SHAPE OF THE NUCLEAR MAGNETIC RESONANCE SIGNAL FROM SUPERCONDUCTING ALLOYS NEAR THE SECOND CRITICAL FIELD

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The hypothetical shape of the nuclear magnetic resonance signal from type II conductors near the second critical field is determined along with its first four central moments.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

ABRIKOSOV [2] has examined the behavior of a superconducting alloy (a type II superconductor) in an external magnetic field on the basis of the Ginzburg-Landau theory.[1] When a long cylinder (with a negligibly small demagnetization factor) is placed in a longitudinal field H_0, the field begins to penetrate the sample when the first critical field H_{c1} is reached and completes its penetration at the second critical field strength H_{c2}.[2] For $H_0 > H_{c2}$ the sample is in a normal state with complete penetration by the field.

The foregoing pertains to type II superconductors, i.e. to metals and alloys having the Ginzburg-Landau parameter value $\kappa > 1/\sqrt{2}$. A mixed state exists for any magnetic field $H_0 \lesssim H_{c2}$, which penetrates the sample with a magnitude that is a periodic function (with a period $\sim 10^{-6}$ cm) of the coordinates x and y; the magnetic field is taken to be parallel to the z axis. If the field H_0 is close to H_{c2} the absolute magnitude of these spatial variations of the internal field is much smaller than H_{c2}.

Since the period of spatial variations of the internal magnetic field is much larger than the alloy lattice constant, the shape of the NMR (nuclear magnetic resonance) signal of any alloy component reflects to some degree the nuclear distribution in the magnetic field.

It is our present problem to determine the NMR line shape for any alloy component and to compute the second, third, and fourth central moments of this line for a long cylindrical sample placed in a longitudinal magnetic field $H_0 > H_{c2}$. It is assumed that we know the shape of the NMR signal from the same sample in fields $H_0 > H_{c2}$ when all atoms of the alloy are in a homogeneous magnetic field H_0.

2. CALCULATION OF CENTRAL MOMENTS OF THE SIGNAL

Let a long cylinder located in a longitudinal magnetic field $H_0 > H_{c2}$ (i.e. in the normal state) give a NMR signal represented by a function $g(\nu - \nu_0)$, where ν_0 is the resonance frequency in the field H_0; the function $g(\nu - \nu_0)$ is defined only for $\nu > 0$. Since this function of the frequency ν differs from zero only in a certain range $\Delta\nu$ about the resonance frequency ν_0, with $\Delta\nu \ll \nu_0$, we can supply a convenient formal definition of the function for $\nu < 0$. We now require that $g(x)$ be a continuous, integrable, and infinitely differentiable function that vanishes along with all its derivatives at $x = \pm\infty$. It is also assumed that $x = 0$ at the center of gravity of the distribution $g(x)$, i.e.

$$\int_{-\infty}^{\infty} xg(x)\,dx = 0, \qquad \int_{-\infty}^{\infty} g(x)\,dx = 1.$$

The magnitude of ν_0 is proportional to the strength of the external magnetic field in which the nuclei are located; therefore the field can replace the frequency as the argument of g. We now express the function as $g(H - H_0)$.

Reducing the external field H_0 somewhat below the second critical field H_{c2}, we now have an inhomogeneous magnetic field $H_0 - \Delta(\mathbf{r})$ inside the sample. Here $\Delta(\mathbf{r})$ denotes the part of the internal field which depends on the coordinates.

The shape of the NMR signal will be given by

$$f(H) = \int g(H - H_0 + \Delta(\mathbf{r}))\,d\tau. \tag{1}$$

The integration extends over the entire volume of the sample, which can be taken as unity. Using the aforementioned properties of $g(x)$, we easily obtain

$$f(H) = \sum_{n=0}^{\infty} \frac{\overline{\Delta^n}}{n!} g^{(n)}(H - H_0), \tag{2}$$

$$\overline{\Delta^n} = \int \Delta^n (\mathbf{r})\, d\tau. \tag{3}$$

In order to calculate the central moments of the curve f(H) we must determine the center of gravity H* of the distribution f(H):

$$H^* = \int_{-\infty}^{\infty} Hf(H)\, dH. \tag{4}$$

Substituting (2) into (4), we obtain $H^* = H - \Delta$.

