ELECTROPRODUCTION OF PIONS AND FERMION REGGE POLES

M. P. REKALO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor June 29, 1963

J. Explt. Theoret. Phys. (U.S.S.R.) **46**, 612-619 (February, 1964)

The production π-mesons at angles $\sim 180°$ in the c.m.s. of the high energy colliding electrons and the nucleons is considered. The longitudinal components of the virtual photon polarization vector do not contribute to the asymptotic quantity under consideration. Asymptotic formulas are derived for the differential cross sections and for the polarization of the recoil nucleons. The correlation quantities are oscillating functions of energy and angle.

1. According to present day thought the asymptotic behavior of the scattering amplitude for strongly interacting particles is determined for backward scattering in the center-of-mass system (c.m.s.) by the fermion Regge poles if the initial and the final states contain each a single fermion. The opposite-parity partial amplitudes have poles at complex conjugate points for the case that the square of the energy in the u-channel is negative.[1]

The process of π-meson production associated with high-energy electron-nucleon scattering, for π-meson emission angles of $\sim 180°$ in the c.m.s. of the π meson and the outgoing nucleon, is of both theoretical and experimental interest. The process of virtual photoproduction of π mesons differs somewhat from the process of π-meson production by real photons and nucleons. First, a virtual photon has a longitudinal polarization in addition to the transverse polarization; second, the square of the four-momentum of a virtual photon differs from zero; third, the π-meson electroproduction cross section averaged over the polarizations of all particles participating in the reaction contains a contribution corresponding to linear photon polarization. Therefore the study of the production of π mesons by unpolarized electrons can give the same information as that obtainable by a study of the π-meson photoproduction by polarized real photons.

In the present paper the process of π-meson electroproduction on nucleons will be investigated on the basis of the concept of moving fermion Regge poles.

2. We define the amplitude for the electroproduction of π mesons on nucleons, A, as follows:

$$S_{if} = \delta_{if} - i\,(2\pi)^4\,\delta^4\,(r_1 + p_1 - r_2 - p_2 - q) \times \frac{mM}{\sqrt{2\omega\varepsilon_1\varepsilon_2 E_1 E_2}} A, \quad (1)$$

where S_{if} is an element of the S-matrix, r_1, m, ε_1 are the four-momentum, the mass, and the energy of the incoming electron respectively; r_2, ε_2 are the four-momentum and the energy of the scattered electron, p_1, M, E, are the four-momentum, the mass, and the energy of the target nucleon, p_2, E_2 are the four-momentum and the energy of the recoiling nucleon, and q, ω are the four-momentum and the energy of the π meson.

We shall consider the electromagnetic interaction in lowest order. Then one can write A as follows:

$$A = ek^{-2}\varepsilon_\mu \langle p_2 q | J_\mu | p_1 \rangle, \quad (2)$$

$$\varepsilon_\mu = \bar{u}\,(r_2)\,\gamma_\mu u\,(r_1), \quad (3)$$

where $k^2 = -(r_1 - r_2)^2$, J_μ is the electromagnetic current operator for the meson-nucleon system.

The quantity $\varepsilon_\mu J_\mu$ can be expanded in terms of the independent invariants M_i:[2]

$$\varepsilon_\mu J_\mu = \sum_{i=1}^{6} A_i\,(s, u)\,M_i\,(\varepsilon, p_1, p_2, q); \quad (4)$$

$$M_1 = i\gamma_5\,(\hat{\varepsilon}\hat{k}), \quad M_2 = 2i\gamma_5\,(\varepsilon P), \quad M_3 = i\gamma_5\,(\varepsilon q),$$
$$M_4 = \gamma_5\hat{\varepsilon}, \quad M_5 = \gamma_5\hat{k}\,(\varepsilon P), \quad M_6 = \gamma_5\hat{k}\,(\varepsilon q), \quad (5)$$

where $P = (p_1 + p_2)/2$, $s = -(k + p_1)^2$, $u = -(k - p_2)^2$ are the independent kinematic variables of the process of π-meson virtual photoproduction.

