THE BEHAVIOR OF THE CROSS SECTION OF THE INELASTIC PROCESS a + b → c + d + e *AT HIGH ENERGIES*

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The asymptotic expression for the amplitude for the transformation of two particles into three at high energy, which has been derived previously [1,2] on the basis of the one-Regge-pole approximation, is utilized here for the computation of the total cross section of this reaction and for the determination of the most likely values of the momenta of the generated particles. It is shown that in the high energy region $s = s_{ab} \to \infty$ the total cross-section of the reaction a + b → c + d + e has an energy dependence of the form $\{c_1 \ln[\ln(s/m^2)] + c_2\}/\ln(s/m^2)$. An investigation of the differential cross section of the reaction a + b → c + d + e has shown that the most probable situation is the case in which all outgoing particles are ultrarelativistic, with two of the particles emitted, in the c.m.s. of the reaction, within a narrow cone in one direction and the third particle emitted in the opposite direction. The largest contribution to the cross-section, of the order of $\{\ln[\ln(s/m^2)]\}/\ln(s/m^2)$ corresponds to the case of so-called "genuine inelastic" collisions, when all three invariants s_{ec}, s_{cd}, and s_{de} are large compared to m^2 as $s \to \infty$. In this case one of the two particles, which are emitted in the same direction, has its momentum considerably in excess of the momentum of the other particle.

If the momenta of these particles are equal their energy in the c.m.s. will not be large. This corresponds to the case of the so-called "almost elastic" collisions; their contribution to the total cross section is of the order $1/\ln(s/m^2)$. If the energy is not extremely large, i.e., $\ln(s/m^2)$ is not much larger than one, then the probability for the generation of the two fast particles and one slow particle, almost isotropically distributed in the c.m.s. angle, is relatively large (of the order of $1/\ln(s/m^2)$).

1. In the present paper an investigation of the differential and total cross sections for the reaction a + b → c + d + e is carried out in the region of very high energies on the basis of the results of the analysis [1,2] of the asymptotic behavior of amplitudes for inelastic processes. In spite of the complicated nature of the problem, the results turn out to be extremely simple and intuitive. They reduce to the assertion that, taking into account the contribution of only one Regge pole—the one situated most to the right, the amplitudes for inelastic collisions (Fig. 1b) have their asymptotic behavior determined by contributions from simple diagrams, represented in Fig. 2b and very similar to Feynman graphs. However, in these diagrams the propagation line for a virtual particle (which we call a "reggeon") is associated to a quantity $I_0(t)s^{j_0(t)}$, depending not only on the square of the "reggeon" momentum $q^2 = t$ (as for usual Feynman graphs), but also on the energy $(s)^{1/2}$ of the colliding particles. Here $j_0(t)$ is the position of the Regge pole situated most to the right and $I_0 = i - \cot(\pi j_0(t)/2)$.

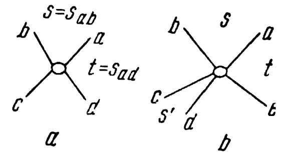

FIG 1

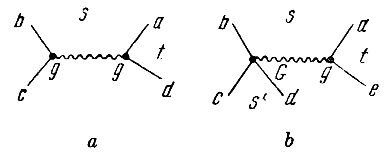

FIG 2

Similar results have been obtained subsequently also by other authors. Some have formulated such results in the form of assumptions [3], others have derived them on the basis of arguments [4] which in our opinion are not convincing.

In the above-mentioned work [1,2] it has been shown that, in the same manner as for reactions of the type a + b → c + d [5] (Fig. 1a) the asymptotic behavior of the amplitude is determined by the contribution of the diagram in Fig. 2a:

$$A\,(2 \leftarrow 2) = g\,(t)\, g\,(t)\, I_0\,(t)\, \{s/m^2\}^{j_0(t)}, \tag{1}$$

the amplitude for the reaction a + b → c + d + e (Fig. 1b), for $s_{ab} \gg m^2$, is determined by the contribution of the diagram represented in Fig. 2b:

$$A\,(3 \leftarrow 2) = G\,(k', t)\, g\,(t)\, I_0\,(t)\, \{s/m^2\}^{j_0(t)}. \tag{2}$$

Here k′ is the momentum of the relative motion of the particles c and d in their center of mass system (c.m.s.) (in what follows, all quantities defined in such a frame of reference will be denoted by one or two primes). This expression for the amplitude of the inelastic process (2) has been written under the assumption that the energy $(s_{cd})^{1/2} = 2(m^2 + k'^2)^{1/2}$ of the particles c and d is small in the c.m.s. [1]), i.e., $s_{cd} \approx s' \sim m^2$. The vertex function g(t) in (2) corresponds to the absorption of a reggeon, and the vertex function G(k′, t) corresponds to the emission of a reggeon in the transition from particle b to particles c and d. This vertex function is a "four-point function" and therefore, besides the square of the reggion four-momentum, it also depends on the components of the momentum **k′** of the relative motion of the particles c and d.

