RADIATION FROM FAST ELECTRONS WITH ORIENTED SPINS IN A MAGNETIC FIELD

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The polarization properties of the radiation from relativistic electrons with oriented spins in a magnetic field are investigated. Two cases of electron spin polarization are considered: polarization along the direction of motion and polarization along the magnetic field intensity vector.

As is well known, the radiation of fast electrons moving in a magnetic field is strongly polarized[1]. Seven eighths of the total intensity of radiation belongs to the σ component of the linear polarization (the electric vector of the radiation field lies in the plane of the electron orbit and is directed along the radius towards its center), and $1/8$ of the total intensity pertains to the π component (the electric vector of the radiation field is practically perpendicular to the plane of the electron orbit). The σ and π components of the linear polarization differ in angular distribution: the σ component is characterized by the presence of two maxima, which are symmetrical relative to the plane of the orbit $\theta = \pi/2$, and the σ component has a maximum in the plane of the orbit. These deductions of the classical theory were confirmed experimentally by an investigation of the polarization properties of synchrotron radiation[2].

In the present paper we wish to investigate the properties of the radiation of fast electrons in a magnetic field by the methods of quantum theory, including in the analysis the polarization of electron spin.

1. WAVE FUNCTIONS OF AN ELECTRON MOVING IN A CONSTANT AND HOMOGENEOUS MAGNETIC FIELD

To investigate the motion of a relativistic electron in a constant and homogeneous magnetic field

$$A_x = -\tfrac{1}{2} yH, \quad A_y = \tfrac{1}{2} xH, \quad A_z = 0 \tag{1}$$

we require that the wave function obey the Dirac equation

$$i\hbar\partial\psi/\partial t = \mathscr{H}\psi; \quad \mathscr{H} = c\,(\boldsymbol{\alpha}\mathbf{P}) + \rho_3 m_0 c^2, \quad \mathbf{P} = \mathbf{p} - e\mathbf{A}/c, \tag{2}$$

and that for operators commuting with the Hamiltonian it be an eigenfunction for the operators of projection of momentum and total angular momentum on the direction of the magnetic field, i.e., on the z axis. In a cylindrical coordinate system r, φ, z, which is naturally connected with the character of motion of the electron, the wave function is of the form

$$\psi_{1,3} = e^{-i\varepsilon cKt}\frac{e^{ik_3 z}}{\sqrt{L}}\frac{e^{i(l-1)\varphi}}{\sqrt{2\pi}} f_{1,3}(\rho);$$

$$\psi_{2,4} = e^{-i\varepsilon cKt}\frac{e^{ik_3 z}}{\sqrt{L}}\frac{e^{il\varphi}}{\sqrt{2\pi}} f_{2,4}(\rho), \tag{3}$$

$$E = \varepsilon c\hbar K = \varepsilon c\hbar\sqrt{k_0^2 + 4\gamma n + k_3^2}, \quad \varepsilon = \pm 1 \tag{4}$$

Here $\rho = \gamma r^2$, $\gamma = e_0 H/2c\hbar$, $e_0 = -e$.

The radial functions are of the form

$$f_{1,2,3,4} = \sqrt{2\gamma}\begin{Bmatrix} C_1 I_{n-1,s}(\rho) \\ iC_2 I_{n,s}(\rho) \\ C_3 I_{n-1,s}(\rho) \\ iC_4 I_{n,s}(\rho) \end{Bmatrix}. \tag{5}$$

Here $I_{n,s}(\rho)$ are connected with the Laguerre polynomials:

$$I_{n,s}(\rho) = \frac{1}{\sqrt{n!\,s!}} e^{-\rho/2}\rho^{(n-s)/2} Q_s^{n-s}(\rho), \tag{6}$$

with $n = l + s = 0, 1, 2 \ldots$ the principal quantum numbers and, $s = 0, 1, 2 \ldots$ the radial ones, and $l = 0, \pm 1, \pm 2 \ldots$ $(-\infty \le l \le n)$ are the azimuthal ones. The spin coefficients C_i obey the Dirac equation (2) and are interrelated by the normalization condition

$$\sum_{i=1}^{4} |C_i|^2 = 1, \tag{7}$$

but their form nevertheless remains indeterminate. For a complete determination of the wave function it is necessary to introduce a fourth operator, which commutes with the Hamiltonian and characterizes the polarization of the electron spin.