3. We consider this process in the u-channel. In order to obtain the amplitude in the u-channel one must replace in (4) and (5) $k \to -k$ and

$q \to -q$. We now change in (4) and (5) to two-component nucleon spinors, using the explicit form of the four-component nucleon spinors. We further eliminate the time-like component of the vector ε_μ, using the charge conservation of the electric current

$$k_\mu \bar{u}\gamma_\mu u = k_\mu \varepsilon_\mu = 0. \tag{6}$$

This way we obtain

$$F_u = i\sigma\varepsilon F_{1u} + i(\sigma\hat{k})(\sigma\varepsilon)(\sigma\hat{q})F_{2u} + i(\sigma\hat{k})(\varepsilon\hat{q})F_{3u}$$
$$+ i(\sigma\hat{q})(\varepsilon\hat{q})F_{4u} + i(\sigma\hat{k})(\varepsilon\hat{k})F_{5u} + i(\sigma\hat{q})(\varepsilon\hat{k})F_{6u}, \tag{7}$$

where the amplitudes F_u except for kinematic factors are equal to $\varepsilon_\mu J_\mu$ in the c.m.s. in the u-channel; $\hat{k}$ and $\hat{q}$ are unit vectors in the direction of the momentum of the recoiling and the target nucleon, respectively, and ε are the space-like components of ε_μ:

The amplitudes F_{iu} are connected with the invariant amplitudes:

$$F_{1u} = \frac{\sqrt{(E_1+M)(E_2+M)}}{8\pi w}[-(w-M)A_1 - A_4],$$

$$F_{2u} = \frac{\sqrt{(E_1-M)(E_2-M)}}{8\pi w}[(w+M)A_1 - A_4],$$

$$F_{3u} = \frac{(E_1+M)\sqrt{(E_1-M)(E_2-M)}}{8\pi w}$$
$$\times [A_2 + A_3 - (w+M)(A_5+A_6)],$$

$$F_{4u} = \frac{(E_1-M)\sqrt{(E_1+M)(E_2+M)}}{8\pi w}$$
$$\times [-A_2 - A_3 - (w-M)(A_5+A_6)],$$

$$F_{5u} = \frac{(E_2-M)\sqrt{(E_1+M)(E_2+M)}}{8\pi w k_0}[(w+M)A_1 - A_4$$
$$+ (w+E_1)(A_2 - (w+M)A_5) - \omega(A_3 - (w+M)A_6)],$$

$$F_{6u} = \frac{(E_2+M)\sqrt{(E_1-M)(E_2-M)}}{8\pi w k_0}[-(w-M)A_1 - A_4$$
$$+ (w+E_1)(A_2 - (w-M)A_5)$$
$$+ \omega(A_3 + (w-M)A_6)], \tag{8}$$

where w is the total energy, k_0 is the energy of the virtual photon, and ω is the energy of the π meson in the u-channel. Then from the equations (8) follow the crossing relations:

$$F_{1u}(w) = -F_{2u}(-w), \quad F_{3u}(w) = -F_{4u}(-w),$$
$$F_{5u}(w) = -F_{6u}(-w). \tag{9}$$

These relations will be important later.

We further introduce helicity amplitudes in the u-channel:[3]

$$f_1 \equiv \left(\tfrac{1}{2}, 1 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_1^j(w) d^j_{1/2,-1/2}(z)$$
$$= -2\sin\frac{\theta}{2}\sum_j f_1^j [P'_{j+1/2} + P'_{j-1/2}],$$

$$f_2 \equiv \left(-\tfrac{1}{2}, -1 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_2^j(w) d^j_{1/2,1/2}(z)$$
$$= 2\cos\frac{\theta}{2}\sum_j f_2^j [P'_{j+1/2} - P'_{j-1/2}],$$

$$f_3 \equiv \left(\tfrac{1}{2}, -1 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_3^j(w) d^j_{1/2,3/2}(z)$$
$$= 2\sin\frac{\theta}{2}\sum_j f_3^j \left[\sqrt{\frac{2j-1}{2j+3}}P'_{j+1/2} + \sqrt{\frac{2j+3}{2j-1}}P'_{j-1/2}\right],$$

$$f_4 \equiv \left(-\tfrac{1}{2}, 1 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_4^j(w) d^j_{1/2,-3/2}(z)$$
$$= -2\cos\frac{\theta}{2}\sum_j f_4^j \left[\sqrt{\frac{2j-1}{2j+3}}P'_{j+1/2} - \sqrt{\frac{2j+3}{2j-1}}P'_{j-1/2}\right],$$