If the momenta of the particles c and d are such that the invariant $s_{cd} = s'$ is not small, i.e., if $s \gg m^2$ and $s' \gg m^2$, then the amplitude of the process described by Fig. 1b is determined by the contribution of the pole diagram of Fig. 3, with two reggeons and which in the asymptotic region can be represented in the form

$$A\,(3 \leftarrow 2) \approx g\,(t_1)\, \gamma\,(t_1, t)\, g\,(t)\, I_0\,(t_1)\, (s_{cd}/m^2)^{j_0(t_1)}\, I_0\,(t) (s_{de}/m^2)^{j_0(t)}. \tag{3}$$

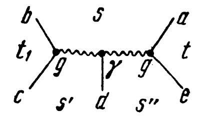

FIG 3

Here $\gamma(t_1, t)$ is the vertex function (Fig. 3) corresponding to the emission of the particle d by the reggeon, $t_1 = (p_b - p_c)^2$.

In order for Eq. (3) to be true, it is necessary that as $s_{ab} = s \to \infty$ the inequalities $s_{de} \gg m^2$ and $s_{dc} \gg m^2$ be satisfied and the squares of the momentum transfers t_1 and t be small. When the magnitude of the invariant s_{cd} decreases Eq. (3) goes over into Eq. (2), since for small $s' = s_{cd}$ the magnitudes of the invariants $s_{ab} = s$ and s_{de} differ by a factor which depends only on k′ (this follows from the kinematics which is described in detail below).

The derivation of Eqs. (2) and (3) is based on the assumption that only one Regge pole, the one situated most to the right, is dominently contributing at high energy. If in reality the situation turns out to be more complicated [6], e.g., if the right-most singularity of the partial wave amplitudes is not an isolated pole, but a point of accumulation of singularities [7], then the asymptotic expressions (2), (3) will be essentially modified. Nevertheless, apparently in all cases the asymptotic behavior of the amplitudes of inelastic processes must correspond to the asymptotic behavior of the amplitude of elastic scattering in the same sense as (2) and (3) correspond to (1).

Keeping this in mind, it seems interesting to us to carry out an investigation of the differential and total cross sections for the reaction a + b → c + d + e on the basis of Eqs. (2) and (3) and to determine the most likely configurations of the momenta of the produced particles. In our opinion, the results of such an investigation have a quite general character and will hardly be modified in their essence, should it turn out that the assumption of the dominance of a single Regge pole is incorrect.

2. As is well known, the cross section for the reaction a + b → c + d + e is connected with the amplitude (2) or (3) A(3 ← 2) through the relation

$$d\sigma\,(3 \leftarrow 2) = \frac{1}{4\varepsilon_a \varepsilon_b F_0} |A\,(3 \leftarrow 2)|^2\, d\tau_3, \tag{4}$$

where $F_0 = dW_0/dp_a \equiv p_a(s)^{1/2}/\epsilon_a\epsilon_b$ is the flux of incoming particles a and b (all quantities are in the c.m.s. of the reaction),

$$W_0 = \sqrt{m_a^2 + \mathbf{p}_a^2} + \sqrt{m_b^2 + \mathbf{p}_b^2} = 2\sqrt{m^2 + \mathbf{p}_a^2}$$

is the total energy, $W_0 = (s)^{1/2}$, and

$$d\tau_3 = \frac{(2\pi)^4}{3!}\,\delta^4\,(P_0 - P)\,\frac{d^3p_c}{(2\pi)^3\,2\varepsilon_c}\,\frac{d^3p_d}{(2\pi)^3\,2\varepsilon_d}\,\frac{d^3p_e}{(2\pi)^3\,2\varepsilon_e} = \frac{1}{3!}\,\frac{4}{(4\pi)^5}\,\frac{p_e^2}{\varepsilon_c\varepsilon_e\,(dw/dp_e)}\,\frac{d^3p_d\,dn_e}{\varepsilon_d} \tag{5}$$

is the statistical weight of the final state of the particles c, d, e; $P_0 = (W_0, \mathbf{P}_0)$ is the total four-momentum ($\mathbf{P}_0 = 0$) of the initial state, similarly $P = (W, \mathbf{P})$ is the total four-momentum of the final state, and

$$W = \sqrt{m^2 + (\mathbf{p}_d + \mathbf{p}_e)^2} + \sqrt{m^2 + \mathbf{p}_d^2} + \sqrt{m^2 + \mathbf{p}_e^2}. \tag{6}$$

The statistical weight (phase space factor) (5) con-

[1])In order to simplify the exposition we consider the case when the masses of all particles are equal. The case of unequal masses leads to some complication in writing, which is in principle inessential.

tains the factor 1/3! since we assume that the particles are all identical.