In the investigation of the longitudinal polarization of the electron spin, it is advantageous to use the polarization 4-vector

$$T_\mu = {}^1/_2 (P_4\sigma_\mu + \sigma_\mu P_4), \tag{8}$$

where $P_4 = (i/c)(\hat{\mathcal{H}} - e\varphi)$ (in our problem the scalar potential φ is equal to zero), and $\sigma_\mu = \{\sigma, i\rho_1\}$ are Dirac matrices (see [3,4]). The time-dependent component of this vector

$$T_4 = -\boldsymbol{\sigma}\mathbf{P} \tag{9}$$

is an integral of the motion and describes the projection of the spin on the direction of motion. We stipulate that the wave function be an eigenfunction for this operator, i.e.,

$$(\boldsymbol{\sigma}\mathbf{P})\psi = \hbar\tilde{k}\tilde{\zeta}\psi. \tag{10}$$

Then simultaneous solution of (2) and (10) for the coefficients C_i yields the following expression (see [5]):

$$C_1 = \tilde{\zeta}\tilde{\alpha}\tilde{A},\ C_2 = \tilde{\alpha}\tilde{B},\ C_3 = \varepsilon\tilde{\beta}\tilde{A},\ C_4 = \varepsilon\tilde{\zeta}\tilde{\beta}\tilde{B},$$

$$\tilde{A} = \sqrt{{}^1/_2 (1 + \tilde{\zeta}k_3/\tilde{k})},\ \tilde{B} = \sqrt{{}^1/_2 (1 - \tilde{\zeta}k_3/\tilde{k})},$$

$$\tilde{\alpha} = \sqrt{{}^1/_2 (1 + \varepsilon k_0/K)},\ \tilde{\beta} = \sqrt{{}^1/_2 (1 - \varepsilon k_0/K)}. \tag{11}$$

In these formulas $\tilde{k} = \sqrt{K^2 - k_0^2}$; $\tilde{\zeta} = \pm 1$ corresponds to the spin polarization with and against the motion.

To investigate the polarization of the electron spin in the direction of the magnetic field, the polarization 4-vector (8) is of little use. Its space-dependent component

$$T_3 = m_0c^2\rho_3\sigma_3 + c\rho_1P_3 \tag{12}$$

is also an integral of the motion, but the presence of the electron rest energy in front of the spin matrix raises practical difficulties in the case of large energies $E \gg m_0c^2$, since the term containing the spin matrix becomes small. In this respect it is more advantageous to introduce the polarization tensor (see [6,7])

$$\Pi_{\mu\nu} = \int \psi^+ F_{\mu\nu 4}\psi\, d^3x,\quad F_{\mu\nu\lambda} = \frac{c}{2}(P_\lambda\alpha_{\mu\nu} + \alpha_{\mu\nu}P_\lambda), \tag{13}$$

where

$$\alpha_{\mu\nu} = \begin{pmatrix} \alpha_{23} & \alpha_{31} & \alpha_{12} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} \end{pmatrix} = \begin{pmatrix} \rho_3\sigma_1 & \rho_3\sigma_2 & \rho_3\sigma_3 \\ i\rho_2\sigma_1 & i\rho_2\sigma_2 & i\rho_2\sigma_3 \end{pmatrix} \tag{14}$$

is the tensor of the magnetic and electric moments. The quantity

$$\Pi_{12} = m_0c^2\sigma_3 + c\rho_2[\boldsymbol{\sigma}\mathbf{P}]_3 \tag{15*}$$

is also an integral of motion. Subjecting the wave function to the requirement that it be an eigenfunction of the operator (15):

$$\Pi_{12}\psi = c\hbar k\zeta\psi \tag{16}$$

and solving (2) and (16) simultaneously, we obtain the coefficients (5) in the form

$$C_1 = aA,\ C_2 = -\zeta bB,\ C_3 = bA,\ C_4 = \zeta aB,$$
$$A = \sqrt{{}^1/_2 (1 + \zeta k_0/k)},\ B = \sqrt{{}^1/_2 (1 - \zeta k_0/k)},$$

$$a = {}^1/_2 (\sqrt{1 + \varepsilon k_3/K} + \varepsilon\zeta\sqrt{1 - \varepsilon k_3/K}),$$
$$b = {}^1/_2 (\sqrt{1 + \varepsilon k_3/K} - \varepsilon\zeta\sqrt{1 - \varepsilon k_3/K}), \tag{17}$$

where $k = \sqrt{K^2 - k_3^2}$, and $\zeta = \pm 1$ characterizes the state of the spin polarization relative to the direction of the magnetic field: $\zeta = 1$ along the field and $\zeta = -1$ against the field.