$$f_5 \equiv \left(-\tfrac{1}{2}, 0 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_5^j(w) d^j_{1/2,-1/2}(z)$$
$$= -2\sin\frac{\theta}{2}\sum_j f_5^j [P'_{j+1/2} + P'_{j-1/2}],$$

$$f_6 \equiv \left(\tfrac{1}{2}, 0 | F_u | \tfrac{1}{2}, 0\right) = \sum_j (2j+1) f_6^j(w) d^j_{1/2,1/2}(z)$$
$$= 2\cos\frac{\theta}{2}\sum_j f_6^j (P'_{j+1/2} - P'_{j-1/2}], \tag{10}$$

where $z = \cos\theta$ and θ is the photon emission angle in the reaction $\pi + N \to N + \gamma^*$.

The helicity amplitudes can be expressed in terms of the amplitudes F_{iu} as follows:

$$f_1 = 2^{-1/2}\sin(\theta/2)[-2(F_{1u}+F_{2u})$$
$$-(1+\cos\theta)(F_{3u}+F_{4u})],$$

$$f_2 = 2^{-1/2}\cos(\theta/2)[-2(F_{1u}-F_{2u})$$
$$+(1-\cos\theta)(F_{3u}-F_{4u})],$$

$$f_3 = 2^{-1/2}\sin(\theta/2)(1+\cos\theta)(F_{3u}+F_{4u}),$$

$$f_4 = -2^{-1/2}\cos(\theta/2)(1-\cos\theta)(F_{3u}-F_{4u}),$$

$$f_5 = -\sin(\theta/2)[(F_{1u}+\cos\theta F_{3u}+F_{5u})$$
$$-(F_{2u}+\cos\theta F_{4u}+F_{6u})],$$

$$f_6 = -\cos(\theta/2)[(F_{1u}+\cos\theta F_{3u}+F_{5u})$$
$$+(F_{2u}+\cos\theta F_{4u}+F_{6u})]. \tag{11}$$

The partial helicity amplitudes $f^j_\alpha(w)$ are connected with the partial amplitudes with definite parity by the relations

$$f_1^j = \tfrac{1}{2}(h_1^j + h_2^j), \qquad f_3^j = \tfrac{1}{2}(h_3^j + h_4^j), \qquad f_5^j = \tfrac{1}{2}(h_5^j + h_6^j),$$

$$f_2 = \frac{1}{2}(h_1^j - h_2^j), \quad f_4^j = \frac{1}{2}(h_3^j - h_4^j), \quad f_6^j = \frac{1}{2}(h_5^j - h_6^j). \tag{12}$$

The partial amplitudes h_1^j, h_3^j, h_5^j have the same parity, opposite to that of the amplitudes h_2^j, h_4^j, h_6^j.

4. The crossing relations (9) for the amplitudes F_{iu} together with (10), (11), and (12) show that the partial amplitudes $h_\alpha^j(w)$ obey the following relations:

$$h_1^j(w) = -h_2^j(-w), \quad h_3^j(w) = -h_4^j(-w),$$
$$h_5^j(w) = h_6^j(-w). \tag{13}$$

These equations show that for $u < 0$ (w purely imaginary) the amplitudes with opposite parity are related so that if $h_1^j(w)$ as a function of j has poles then $h_2^j(w)$ has poles at complex conjugate points. This is similarly the case for the pairs h_3^j and h_4^j, and h_5^j and h_6^j.

5. We now consider the asymptotic behavior for $s \to \infty$, $u < 0$, i.e., we go to the physical region of the s-channel corresponding to high energies and π-meson emission angles $\sim 180°$ (this corresponds in (10) and (11) to $\cos\theta \to \infty$). To this end we rewrite the sum in (10) as an integral by means of the Sommerfeld-Watson transformation, and then we deform the integration contour. Assuming that the extreme-right singularity of the amplitudes h_α^j is a pole, and retaining asymptotically for large s only the contribution from this pole we obtain