The c.m.s. momentum configuration of all five particles (i.e., in the c.m.s. of the particles a and b) is represented in Fig. 4. For large energies and large values of the invariants s_{cd} and s_{de} the amplitude (3) is not small only for small absolute values of the squares of the momentum transfers $t_1 = (p_b - p_c)^2$ and $t = (p_a - p_e)^2$.

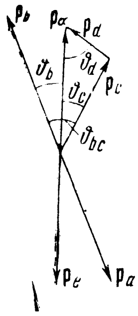

FIG. 4

Indeed, in the asymptotic region the amplitude of the reaction a + b → c + d + e is proportional to the quantities $\exp[j_0(t_1)\ln(s_{cd}/m^2)]$ and $\exp[j_0(t)\ln(s_{de}/m^2)]$. We expand the functions $j_0(t)$ and $j_0(t_1)$ around the points $t = 0$ and $t_1 = 0$:

$$j_0(t) = j_0(0) + j_0'(0)\,t, \qquad j_0(t_1) = j_0(0) + j_0'(0)\,t_1. \quad (7)$$

Here, as is well known,

$$(dj_0(t)/dt)_{t=0}, \qquad (dj_0(t_1)/dt_1)_{t_1=0}$$

are quantities of the order $1/m^2$ and $j_0(0) = 1$ (the latter condition is a well known property of the vacuum trajectory $j_0(t)$, which guarantees the constancy of the total cross section of the particle interaction at extremely high energies).

Since in the physical region of the reaction a + b → c + d + e the quantities t_1 and t are negative, the exponent of the factors $\exp[j_0(t_1)\ln(s_{cd}/m^2)]$, $\exp[j_0(t)\ln(s_{de}/m^2)]$ decreases as t and t_1 increase. It can be seen from this, that at high energies and large values of s_{cd} and s_{de} the amplitude (2) and (3) is not small only in the region of small t and t_1. In order that t and t_1 be small it is necessary that the angles ϑ_b and ϑ_{bc} in Fig. 4 be very small, and the momenta p_c and p_d be large, i.e., of the same order of magnitude as the momenta $p_a \approx p_b \approx (s)^{1/2}/2$ of the colliding particles.

Without loss of generality we can assume that among the momenta p_c, p_d, and p_e, the momentum p_d is the small one, i.e., that the momenta p_c and p_d are almost parallel (if p_d is not very small) and are directed oppositely to p_e.

The amplitude for the reaction a + b → c + d + e consists of the contributions of six diagrams of the type represented in Fig. 3, and obtained by carrying out all possible permutations of the particles c, d, and e. To each of these diagrams correspond obviously six configurations of momenta for the particles produced in the reaction, analogous to the configurations in Fig. 4, but differing from it by interchanges of the particles c, d, and e. However, only one of the configurations makes a noticeable contribution to the amplitude, since the momentum transfers in the other five will be large. Therefore the amplitude of the reaction a + b → c + d + e will consist of six equal contributions, and thus the factor 1/3! in $d\tau_3$ will cancel out.

The conservation laws for the energy and the projection of the total momentum on the direction of p_e yield

$$\varepsilon_e + \varepsilon_c + \varepsilon_d = \sqrt{s}, \qquad k_c + k_d = p_e, \quad (8)$$

where k_c and k_d denote the projections of p_c and and p_d on the direction of p_e (cf. Fig. 4). For the following it will be convenient to characterize the momenta p_c and p_d by their projections k_c, k_d and the two-dimensional projection κ on the plane perpendicular to p_e, i.e., $p_c = (k_c, \kappa)$, $p_d = (k_d, -\kappa)$. In particular $p_c \cdot p_d = k_c k_d - \kappa^2$. It will be shown below that the quantity κ^2 is always small, [of the order of $m^2/\ln(s/m^2)$]. Therefore if p_c and p_d are ultrarelativistic,

$$\varepsilon_c = \sqrt{m^2 + \varkappa^2 + k_c^2} \approx k_c, \quad \varepsilon_e = \sqrt{m^2 + k_e^2} \approx k_e,$$
$$\varepsilon_d = \sqrt{m^2 + \varkappa^2 + k_d^2}.$$

Thus the conservation laws (8) can be put in the form

$$p_e + k_c = \sqrt{s} - \varepsilon_d, \qquad p_e - k_c = k_d. \quad (9)$$

Adding these equalities, we obtain $2p_e = (s)^{1/2} - (\epsilon_d - k_d)$.