The coordinate origin is now transferred to the point H*, i.e., we introduce the new coordinate $h = H - H^*$. The NMR signal will then be given by

$$f(H) = f(h + H^*) = F(h), \tag{5}$$

$$F(h) = \sum_{n=0}^{\infty} \frac{\overline{\Delta^n}}{n!} g^{(n)} (h - \overline{\Delta}). \tag{6}$$

Expanding $g^{(n)}(h - \overline{\Delta})$ in powers of $\overline{\Delta}$ and collecting terms, we finally obtain

$$F(h) = \sum_{n=0}^{\infty} \frac{1}{n!} \overline{(\Delta - \overline{\Delta})^n} g^{(n)}(h). \tag{7}$$

This form of F(h) is convenient for calculating its central moments.

By definition the m-th central moment is

$$\mu_m = \int_{-\infty}^{\infty} h^m F(h)\, dh.$$

Integrating by parts and using (7), we obtain the first four central moments of the NMR signal:

$$\begin{gathered} \mu_0 = 1, \quad \mu_1 = 0, \quad \mu_2 = \mu_{20} + \overline{(\Delta - \overline{\Delta})^2}, \\ \mu_3 = \mu_{30} + \overline{(\Delta - \overline{\Delta})^3}, \\ \mu_4 = \mu_{40} + 6\mu_{20}\overline{(\Delta - \overline{\Delta})^2} + \overline{(\Delta - \overline{\Delta})^4}; \\ \mu_{n0} = \int_{-\infty}^{\infty} h^n g(h)\, dh, \end{gathered} \tag{8}$$

i.e., the corresponding central moments of the signal g in a field $H_0 > H_{c2}$.

Equation (8) shows that by investigating the experimental shape of the NMR signal from any one of the components of a superconducting alloy in an external field $H_0 < H_{c2}$, we can obtain information regarding the spatial variations $\Delta(\mathbf{r})$ of the magnetic field inside the superconducting alloy.

3. ON THE MEAN SPATIAL VARIATIONS OF A MAGNETIC FIELD IN A TYPE II SUPERCONDUCTOR

Let us now consider the mixed-state region, with the external field H_0 (along the z axis) close to H_{c2}. Then, according to [2], the magnetic field H in the superconductor will be

$$\begin{gathered} H = H_0 - |\Psi|^2/2\varkappa; \\ |\Psi|^2 = |C|^2 \Delta_1(x, y). \end{gathered} \tag{9}$$

We have here introduced the notation

$$\Delta_1(x, y) = \exp(-\varkappa^2 x^2) \sum_{n, m=-\infty}^{\infty} \exp[-\pi^2(m^2 + n^2) + \sqrt{2\pi}\varkappa (n + m) x + i\sqrt{2\pi}\varkappa (n - m) y]. \tag{10}$$

The constant $|C|^2$ can be expressed in terms of the magnetic induction B. Indeed, it can easily be shown that $\overline{|\Psi|^2} = |C|^2/\sqrt{2}$. Also, $B = \overline{H} = H_0 - \overline{|\Psi|^2}/2\kappa$; therefore $|C|^2 = 2\sqrt{2\kappa}\,(H_0 - B)$. Eq. (9) can now be rewritten as

$$H = H_0 - \sqrt{2}(H_0 - B)\Delta_1(x, y). \tag{11}$$

However, the internal magnetic field was expressed in the preceding section as $H = H_0 - \Delta(\mathbf{r})$, so that

$$\Delta(x, y) = \sqrt{2}(H_0 - B)\Delta_1(x, y). \tag{12}$$

We shall now determine $\overline{(\Delta - \overline{\Delta})^2}$, $\overline{(\Delta - \overline{\Delta})^3}$, and $\overline{(\Delta - \overline{\Delta})^4}$. According to [2], $\Delta_1(x, y)$ is periodic in the (x, y) plane with the period $\sqrt{2\pi/\kappa}$ along the x and y axes. The Fourier series expansion of this function is

$$\Delta_1(x, y) = \sum_{m, n=0}^{\infty} A_{mn} \cos m\sqrt{2\pi}\varkappa x \cos n\sqrt{2\pi}\varkappa y. \tag{13}$$

Utilizing the rapid convergence of (10), we obtain the first few Fourier coefficients A_{mn}:

$$\begin{gathered} A_{00} = 0.707, \quad A_{10} = A_{01} = 0.294, \\ A_{11} = -0.122, \quad A_{20} = A_{02} = 0.00243, \\ A_{12} = A_{21} = 0.00137, \quad A_{22} = -0.0001. \end{gathered}$$