$$\frac{f_1}{2\sin(\theta/2)} + \frac{f_2}{2\cos(\theta/2)} = \frac{2\alpha^*}{\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$\frac{f_1}{2\sin(\theta/2)} - \frac{f_2}{2\cos(\theta/2)} = -\frac{2\alpha}{\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}],$$
$$\frac{f_3}{2\sin(\theta/2)} + \frac{f_4}{2\cos(\theta/2)} = -\frac{2\beta^*}{\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$\frac{f_3}{2\sin(\theta/2)} - \frac{f_4}{2\cos(\theta/2)} = \frac{2\beta}{\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}],$$
$$\frac{f_5}{2\sin(\theta/2)} + \frac{f_6}{2\cos(\theta/2)} = -\frac{2\gamma^*}{\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$\frac{f_5}{2\sin(\theta/2)} - \frac{f_6}{2\cos(\theta/2)} = -\frac{2\gamma}{\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}], \tag{14}$$

where α, β, and γ are the residues of the amplitudes h_1^j, h_3^j, and h_5^j, and the signs $\pm$ denote the different signatures.

Utilizing (14) one can obtain from (11) asymptotic expressions for the amplitudes F_{iu}. From these equations it follows immediately that for large s, i.e., for large $\cos\theta$, the amplitude F_{3u} and F_{4u} are asymptotically smaller than F_{1u}, F_{2u}, F_{5u}, or F_{6u}. With these asymptotic expressions for the amplitudes F_{iu} one can determine from (8) the asymptotic expressions for the invariant amplitudes:

$$A_1 = \frac{1}{2}u^{-1/2}(F_2 - F_1),$$
$$A_4 = \frac{1}{2}u^{-1/2}[F_1(\sqrt{u} + M) + F_2(\sqrt{u} - M)],$$
$$A_2 = \frac{1}{4}u^{-1}[(F_5 - F_2 + \omega F_3)(\sqrt{u} - M)$$
$$- (F_6 - F_1 + \omega F_4)(\sqrt{u} + M)],$$
$$A_3 = -\frac{1}{4}u^{-1}[(F_5 - F_2 + (\sqrt{u} + E_1)F_3)(\sqrt{u} - M)$$
$$- (F_6 - F_1 + (\sqrt{u} + E_1)F_4)(\sqrt{u} + M)],$$
$$A_5 = -\frac{1}{4}u^{-1}[F_5 - F_2 + F_6 - F_1 + \omega(F_3 + F_4)],$$
$$A_6 = \frac{1}{4}u^{-1}[F_5 - F_2 + F_6 - F_1 - (\sqrt{u} + E_1)(F_3 - F_4)], \tag{15}$$

where, in turn, the quantities F_i have the following structure:

$$F_1 = \frac{a^*}{\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$F_2 = \frac{a}{\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}],$$
$$F_3 = -\frac{b^*}{s\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$F_4 = \frac{b}{s\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}],$$
$$F_5 = \frac{c^*}{\cos\pi j^*}[s^{j^*-1/2} \mp (-s)^{j^*-1/2}],$$
$$F_6 = \frac{c}{\cos\pi j}[s^{j-1/2} \mp (-s)^{j-1/2}]. \tag{16}$$

Here a, b, c are certain combinations of the residues α, β, and γ multiplied by known kinematic factors which depend only on u.

If one neglects in (15) the contributions which are proportional to F_3 and F_4, since according to (16) they asymptotically are smaller than the other terms, we obtain

$$A_2 = -A_3, \quad A_5 = -A_6. \tag{17}$$

Thus it turns out that all six invariant amplitudes depend on two complex quantities, a and c. This dependence is such that it is possible to write the complete amplitude for the π-meson production by virtual photons in the following form:

$$\varepsilon_\mu J_\mu = M_+ + M_-, \tag{18}$$
$$M_+ = \varepsilon_\mu[\gamma_\mu a_1(\sqrt{u}) + i\gamma_\mu \hat{k} a_2(\sqrt{u})]$$
$$\times (i\hat{f} - \sqrt{u})\gamma_5(s^{j-1/2} \mp (-s)^{j-1/2})/\cos\pi j,$$
$$M_- = \varepsilon_\mu[\gamma_\mu a_1^*(\sqrt{u}) + i\gamma_\mu \hat{k} a_2^*(\sqrt{u})]$$
$$\times (i\hat{f} + \sqrt{u})\gamma_5(s^{j^*-1/2} \mp (-s)^{j^*-1/2})/\cos\pi j^*, \tag{19}$$

where $f = p_2 - k$, and a_1 and a_2 are given in terms of a and c by the relation

$$a_1 = (a - c)(\sqrt{u} + M)/4u,$$
$$a_1 + a_2(\sqrt{u} + M) = a/2\sqrt{u}. \tag{20}$$