For $(s)^{1/2} \gg m$, the difference $\epsilon_d - k_d$ is negligibly small compared to $(s)^{1/2}$. Indeed, if k_d is of the order of m, this difference is a quantity of the order of m; if $k_d \gg m$, $\epsilon_d - k_d \approx m^2/2k_d$. Thus, up to terms of the order $m/(s)^{1/2}$ we obtain that $p_e \approx (s)^{1/2}/2$, i.e.,

$$k_c + k_d = \sqrt{s}/2. \quad (10)$$

After these remarks we proceed to compute $d\tau_3$. Differentiating the energy (6) with respect to p_e, we obtain

$$dW/dp_e = (p_e - z_d p_d)/\varepsilon_c + p_e/\varepsilon_e,$$

where $z_d = \cos\vartheta_d$ (cf. Fig. 4), i.e., $z_d p_d = k_d$.

But $p_e - k_d = k_e$, therefore

$$\varepsilon_e \varepsilon_c \, dW/dp_e = \varepsilon_e k_c + \varepsilon_c p_e.$$

Thus

$$\frac{6 d\tau_3}{4\varepsilon_a \varepsilon_b F_0} = \frac{1}{(4\pi)^5} \frac{p_e^2}{p_a \sqrt{s}\,(\varepsilon_e k_c + \varepsilon_c p_e)} d\mathbf{n}_e \frac{d\mathbf{p}_d}{\varepsilon_d}.$$

This is the exact expression. For large energies, when the particles a, e, and c are ultrarelativistic, one may set $p_a \approx (s)^{1/2}/2$, $p_e \approx \epsilon_e \approx (s)^{1/2}/2$, $\epsilon_c \approx k_c$, i.e.,

$$\frac{6 d\tau_3}{4\varepsilon_a \varepsilon_b F_0} \approx \frac{1}{(4\pi)^5} \frac{\sqrt{s}}{2k_c} \frac{d\mathbf{n}_e \, d\mathbf{p}_d}{s\varepsilon_d}.$$

Substituting into (4) the expression for $d\tau_3$ and the asymptotic expression (3) for the amplitude (since in the case $s \to \infty$ both invariants $s' = s_{cd}$ and $s'' = s_{de}$ are large, this is the most interesting case, and in fact, it will turn out to be the general case), we obtain

$$d\sigma(3 \leftarrow 2) = \frac{1}{4} \frac{g^2(t_1)}{4\pi} \frac{g^2(t)}{4\pi} \frac{\gamma^2(t_1 t)}{4\pi} \left(\frac{s'}{m^2}\right)^{2j_0(t_1)} \left(\frac{s''}{m^2}\right)^{2j_0(t)} \frac{\sqrt{s}}{2k_c} \frac{d\mathbf{n}_e d\mathbf{p}_d}{4\pi^2 \varepsilon_d s}. \quad (11)$$

As will be shown, for very large energies the values $t_1 = 0$ and $t = 0$ are essential, therefore in (11) the factor $[I_0(t) I_0(t_1)]^2$, which is unity for $t = 0$ and $t_1 = 0$, has been omitted.

The quantities $g(t_1)$, $g(t)$, and $\gamma(t_1 t)$ which have no singularities for $t_1 = 0$ and $t = 0$ will be replaced by their values in these points, i.e., $g_0 = g(0)$, and $\gamma_0 = \gamma(00)$.

According to Fig. 4, the invariants $s' = s_{cd} = (p_c + p_d)^2$, $s'' = s_{de} = (p_d + p_e)^2$ and $t_1 = (p_b - p_c)^2$, $t = (p_a - p_c)^2$ have the following values:

$$s' \approx 2p_c p_d = 2(\varepsilon_c \varepsilon_d - k_c k_d + \varkappa^2) \approx 2k_c(\varepsilon_d - k_d),$$
$$s'' \approx 2p_d p_e \approx 2(\varepsilon_d \varepsilon_e + k_d k_e) \approx \sqrt{s}(\varepsilon_d + k_d); \quad (12)$$

$$-t_1 = -2m^2 + 2\varepsilon_b \varepsilon_c - 2p_b p_c z_{bc} \approx \tau_1^0 + \sqrt{s} k_c \vartheta_{bc}^2/2,$$
$$-t = p_a p_b \vartheta_b^2 = s\vartheta_b^2/4. \quad (13)$$

