zero scattering angle $f_0 = -f_2 = 1$ and

$$|M|^2 = \tfrac{1}{4}\,(1+ll')\,(1-s_+s_-). \tag{5}$$

But in the case of ultrarelativistic electrons and positrons ($\beta \to 1$), when the pair is emitted at a large angle ($1 - \cos\theta \gg 1-\beta$), $|M|^2$ takes on the following form

$$|M|^2_{\gamma p} = (1+ll')\,(1-s_+s_-)\,(1-\beta\cos\theta)^{-2}. \tag{6}$$

From (5) and (6) it can be seen that a pair of any energy emitted at zero angle, or an ultrarelativistic pair emitted at large angles, will always be in a singlet state, i.e., in these limiting cases the spins of the electron and of the positron will turn out to be antiparallel ($s_+ = -s_- = \pm 1$). But in all other cases the pair can also be in the triplet state ($s_+ = s_- = \pm 1$).

Figure 1 shows the angular dependence of the cross section for the process (4) for values of the energy $\mathcal{E}_\gamma = 2\mathcal{E}_{\gamma'} = 4E = 2\times 10^3 m_0c^2$ for $Z = 1$.

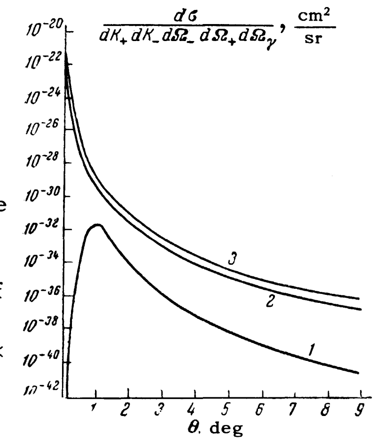

FIG. 1. The angular dependence of the cross section for the process (4): curve 1 characterizes the angular distribution of the pairs in the triplet state ($s_-s_+ = 1$), curve 2 – in the singlet state ($s_-s_+ = -1$) and curve 3 – in the case of unpolarized particles for $\epsilon_\gamma = 2\epsilon_{\gamma'} = 4E = 2\times 10^3 \times m_0c^2$.

Curves 1 and 2 refer to cases when the pair is respectively in the triplet ($s_-s_+ = 1$) or in the singlet state ($s_-s_+ = -1$); curve 3 characterizes the effective cross section averaged over the polarizations of the incident quanta and summed over the polarizations of the remaining particles. From this it can be seen that for given values of the energies of the particles the singlet state will always predominate over the triplet state.

We also note that the effective cross section is very sensitive to the angle of emission of the pair and falls off sharply as the angle increases. It has the greatest value for zero angle (curves 2, 3) with exception of the triplet state (curve 1). At particle energies chosen above for the largest value of $d\sigma(\theta = 0)$ we obtain from (4)

$$\frac{1}{2}\sum_{l,l',s_-,s_+}\frac{d\sigma}{dK_-dK_+d\Omega_{\gamma'}d\Omega_-d\Omega_+} = 5\cdot 10^{-22}\,Z^2\,\frac{\text{cm}^2}{\text{sr}^3}.$$

The curves in Fig. 2 characterize the energy dependence of the cross section for the process involving unpolarized particles for a fixed value of the energy of the incident γ-quanta $\mathcal{E}_\gamma = 2\times 10^3$ (in units of m_0c^2) and of the angle θ equal to 2 and 10°. Along the x axis we have plotted the ratio of the energies of the scattered and the incident γ quanta $\mathcal{E}_{\gamma'}/\mathcal{E}_\gamma = 1 - (E_- + E_+)/\mathcal{E}_\gamma$: From these curves it follows that as the fraction of the energy carried by the scattered γ quantum

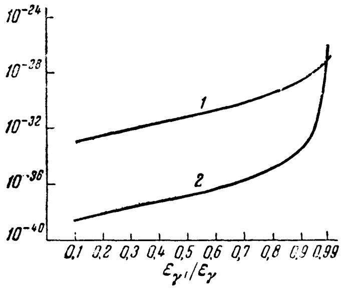

FIG. 2. Dependence of the cross section (4) for unpolarized particles on the energy of the scattered γ-quantum; curve 1 corresponds to $\theta = 2°$, curve 2 corresponds to $\theta = 10°$.

increases (as the fraction of the energy characterizing the pair diminishes) the effective cross section increases. Here there is a certain analogy with the processes of the Compton effect and of the creation of a pair by a γ quantum in the field of a nucleus, where the corresponding effective cross sections attain their maximal values respectively as $\mathcal{E}_{\gamma'}/\mathcal{E}_\gamma \to 1$ and $(E_\pm - m_0c^2)/\mathcal{E}_\gamma \to 0$. Therefore, we can say that the process under investigation involving the creation of an electron-positron pair of low energy (i.e., $E_\pm/\mathcal{E}_\gamma \ll 1$) is more probable.