These expressions give the asymptotic amplitude in factorized form. They correspond to second order Feynman graphs in which a "reggeon" is exchanged between the photon and the final nucleon on the one hand and the π meson and the initial nucleon on the other hand. The quantities $i\hat{f} \pm u^{1/2}$ correspond to propagation of the "reggeon" between the vertices, and γ_5 corresponds to the vertex $\pi N \rightarrow$ "reggeon", $\varepsilon_\mu \gamma_\mu [a_1 + ia_2\hat{k}]$ to the vertex $\gamma^* N \rightarrow$ "reggeon".

6. In order to evaluate the differential cross section and the quantities which characterize the different polarization effects in the π-meson electroproduction it is useful to employ the helicity amplitudes in the s-channel. We write out the final expressions for the asymptotic helicity amplitudes of the virtual photoproduction of π mesons:

$$\left(\frac{1}{2}, 0 \,|F_s|\, \frac{1}{2}, \pm 1\right) = -\frac{1}{16\pi\sqrt{2}}[F_1 + F_2 \mp (F_5 + F_6)],$$

$$\left(\frac{1}{2}, 0 \,|F_s|\, -\frac{1}{2}, \mp 1\right) = \frac{i}{16\pi\sqrt{2}}[-F_1 + F_2 \mp (F_5 - F_6)],$$

$$\left(\frac{1}{2}, 0 \,|F_s|\, \frac{1}{2}, 0\right) = \frac{i\sqrt{s}}{16\pi}(F_3 + F_4),$$

$$\left(\frac{1}{2}, 0 \,|F_s|\, -\frac{1}{2}, 0\right) = -\frac{\sqrt{s}}{16\pi}(F_3 - F_4), \tag{21}$$

where F_i are given by (16) and F_s is the amplitude in the c.m.s. of the s-channel; it is defined in an analogous manner like the amplitude F_u is defined in the u-channel. Inserting (2) and (3) in (1) we obtain

$$S_{if} = -i(2\pi)^4 \delta \times (r_1 + p_1 - r_2 - p_2 - q)\frac{e}{k^2}\frac{m}{\sqrt{\varepsilon_1\varepsilon_2}}\frac{M}{\sqrt{2\omega E_1 E_2}}\bar{u}\gamma_\mu u \times \langle p_2, q\,|J_\mu|\,p_1\rangle = \frac{e}{k^2}\frac{m}{\sqrt{\varepsilon_1\varepsilon_2}}\sqrt{2k_0}A(\gamma^* + N \rightarrow N + \pi), \tag{22}$$

where $A(\gamma^* + N \rightarrow N + \pi)$ is the S-matrix element for the virtual photoproduction, where the role of the photon polarization vector is taken by $\bar{u}\gamma_\mu u$; the factor $(2k_0)^{1/2}$ results from the normalization of $A(\gamma^* + N \rightarrow N + \pi)$.

Using (22) we obtain for the differential π-meson electroproduction cross section the expression

$$d\sigma = \frac{e^2}{(2\pi)^3}\frac{m^2}{\varepsilon_1\varepsilon_2 k^4}\frac{2k_0(r_1p_1)}{\sqrt{(r_1p_1)^2 - m^2M^2}} \times \frac{\sqrt{(kp_1)^2 - k^2M^2}}{(kp_1)} d\sigma(\gamma^* + N \rightarrow N + \pi)\, d^3\mathbf{r}_2$$

$$= \frac{\alpha}{\pi^2}\frac{k_0 m^2}{\varepsilon_1\varepsilon_2 k^4}\frac{(r_1p_1)}{(kp_1)}\sqrt{\frac{(kp_1)^2 - k^2M^2}{(r_1p_1)^2 - m^2M^2}}\, d\sigma \times (\gamma^* + N \rightarrow N + \pi)\, d^3\mathbf{r}_2. \tag{23}$$

The quantity $[(r_1p_1)^2 - m^2M^2]^{1/2}/(r_1p_1)$ represents the relative velocity of the incoming electron and the target nucleon, while $[(kp_1)^2 - k^2M^2]^{1/2}/(kp_1)$ represents the relative velocity of the virtual photon and the target nucleon which is introduced so that $|A(\gamma^* + N \rightarrow N + \pi)|^2$ goes over into $d\sigma(\gamma^* + N \rightarrow N + \pi)$.