In (12), terms of the order m^2 have been neglected (since the case $s' \gg m^2$, $s'' \gg m^2$ has been kept in mind), and in (13) the quantity $(s)^{1/2}$ has been substituted for $p_a = p_b$ and it has been taken into account that $\epsilon_c \approx k_c$, $\epsilon_e \approx p_e \approx (s)^{1/2}/2$. The cosines of the angles ϑ_{bc} and ϑ_b (Fig. 4) have been written in (13) in the form $z_{bc} = 1 - \vartheta_{bc}^2/2$, $z_b = 1 - \vartheta_b^2/2$, since in what follows only the region of very small values of these angles will be of interest. The part of $(-t_1)$ which does not depend on the directions of the momenta is denoted by τ_1^0:

$$\tau_1^0 = 2(\varepsilon_b \varepsilon_c - p_b p_c - m^2) = m^2(p_b/k_c + k_c/p_b - 2)$$
$$\approx m^2(\sqrt{s}/2k_c + 2k_c/\sqrt{s} - 2).$$

Introducing the notation $y = 2k_d/(s)^{1/2}$ and taking into account the equality (10) one can represent the quantity τ_1^0 in the form

$$\tau_1^0 = m^2 y^2/(1-y). \quad (14)$$

We remark that the angles ϑ_b, ϑ_c, and ϑ_{bc}, as can be seen from Fig. 4, are connected by the relation

$$z_{bc} = z_b z_c + \sqrt{(1 - z_b^2)(1 - z_c^2)} \cos\varphi,$$

where φ is the azimuthal angle between the planes which contain the vectors $\mathbf{p}_e$, $\mathbf{p}_b$ and $\mathbf{p}_c$, $\mathbf{p}_d$, respectively, and $z_{bc} \approx 1 - \vartheta_{bc}^2/2$, $z_b \approx 1 - \vartheta_b^2/2$, and $z_c \approx 1 - \vartheta_c^2/2$ are the cosines of the corresponding angles. From here it follows for ϑ_{bc}

$$\vartheta_{bc}^2 = \vartheta_b^2 + \vartheta_c^2 - 2\vartheta_b \vartheta_c \cos\varphi. \quad (15)$$

Taking into account these values of the invariants, in particular that according to (12)

$$\frac{s's''}{m^2} = 2k_c \sqrt{s}\left(1 + \frac{\varkappa^2}{m^2}\right),$$

and substituting (8) into (11) we write the cross section in the form

$$d\sigma(3 \leftarrow 2) = \left(\frac{g_0^2}{4\pi}\right)^2 \frac{\gamma_0^2}{4\pi} \exp\left(-2j_0'' m^2 \frac{y^2 \xi'}{1-y}\right) \left(\frac{2k_c}{\sqrt{s}}\right)^3 dk_d \, dF, \quad (16)$$

where

$$dF = \frac{(1 + \varkappa^2/m^2)^2}{\varepsilon_d (2m)^4} \exp\left\{-j_0'' \xi' \left(\frac{2k_c}{\sqrt{s}}\right) \frac{s\vartheta_{bc}^2}{2} - j_0'' \xi'' \frac{s}{2} \vartheta_b^2\right\}$$
$$\times d\left(\frac{s\vartheta_b^2}{2}\right) d\left(\frac{s\vartheta_{bc}^2}{2}\right) \left(\frac{d\varphi}{2\pi}\right), \quad (17)$$

with

$$\xi' = \ln\frac{s'}{m^2} = \ln\frac{2k_c}{m^2}(\varepsilon_d - k_d),$$
$$\xi'' = \ln\frac{s''}{m^2} = \ln\frac{\sqrt{s}}{m^2}(\varepsilon_d + k_d). \quad (18)$$

In (16) and (17) it has been taken into account that the cross section (11) does not depend on the azimuthal angle of the vector $\mathbf{n}_e = \mathbf{p}_e/p_e$ (since a variation of this angle corresponds to a rotation as a whole of the ensemble of vectors represented in Fig. 4), therefore the differential $d\mathbf{n}_e$ can be represented in the form

$$d\mathbf{n}_e = 2\pi d(\cos\vartheta_b) \approx 2\pi d(\vartheta_b^2/2).$$

Besides, the differential $d\mathbf{p}_d$ has been written in the form $d\mathbf{p}_d = dk_d d\boldsymbol{\varkappa}$, with $d\boldsymbol{\varkappa} = \frac{1}{2} d\varkappa^2 d\varphi = k_c^2 \times d(\vartheta_c^2/2) d\varphi$. Since according to Fig. 4, $\varkappa \approx \vartheta_c k_c$, in the variables of Eq. (17) the quantity $\varkappa^2$ can be rewritten in the form

$$\varkappa^2 = \frac{1}{2}\left(\frac{2k_c}{\sqrt{s}}\right)^2 \frac{s\vartheta_c^2}{2}.$$