On defining the degree of longitudinal polarization of the electron in the usual manner, we obtain for it from (4) the following expression:

$$P = s_+ f_2/f_0. \tag{7}$$

Since the process of creation of a pair of relatively low energy is more probable, it is sensible to investigate the angular dependence of the degree of longitudinal polarization of electrons for various values of their velocities.

In Fig. 3 curves 1—5 characterize the angular dependence of the degree of longitudinal polarization of the electrons for the following values of the velocity β respectively: 0.01; 0.1; 0.4; 0.9; 0.99 (the curves are drawn for $s_+ = 1$, therefore (7) can also characterize the degree of polarization of the pair).

For slow electrons (curve 1) the degree of longitudinal polarization increases monotonically from -1 to $+1$ as the angle θ increases from 0 to 90°, passing through zero at $\theta = 45°$. As the speed increases (curves 2—5) the monotonic

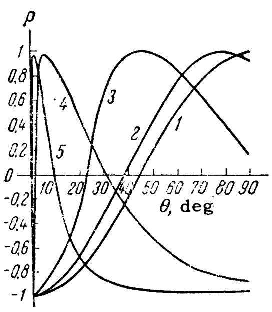

FIG. 3. Angular dependence of the degree of longitudinal polarization of pairs for different values of the electron velocity: $1 - \beta = 0.01$; $2 - \beta = 0.1$; $3 - \beta = 0.4$; $4 - \beta = 0.9$ and $5 - \beta = 0.99$.

nature of the curves is destroyed, and P has a maximum at values of θ equal to 75; 45; 15 and 2°, i.e., a displacement of the maxima towards smaller angles is observed. For very high velocities ($\beta \sim 1$) an increase in the angle at first leads in the region of small angles ($\theta \sim 1°$) to a sharp rise from -1 to $+1$, and then to a smooth variation in the degree of longitudinal polarization down to -1. Consequently, in the case of nonrelativistic velocities for small and medium angles ($\theta \leq 40°$) the pair will generally be in the singlet state, while for large angles ($\theta > 40°$) it will generally be in the triplet state. As the velocity of the electron is increased up to $\beta = 0.5$ the range of angles for which the singlet state predominates is compressed ($\theta = 0 - 20°$), while the range of angles for which the triplet state predominates is expanded ($\theta = 20 - 90°$). For very fast electrons the opposite behavior is observed, i.e., the pair is generally formed in the singlet state, and only in the narrow range of angles ($\theta = 1 - 15°$) will the triplet state predominate. Such a strong dependence of the spin correlations of the electron-positron pair on the angle θ and on the velocity of the particles is evidently associated with the pair acquiring orbital angular momentum if it is emitted at an angle $\theta \neq 0$.

As has been stated already, the process of the scattering of γ quanta accompanied by the creation of a pair without taking the polarization of the particles into account has been investigated previously [1] for the given special case. However, the final expression for the effective cross section quoted in the article cited (formula (4) from [1]) differs from the corresponding expression obtained by us (formula (4) of the present paper) not only by its awkward structure, but also by the nature of its dependence on the energies of the particles. For the effective cross section at $\mathcal{E}_\gamma = 2\mathcal{E}_{\gamma'} = 4E = 2 \times 10^3 m_0c^2$ and $\theta = 0°$ too high a value is obtained in [1] equal to $d\sigma(\theta = 0°) \sim 10^{-11} Z^2$ cm²/sr³, while for the same parameters we have obtained $d\sigma(\theta = 0°) \sim 5 \times 10^{-22} Z^2$ cm²/sr³. If the result of the paper of DeTollis et al. [1] were valid, then this process would have been found experimentally long ago. Moreover, in the paper quoted [1] the cross section for the process is inversely proportional to the energy of the scattered γ quantum which for $\mathcal{E}_{\gamma'} \to 0$ leads to a (nonlogarithmic) divergence of the type $d\sigma \sim 1/\mathcal{E}_{\gamma'} \to \infty$, and this, apparently, can not occur. In our case the dependence of the effective cross section on the energy of the scattered photon $\mathcal{E}_{\gamma'}$

is of the opposite character, i.e., for $\varepsilon_{\gamma'} \to 0$ we have $d\sigma \sim \varepsilon_{\gamma'} \to 0$. It seems to us that such a behavior of the cross section can be understood physically.