The above introduced amplitude F_3 is associated with the differential cross section by the relation

$$\frac{d\sigma}{d\Omega}(\gamma^* + N \rightarrow N + \pi) = \frac{q}{k}|F_s|^2. \tag{24}$$

After averaging over the initial and summing over the final electron spins we find for the differential virtual photoproduction cross section the expression

$$\frac{d\sigma}{d\Omega}(\gamma^* + N \rightarrow N + \pi) = \frac{q}{4m^2k}\varepsilon_{\alpha\beta}f_{\alpha\beta}, \tag{25}$$

$$-\varepsilon_{\alpha\beta} = 2\hat{k}'_\alpha\hat{k}'_\beta(\hat{\mathbf{k}}'\mathbf{r}_1)^2 - \delta_{\alpha\beta}\frac{k^2}{2} + 2\hat{k}_\alpha\hat{k}_\beta[(\mathbf{k}\mathbf{r}_1) + (\hat{\mathbf{k}}\mathbf{r}_1)^2] + (k_\alpha\hat{k}'_\beta + k_\beta\hat{k}'_\alpha)(k + 2\hat{\mathbf{k}}\mathbf{r}_1), \tag{26}$$

where $\hat{\mathbf{k}}'$ is a unit vector which is orthogonal to $\mathbf{k}$ and lies in the plane determined by the vectors $\mathbf{k}$ and $\mathbf{q}$. In these formulae the term $\hat{k}'_\alpha\hat{k}'_\beta f_{\alpha\beta}$ corresponds to the production of π-mesons by linearly polarized transversal photons, the term $\hat{k}_\alpha\hat{k}_\beta f_{\alpha\beta}$ corresponds to π-meson production by longitudially polarized virtual photons, and the term $(k_\alpha\hat{k}'_\beta + k_\beta\hat{k}'_\alpha)f_{\alpha\beta}$ describes the interference between the π-meson production amplitudes by transverse and longitudinal photons, and, finally, the term proportional to $\delta_{\alpha\beta}$ corresponds to the differential cross section averaged over the polarizations of the virtual photon.

According to (21) the contribution to the cross section of the longitudinal photons is asymptotically smaller than that of the transverse photons. Therefore we shall not consider the longitudinal photons in (26). Inserting in (25) and (26) the asymptotic expressions for the helicity amplitudes we obtain

$$\frac{d\sigma}{d\Omega}(\gamma^* + N \rightarrow N + \pi) = -\frac{k^2}{8m^2}\left[\frac{d\sigma}{d\Omega} - \frac{2\mathbf{r}_1^2\sin^2\sigma}{k^2}\frac{d\sigma}{d\Omega_t}\right], \tag{27}$$

where σ is the angle between the vectors $\mathbf{k}$ and $\mathbf{r}_1$.

The quantities $d\sigma/d\Omega$ and $d\sigma/d\Omega_t$ represent the differential π-meson photoproduction cross sections by unpolarized and linearly polarized photons, respectively. Asymptotically they are given by the expressions:

$$d\sigma/d\Omega = 2\,(1+\alpha_\pm^2)\,(\rho_1^2+\rho_2^2)\,s^{2j'-1},$$

$$d\sigma/d\Omega_t = d\sigma/d\Omega + 4\cos 2\varphi\rho_1\rho_2(1-\alpha_\pm^2)\cos(\varphi_1-\varphi_2)\,s^{2j'-1} - 4\sin 2\varphi\rho_1\rho_2\alpha_\pm\sin\beta\sin(\varphi_1-\varphi_2)\,s^{2j'-1}, \qquad (28)$$

where*

$$\alpha_\pm^2 = \frac{\operatorname{ch}\pi j'' \mp \sin\pi j'}{\operatorname{ch}\pi j'' \pm \sin\pi j'}, \qquad \operatorname{tg}\beta = \frac{\operatorname{sh}\pi j''}{\cos\pi j'},$$

$$\rho_1 e^{i\varphi_1} = \frac{a-c}{16\pi\sqrt{2}}, \qquad \rho_2 e^{i\varphi_2} = \frac{a+c}{16\pi\sqrt{2}},$$

φ is the azimuthal angle of the emitted π meson, while j' is the real and j'' the imaginary part of the function $j = j(u)$ which describes the trajectory of the pole.