In order to compute the total cross section we first integrate in (17) over ϑ_b, ϑ_c, and φ. To do this, we introduce two-dimensional vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, situated in a plane perpendicular to $\mathbf{p}_e$, as is also $\boldsymbol{\kappa}$, and having the values $\beta^2 = s\vartheta_b^2/2$ and $\gamma^2 = s\vartheta_c^2/2$. We assume that the vector $\boldsymbol{\gamma}$ is parallel to $\boldsymbol{\kappa}$ (its square differs from κ^2 only by the factor $\frac{1}{2}(2k_c/(s)^{1/2})^2$) and $\boldsymbol{\beta}$ is situated in the plane in which the angle ϑ_b is measured, i.e., $\boldsymbol{\gamma}\cdot\boldsymbol{\beta} = |\gamma||\beta|\cos\varphi$. According to (15), we have $s\vartheta_{bc}^2/2 = (\boldsymbol{\gamma}-\boldsymbol{\beta})^2$, and (17) can be represented in the form

$$dF = \frac{(1+\varkappa^2/m^2)^2}{\pi^2(2m)^4\varepsilon_d} \times \exp\left\{-j_0'\left[\left(\frac{2k_c}{\sqrt{s}}\right)(\gamma-\beta)^2\xi' + \xi''\beta^2\right]\right\}d^2\beta d^2\gamma. \quad (19)$$

In the expression (19), one has to consider that $\xi' > 1$, and $\xi'' > 1$, since the asymptotic expression (3) of the amplitude $A(3 \leftarrow 2)$ which has been used in (17) is valid only in this region ((18) implies that $\xi' > 1$ only if $k_c \gg k_d$). For $\xi' > 1$ and $\xi'' > 1$ the integration over $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ which is used in the computation of the total cross section can be extended to infinity. Introducing in place of $\boldsymbol{\gamma}$ the vector $\boldsymbol{\alpha} = \boldsymbol{\gamma} - \boldsymbol{\beta}$, we obtain

$$F = \frac{1}{(8\pi m^2)^2}\int_0^\infty d\alpha^2\int_0^\infty d\beta^2\frac{(1+\varkappa^2/m^2)^2}{\sqrt{m^2+\varkappa^2+k_d^2}} \times \exp\left\{-j_0'\left[\xi'\frac{2k_c}{\sqrt{s}}\alpha^2+\xi''\beta^2\right]\right\} \approx \frac{1}{(8\pi j_0' m^2)^2}\left(\frac{\sqrt{s}}{2k_c}\right)\frac{1}{\xi'\xi''\sqrt{m^2+k_d^2}}. \quad (20)$$

In calculating the integral it has been taken into account that the values $\alpha^2 \sim 1/j_0'\xi' \approx m^2/\xi'$ (for $2k_c \sim (s)^{1/2}$) and $\beta^2 \sim 1/j_0'\xi'' \sim m^2/\xi''$ are essential. Therefore for $\xi' > 1$ and $\xi'' > 1$ the values $\kappa^2 = \frac{1}{2}(2k_c/(s)^{1/2})^2 \times (\boldsymbol{\alpha}+\boldsymbol{\beta})^2$, which are essential in the integral, are also small compared to m^2, and one can neglect the quantity κ^2/m^2.

According to (19) and (20) the distribution of the generated particles c and d in their momenta ($k_c + k_d = (s)^{1/2}/2$) has the form

$$d\sigma(3 \leftarrow 2) = \sigma_0(y)\frac{dk_d}{\sqrt{m^2+k_d^2}\,\xi'\xi''}, \quad (21)$$

where the quantity

$$\sigma_0(y) = \frac{1}{(8\pi m^2 j_0')^2}\left(\frac{g_0^2}{4\pi}\right)^2\frac{\gamma^2}{4\pi}(1-y)^2\exp\left(-j_0'\frac{2m^2y^2}{1-y}\xi'\right) \quad (22)$$

has in fact a constant value, corresponding to $y = 0$:

$$\sigma_0(y) \approx \sigma_0 = \frac{1}{(8\pi m^2 j_0')^2}\left(\frac{g_0^2}{4\pi}\right)^2\frac{\gamma_0^2}{4\pi}. \quad (23)$$

In the region where $k_d^2 \sim m^2$, this assertion is obviously true, since the quantity $y = 2k_d/(s)^{1/2}$ is negligibly small in this region. We consider the region $k_d^2 \gg m^2$. Since the value of the cross section (16)—(17) has been obtained under the assumption $\xi' > 1$ and $\xi' = \ln(k_c/k_d) \approx (1/y - 1)$, the inequality $y \ll 1$ must hold. Consequently, in both cases the quantity $y^2\xi' \approx -y^2\ln y$ in the exponent of (22) is negligibly small, and hence taking into account the dependence of σ_0 on y means going beyond the assumed approximation. In the region where $k_c \sim k_d$, i.e., $y \sim 1$ and $s' \sim m^2$, one should use the asymptotic expression (2) for the amplitude $A(3 \leftarrow 2)$.