2. We now go on to the second special case when the pair is emitted at zero angle, while the γ-quantum is scattered through an angle θ, i.e., $\varkappa \parallel \mathbf{k}_+$, $\mathbf{k}_+ = \mathbf{k}_-$, $\varkappa\varkappa' = \varkappa\varkappa' \cos\theta$.

For this case (2) can be conveniently written as follows:

$$|M|^2 = (4Q)^{-1}\{(1 - s_-s_+)[\varphi_1 + (1 + ll')\varphi_2] + s_-s_+ \times [\varphi_3 + (1 + ll')\varphi_4] + (s_+ + s_-)(l\varphi_5 + l'\varphi_6)\}; \quad (8)$$

$$\varphi_1 = 8(1 - \beta^2)y^2[2(1 - \beta^2)\eta^2 + y(\zeta - 2)(2\beta^2(\zeta - 2) + (1 - \beta^2)\gamma) - \beta^2(\zeta - 2)^2 y^2],$$

$$\begin{aligned}\varphi_2 = 2\{&16\eta^6 + 8\eta^2 y[\eta^2(2\delta - 2\eta^2 - \rho) + (1 - \beta^2)(\zeta - 2)^2] \\ &+ \eta y^2[8\eta^3(\rho - 2\delta - 2\beta) + 4\eta\delta(\delta - \rho) + \eta\rho^2 - 4(1 - \beta^2)(\zeta - 2)^2(2\gamma + 2\beta + 3\eta)] \\ &+ y^3[-\eta^2\rho^2 + 4\delta\eta^2(\rho - \delta) + 8\eta^2(\zeta - 2)(\gamma\eta - 1 + \beta^2) + 4(1 - \beta^2)(\zeta - 2)^2(\gamma\zeta + 2\eta\zeta - 3\eta\beta)] \\ &- y^4(\zeta - 2)^2\gamma[(1 - \beta^2)(\gamma + 4\beta) + 2\eta\beta\gamma]\},\end{aligned}$$

$$\varphi_3 = 8(1 - \beta^4)(\zeta - 2)^2(2 - y)y^3,$$

$$\varphi_4 = 8(1 - \beta^2)(\zeta - 2)^2 y\{4\eta^2 - 2\eta y(3\eta + 2\gamma + 2\beta) + 2y^2[\eta(\eta + 2\zeta - 3\beta) + \gamma\zeta] - \gamma(\zeta + \eta)y^3\},$$

$$\varphi_5 = 4(1 - \beta^2)(\zeta - 2)^2 y^2[-4\eta(\gamma + \beta) + 2(2\gamma\eta + \beta\eta + \gamma\zeta + 2\beta)y - (\gamma\zeta + \gamma\eta + 2\beta)y^2],$$

$$\varphi_6 = 4(1 - \beta^2)(\zeta - 2)^2 y^2[-4\eta(\gamma + \beta) + 2(2\eta\gamma + \beta\eta + \gamma\zeta)y - (\gamma\zeta + \gamma\eta)y^2].$$

$$Q = 32\eta^2(\eta + \beta y)^2\left[\eta + \tfrac{1}{2}(\zeta - 2)(\gamma + \beta)y\right]^2,$$

$$q^2 = 2K^2[2\eta^2 + \gamma(\zeta - 2)y],$$

$$\eta = 1 - \beta, \quad \gamma = \zeta - 2\beta, \quad \delta = \zeta - 2 + \beta,$$

$$\rho = 2(\zeta\beta - \beta - \zeta^2 + 2\zeta),$$

$$\zeta = \varkappa/K = \varepsilon_\gamma/E, \quad \beta = k/K, \quad y = 1 - \cos\theta. \quad (9)$$

The functions f_i are in the present cases expressed in terms of φ_i in the following manner:

$$f_0 = \varphi_1 + \varphi_2, \quad f_1 = \varphi_2, \quad f_2 = \varphi_3 + \varphi_4 - \varphi_1 - \varphi_2,$$

$$f_3 = \varphi_4 - \varphi_2, \quad f_4 = f_6 = \varphi_5, \quad f_5 = f_7 = \varphi_6. \quad (10)$$

As should be expected, for $\theta = 0°$ (i.e., $y = 0$) we obtain from (8) expression (5). After averaging (8) over the initial spin states, and summing over the final spin states of the particles we obtain for the effective cross section

$$\frac{1}{2}\sum_{l,l',s_-,s_+} \frac{d\sigma}{dK_+ dK_- d\Omega_{\gamma'} d\Omega_- d\Omega_+} = 8(2\pi)^{-4}\alpha^2 r_0^2 Z^2 \frac{\varkappa'\beta^2(\varphi_1 + \varphi_2)}{\varkappa q^4 Q}. \quad (11)$$

Here the energies of all the particles are expressed in units of m_0c^2. The angular dependence of the cross section for the process (11) is shown in Fig. 4 for $\varepsilon_\gamma = 2\varepsilon_{\gamma'} = 2 \times 10^3$, $Z = 1$.