One sees from (28) that the averaged over nucleon polarizations differential cross section does not oscillate at large energies.

7. We now determine the polarization of the recoil nucleons. It can be represented as follows:

$$\mathbf{P}\frac{d\sigma}{d\Omega}(\gamma^* + N \to N + \pi) = -\frac{k^2}{8m^2}\left[\mathbf{P}'\frac{d\sigma}{d\Omega} - \frac{2r_1^2\sin^2\sigma}{k^2}\mathbf{P}''\frac{d\sigma}{d\Omega_t}\right]. \qquad (29)$$

Here $\mathbf{P}'$ is the polarization of the recoil nucleon corresponding to unpolarized virtual photons and $\mathbf{P}''$ is the polarization of the recoil nucleons for polarized photons. In the considered asymptotic region these quantities have the form

$$P'_x = P'_z = 0,$$

$$P'_y d\sigma/d\Omega = 4\alpha_\pm \sin\beta\cos(\varphi_1-\varphi_2)\,s^{2j'-1}$$

$$P''_y d\sigma/d\Omega_t = P'_y d\sigma/d\Omega_t - \cos\varphi R_2 - \cos 3\varphi R_1,$$

$$P''_x d\sigma/d\Omega_t = \sin\varphi R_2 - \sin 3\varphi R_1, \qquad (30)$$

where, in turn,

$$R_1 = \alpha_\pm \sin\beta\rho_1^2 s^{2j'-1}, \qquad R_2 = \alpha_\pm \sin\beta\rho_2^2 s^{2j'-1}.$$

The coordinate system x, y, z is associated with the recoil nucleon; in it the axis z is parallel to the nucleon momentum and x is in the plane defined by **k** and **q**, the axis y is perpendicular to that plane.

As can be seen from (28) and (30), the differential cross section and the polarization of the recoil nucleon are monotonic functions of the energy and angle, the oscillatory character of the asymptotic amplitudes notwithstanding. Oscillations appear only in the quantities which describe correlations between the polarizations of the initial and final nucleons. For example, the y component of the polarization of the recoil nucleon is given for the case that the initial nucleon is polarized parallel to the momentum (for unpolarized photons) by the expression

$$P_{yz}d\sigma/d\Omega_z = \alpha_\pm \sin\beta\,(\rho_1^2+\rho_2^2)\,s^{2j'-1}\cos\varphi + \tfrac{1}{2}\sin\varphi\,\{[\alpha_\pm^2\sin 2(j''\xi+\varphi_1\mp\beta) - \sin 2(j''\xi+\varphi_1)]\,\rho_1^2 + [\alpha_\pm^2\sin 2(j''\xi+\varphi_2\mp\beta) - \sin 2(j''\xi+\varphi_2)]\,\rho_2^2\}\,s^{2j'-1},$$

$$\xi = \ln s. \qquad (31)$$

Thus the π-meson production process in the interaction of high energy electrons and nucleons for π-meson production angles of $\sim 180°$ is given by two complex quantities, a and c, while the amplitude of the process has a factorized structure, see (18) and (19). The oscillatory behavior which results from the Fermi nature of the Regge poles exists only in quantities that describe correlations. Their experimental observation is hardly possible at the present time.

In conclusion I consider it my pleasant duty to thank A. I. Akhiezer for many important discussions.

*tg = tan, sh = sinh, ch = cosh.

[1] V. N. Gribov, JETP 43, 1529 (1962), Soviet Phys. JETP 16, 1080 (1963).
[2] I. M. Barbour, Nuovo cimento 27, 1382 (1963).
[3] M. Jacob and G. C. Wick, Ann. Phys 7, 404 (1959).

Translated by M. Danos
87