In integrating (21) with respect to k_d (with $k_d \ll (s)^{1/2}/2$) we will distinguish between two regions: (1) $k_d \sim m$, and (2) $k_d \gg m$, or more precisely (1) $0 \le k_d \le N'm$ and (2) $N'm \le k_d \le s/2N''$, where N' and N'' are constants which are much larger than unity and do not depend on s (the result depends very weakly on the particular choice of the values of these numbers). The contribution from the first region to the total cross section is small. In this region

$$\xi' \approx \ln\frac{2k_c}{m^2}(\varepsilon_d - k_d) = \frac{1}{2}\xi + \ln\left(\frac{2k_c}{\sqrt{s}}\frac{\varepsilon_d - k_d}{m}\right) \approx \frac{1}{2}\xi,$$

$$\xi'' = \ln\frac{\sqrt{s}}{m^2}(\varepsilon_d + k_d) \approx \frac{1}{2}\xi + \ln\left(\frac{\varepsilon_d + k_d}{m}\right) \approx \frac{1}{2}\xi,$$

where $\epsilon_d = (m^2 + k_d^2)^{1/2}$ and $\xi = \ln(s/m^2)$. We assume that the quantity $\xi = \ln(s/m^2)$ is so large that one can neglect $\ln 2N'$ compared to ξ.

In the first region both logarithms which enter into the expressions for ξ' and ξ'' are smaller than $\ln N'$, and therefore

$$\sigma_1 = \int_{k_d=0}^{k_d=N'm} d\sigma(3 \leftarrow 2) = \frac{4\sigma_0}{\xi^2}\int_0^{N'm}\frac{dk_d}{\sqrt{m^2+k_d^2}} \approx \frac{4\sigma_0\ln 2N'}{\xi^2}. \quad (24)$$

In the second region ($N'm \le k_d \le (s)^{1/2}/2N''$) we have $\epsilon_d - k_d \approx m^2/2k_d$, therefore

$$\xi' \approx \ln\frac{k_c}{k_d} = \ln\left(\frac{\sqrt{s}}{2k_d} - 1\right).$$

The condition $\xi' > 1$ will be satisfied in this whole region only if N'' is sufficiently large, so that $\ln N'' > 1$. In this case we obtain from (18) and (21)

$$\sigma_2 = \int_{k_d=N'm}^{k_d=\sqrt{s}/2N''} d\sigma(3 \leftarrow 2) = \sigma_0\int_{N'm}^{\sqrt{s}/2N''}\frac{dk_d}{k_d\ln(\sqrt{s}/2k_d)\,[\xi - \ln(\sqrt{s}/2k_d)]}$$

or, introducing the notation $z = \ln[(s)^{1/2}/2k_d]$,

$$\sigma_2 = \sigma_0 \int\limits_{\ln N''}^{\xi/2} \frac{dz}{z(\xi - z)} = \frac{\sigma_0}{\xi} \ln\left[\frac{\xi}{2\ln N''} - 1\right].$$

In the region of very large energies, for $\xi > 2 \ln N''$, we obtain

$$\sigma_2 = \sigma_0 \frac{\ln \xi}{\xi}, \quad \xi = \ln \frac{s}{m^2}. \tag{25}$$

In this region σ_2 is much larger than the cross section (24), corresponding to the creation of one of the three particles with a small momentum.

We note that if the cross section (11) is written (taking (7) into account) in the form

$$d\sigma(3 \leftarrow 2) = \frac{1}{4m^4}\left[\frac{\gamma_0 g_0}{4\pi}\left(\frac{2k_c}{\sqrt{s}}\right)^{(1+j_0'^{t})}\left(1+\frac{\varkappa^2}{m^2}\right)e^{j_0' t_1 \xi'}\right]^2 \times \frac{dp_d}{2\pi\varepsilon_d}\frac{g_0^2}{4\pi}e^{2j_0' t\xi}\left(\frac{\sqrt{s}}{2k_c}\right)d\left(\frac{s\vartheta_b^2}{2}\right), \tag{26}$$

it is easy to follow how the transition from the region of values of $k_d \sim (s)^{1/2}/N'$ to the region of large values of k_d, of the order $(s)^{1/2}/4$, is effected. In the region where k_d is small the value of the invariant $s' = m^2 k_c/k_d$ is small compared to m^2, therefore the asymptotic behavior of the amplitude is determined by Eq. (2) and not by (3). Using (2) we obtain for the cross section the expression

$$d\sigma(3 \leftarrow 2) = \frac{1}{4m^4}\left|\frac{G(k', t)}{4\pi}\right|^2 \frac{dk'}{\pi\sqrt{s}}\frac{g_0^2}{4\pi}e^{2j_0' t\xi}\left(\frac{\sqrt{s}}{2k_c}\right)d\left(\frac{s\vartheta_b^2}{2}\right), \tag{27}$$

which differs from (26) only by the fact, that in place of the term in square brackets in (26), here we have an unknown function $G(k', t)$ (the quantity dp_d/ϵ_d is an invariant with respect to a Lorentz transformation into the c.m.s. of the particles c and d, i.e., $dp_d/\epsilon_d = dk'/\omega'$, where $s' = 4\omega'^2$, therefore $dp_d/2\pi \equiv dk'\pi(s')^{1/2}$).