It is obvious that the remaining coefficients A_{mn} are not needed to calculate the desired mean values. After some elementary but laborious computations we finally obtain

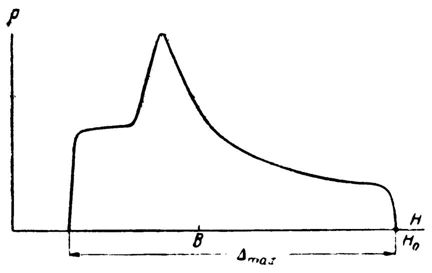
$$\overline{(\Delta - \overline{\Delta})^2} = 0{,}180\,(H_0 - B)^2, \tag{14}$$

$$\overline{(\Delta - \overline{\Delta})^3} = -4{,}38 \cdot 10^{-2}\,(H_0 - B)^3, \tag{15}$$

$$\overline{(\Delta - \overline{\Delta})^4} = 7{,}88 \cdot 10^{-2}\,(H_0 - B)^4. \tag{16}$$

We now introduce the density distribution of nuclei in the magnetic field: $\rho(H) = dN/dH$, where dN is the number of nuclei in the magnetic field between the strengths H and H + dH. For $H(\mathbf{r})$ as represented by (11), this function was obtained by graphic integration and is shown in the accompanying figure.

The quantitative characteristics of this func-



tion can be determined by using Eqs. (14)—(16). We note, to begin with, that the function is asymmetric with an asymmetry coefficient defined by [3]

$$\gamma_1 = \overline{(\Delta - \overline{\Delta})^3} / \sigma^3, \tag{17}$$

where $\sigma = [\overline{(\Delta - \overline{\Delta})^2}]^{1/2}$. Substituting (14) and (15) into (17), we obtain $\gamma_1 = -0.57$.

The sharpness of the distribution near the center of gravity is characterized by the coefficient of excess:[3]

$$\gamma_2 = \overline{(\Delta - \overline{\Delta})^4} / \sigma^4 - 3. \tag{18}$$

If the distribution is normal (Gaussian), we have $\gamma_2 = 0$. When $\gamma_2 > 0$ the distribution has a sharper maximum than a Gaussian curve. When $\gamma_2 < 0$ the distribution is flatter than a Gaussian curve or can even have a minimum (a bimodal distribution) near the center of gravity. The substitution of (14) and (16) into (18) gives $\gamma_2 = -0.57$.

When the width of the NMR signal for $H_0 > H_{c2}$ (i.e., the width of the $g(H - H_0)$ curve) is much smaller than Δ_{max}, the shape of the NMR signal in a field $H_0 < H_{c2}$ will be close to that of the nuclear distribution $\rho(H)$ in the magnetic field.

4. CONCLUSION

If the Ginzburg-Landau theory [2] is valid the nuclear distribution $\rho(H)$ in a magnetic field close to H_{c2} is represented by an asymmetric function with asymmetry and excess coefficients both equal to -0.57. The function $\rho(H)$ is characterized by the values of $\overline{(\Delta - \overline{\Delta})^2}$ $\overline{(\Delta - \overline{\Delta})^3}$, and $\overline{(\Delta - \overline{\Delta})^4}$, which can be determined through the second, third, and fourth central moments of the NMR signal from a type II superconductor in a magnetic field $H_0 \lesssim H_{c2}$. It would be highly desirable to conduct a corresponding experiment for the purpose of testing the theory of [2].

It must be understood, to begin with, that the theory of [2] was developed for homogeneous superconductors containing no macroscopic distortions or mechanical strains. The magnetic field distribution in macroscopically inhomogeneous samples will evidently have a more random character, and the nuclear distribution $\rho(H)$ in a magnetic field will begin to approximate a Gaussian curve. The coefficients of asymmetry and excess, γ_1 and γ_2, will accordingly approach zero.

The proposed investigation could therefore be valuable for studying the magnetic properties of superconducting alloys.

[1] V. L. Ginzburg and L. D. Landau, JETP 20, 1064 (1950).

[2] A. A. Abrikosov, JETP 32, 1442 (1957), Soviet Phys. JETP 5, 1174 (1957).

[3] H. Cramér, Mathematical Methods of Statistics, Princeton U. Press, 1946 (Russ. transl. IIL, 1948, p. 207).

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91