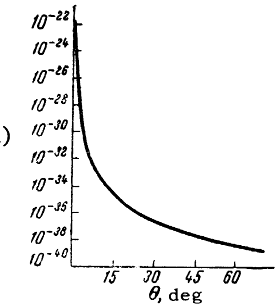

FIG. 4. Angular distribution of scattered γ-quanta (cross section (11)) for $\epsilon_\gamma = 2\epsilon_{\gamma'} = 2 \times 10^3\ m_0c^2$.

It is well known that the degree of polarization of one particle depends on the polarizations of all the other particles. Therefore, for the sake of simplicity, in determining the degree of circular polarization of the scattered γ quanta we shall sum the cross section over the spins of the electrons and the positrons, while in determining the degree of longitudinal polarization of the electrons we shall sum the cross section over the polarizations of the photons. Then for the degree of polarization of the scattered γ quanta and of the electrons we obtain the following expressions:

$$P_{\gamma'} = l\varphi_2/f_0, \qquad P = s_+(\varphi_3 + \varphi_4 - f_0)/f_0. \quad (12)$$

Figure 5 gives the angular dependence of the degree of circular polarization of the scattered quanta for particle energies $\varepsilon_\gamma = 2\varepsilon_{\gamma'} = 4E = 2 \times 10^3$. Here we have assumed, that the incident γ-quanta are right circularly polarized ($l = 1$). As can be seen from Fig. 5, as θ increases from 0 to 5° $P_{\gamma'}$ falls off sharply from +1 to +0.8. But as θ increases further up to 140° the value of $P_{\gamma'}$ remains practically constant. This means that the scattered γ-quanta will also generally be right circularly polarized. Only for very large angles $\theta \approx 180°$ will the scattered photons change

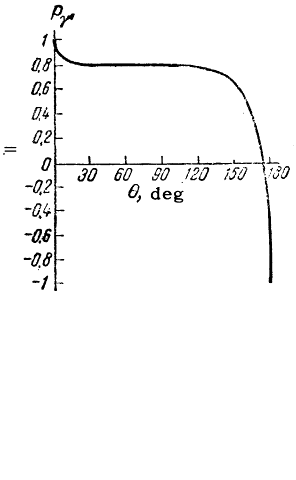

FIG. 5. Angular dependence of the degree of circular polarization of scattered γ-quanta for $\epsilon_\gamma = 2\epsilon_{\gamma'} = 2 \times 10^3 m_0c^2$, $l = 1$.

their direction of polarization, and become left circularly polarized ($l' = -1$).

Figure 6 shows the dependence of the degree of longitudinal polarization of the electrons on the angle θ for $\mathcal{E}_\gamma = 2\mathcal{E}_{\gamma'} = 4E = 2 \times 10^3$, $s_+ = 1$. From this it can be seen that for very small angles θ (0 – 2°) the pair is produced in the singlet state, while for angles $\theta > 2°$ it is produced in the triplet state.

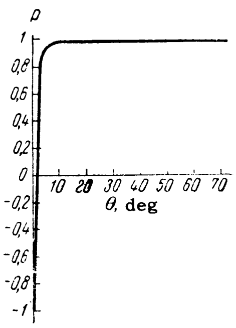

FIG. 6. Dependence of the degree of longitudinal polarization of pairs on the scattering angle for $\epsilon_\gamma = 4E = 2 \times 10^3 m_0c^2$.

It should be noted that the expression for the effective cross section for the case under investigation obtained by DeTollis et al.[1] differs considerably from formula (11) obtained by us (cf., formula (5) in [1]). The effective cross section in the paper cited [1] has (in comparison with (11)) a very awkward form (therefore the authors have presented it in tabular form; it is inversely proportional to $\mathcal{E}_{\gamma'}$ and for $\theta = 0°$ it takes on the excessively high value of $\sim 10^{-11}$ cm^2/sr^3. Moreover, there is another discrepancy between our results and the results of the paper by DeTollis et al.[1]: in our case the maximum power of $\cos\theta$ in expression (11) for $d\sigma$ is equal to four, while in their case it is equal to six.

In conclusion the authors express their gratitude to Professor A. A. Sokolov and B. K. Kerimov for discussions.

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