Integrating (27) with respect to ϑ_b^2 and k′, we obtain for σ_3 the value

$$\sigma_3 = \sigma_0'/\xi,$$

where

$$\sigma_0' = \frac{1}{4\pi m^2 F_0}\int\limits_{s' \leqslant m^2 N}\left|\frac{G(k', 0)}{4\pi}\right|^2\left(\frac{\sqrt{s}}{2k_c}\right)\frac{dk'}{\sqrt{s}}. \tag{28}$$

One can obtain an evaluation of this constant (i.e., its connection with the constant σ_0, which was defined above), by substituting for $|G(k', 0)|$ the asymptotic value of this quantity for $s' > m^2$, i.e., the quantity in the square brackets in Eq. (26).

3. Thus, for very large energies, when not only $s \gg m^2$ but also $\xi = \ln(s/m^2) > 1$, the total cross section for the reaction $a + b \to c + d + e$ consists of the three terms (24), (25), and (28):

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3,$$

i.e.,

$$\sigma = \frac{c_0}{\xi^2} + \frac{\sigma_0}{\xi}\ln\xi + \frac{\sigma_0'}{\xi}, \tag{29}$$

with $c_0 = 4\sigma_0 \ln N'$. The first term determines the contribution to the cross section from small (in particular, nonrelativistic) momenta of the particle d (cf. Fig. 4). If the energy s is not super-high, i.e., ξ is only a little larger than unity, the angular distribution of the slow particles d is almost isotropic (since their transverse momentum κ is a quantity of the order of $m/(\xi)^{1/2}$).

The second term in (29) makes a small contribution to the cross section when the energy is very large and $\xi \gg 1$. This term corresponds to events having a "shower" character, when both ultrarelativistic particles c and d are emitted in a narrow cone in the direction of the colliding particles and the momentum of particle c is much larger than the momentum of particle d. Finally, the last term in (29) corresponds to the case when the momenta of the particles c and d are almost parallel and their magnitudes are of the same order, i.e., the energy $(s)^{1/2}$ of the particles c and d is small in their c.m.s. These are "almost elastic" "shower" collisions, in which one of the colliding particles (in Fig. 4 — particle a) is excited, obtaining a not too large energy $(s')^{1/2} \sim 2m$ and subsequently decays, almost without changing its initial momentum.

The collisions of the first two types are the "genuine inelastic" collisions, since the energies of any pair of generated particles in their c.m.s. are large. In these collisions the transverse momentum κ^2 of the generated particles decreases logarithmically with the increase of energy: $\kappa^2 \sim m^2/\ln(s/m^2)$.

In the case of almost elastic collisions the magnitude of the square of the transverse momentum κ^2 is almost always of the order m^2. This follows from the fact that in the c.m.s. of the particles c and d the transverse momentum is always a quantity of the same order as the longitudinal momentum (since the angular distribution of the momentum k′ is determined in this system by the form of the function $G(k', 0)$, which does not depend on the energy of the incident particles), i.e., is a quantity of the order of m^2. But the transverse momentum does not change under a Lorentz transformation from the c.m.s. of the particles c and d to the c.m.s. of the reaction. Therefore it remains a quantity of the order of m^2 in the c.m.s. of the reaction.

[1] K. A. Ter-Martirosyan, JETP **44**, 341 (1963), Soviet Phys. JETP **17**, 233 (1963); Nuclear Phys. (in press).

[2] A. M. Popova and K. A. Ter-Martirosyan, Nuclear Physics (in press).

[3] S. C. Frautschi, Preprint, 1963.

[4] Amati, Fubini, Stanghellini, and Tonin, Nuovo cimento **22**, 569 (1961). Amati, Fubini, and Stanghellini, Physics Lett. **1**, 29 (1962).

[5] V. N. Gribov and I. Ya. Pomeranchuk, JETP **42**, 1141 (1962), Soviet Phys. JETP **15**, 788 (1962).

[6] K. A. Ter-Martirosyan, Lectures in Nor-Amberd, AN ArmSSR, 1963.

[7] S. Mandelstam, Preprint, 1963.

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