2. SPONTANEOUS TRANSITIONS. INTENSITY OF POLARIZED RADIATION

We shall henceforth follow the method of [1], in which the radiation intensity in spontaneous transition of the electron from the state n, s, k_3, ζ to the state $n' = n - \nu$, s', k_3', ζ',

$$W_i = \frac{ce_0^2}{2\pi}\int d^3\varkappa\,\delta(K - K' - \varkappa)\, S_i, \tag{18}$$

is connected with the quantities S_i, which characterize the polarization of the radiated photons. In particular, we obtain the σ and π components of the linear polarization, if we put

$$S_\sigma = |\bar{\alpha}_1|^2, \tag{19}$$

$$S_\pi = |\bar{\alpha}_2|^2\cos^2\theta + |\bar{\alpha}_3|^2\sin^2\theta - 2|\bar{\alpha}_2||\bar{\alpha}_3|\sin\theta\cos\theta. \tag{20}$$

To investigate circular polarization of the radiation, we must set the quantity S_i equal to

$$S_l = \frac{1}{2}(S_\sigma + S_\pi - il\,[(\bar{\alpha}_1^+\bar{\alpha}_2 - \bar{\alpha}_2^+\bar{\alpha}_1)\cos\theta$$
$$- (\bar{\alpha}_1^+\bar{\alpha}_3 - \bar{\alpha}_3^+\bar{\alpha}_1)\sin\theta\,]), \tag{21}$$

where $l = 1$ corresponds to the right-hand circular polarization (the photon spin is directed with the motion), and $l = -1$ corresponds to left-hand circular polarization (photon spin directed against the motion).

The elements of the Dirac matrices $\bar{\alpha}_n$ take the form

$$\bar{\alpha}_n = \int \psi'^+ e^{-\varkappa i r}\alpha_n\psi\, d^3x. \tag{22}$$

They can be readily obtained with the aid of the functions (3)—(5). To simplify the calculations we set the momentum along the field k_3 in the initial state equal to zero, and confine ourselves to states

*$[\boldsymbol{\sigma}\mathbf{P}] = \boldsymbol{\sigma} \times \mathbf{P}$.

with positive energy $\epsilon = \epsilon' = 1$; then

$$\bar{\alpha}_1 = iI_{ss'}(x)\{(C_1^{+'}C_4 + C_3^{+'}C_2)I_{n,n'-1}(x) - (C_1C_4^{+'} + C_3C_2^{+'})I_{n-1,n'}(x)\}\,\delta_{k_3',-\varkappa\cos\theta},$$

$$\bar{\alpha}_2 = I_{ss'}(x)\{(C_1^{+'}C_4 + C_3^{+'}C_2)I_{n,n'-1}(x) + (C_1C_4^{+'} + C_3C_2^{+'})I_{n-1,n'}(x)\}\,\delta_{k_3',-\varkappa\cos\theta},$$

$$\bar{\alpha}_3 = I_{ss'}(x)\{(C_1^{+'}C_3 + C_3^{+'}C_1)I_{n-1,n'-1}(x) - (C_2^{+'}C_4 + C_4^{+'}C_2)I_{nn'}(x)\}\,\delta_{k_3',-\varkappa\cos\theta}. \quad (23)$$

In this case the Laguerre functions are determined by formula (6), and the argument is equal to

$$x = \varkappa^2\sin^2\theta/4\gamma. \quad (24)$$

From the energy conservation law it follows that the frequency of the radiated photons is

$$\varkappa = K - K' = \sqrt{k_0^2 + 4\gamma n} - \sqrt{k_0^2 + 4\gamma n' + k_3'^2},$$

$$n' = n - \nu, \quad k_3' = -\varkappa\cos\theta. \quad (25)$$

The number ν of the harmonic is best replaced by a new quantity ν' bearing in mind an eventual transition to a continuous spectrum in which summation over n′ is replaced by integrals

$$\nu = \nu'\left(1 - \frac{\nu'}{4n}\beta^2\sin^2\theta\right); \quad (26)$$

and then it follows from (25) that $\kappa = \sqrt{\gamma/n}\,\beta\nu' = \nu'\beta/R$, where R is the radius of the orbit of the electron. The form of the coefficients C_n and C_n' depends on the choice of the state of polarization of the electron spin.

As is well known (see, for example, [8]), the Laguerre function and its derivative can be approximated by the cylindrical K functions uniformly over the entire region of the spectrum

$$I_{n,n'}(x) = \frac{1}{\pi\sqrt{3}}\left(1 - \frac{x}{x_0}\right)^{1/2} K_{1/3}\left\{\frac{2}{3}\sqrt[4]{nn'}\sqrt{x_0}\left(1 - \frac{x}{x_0}\right)^{3/2}\right\}, \quad (27)$$

$$I'_{n,n'}(x) = \frac{1}{\pi\sqrt{3}}\frac{\sqrt[4]{nn'}}{\sqrt{x_0}}\left(1 - \frac{x}{x_0}\right)K_{2/3}\left\{\frac{2}{3}\sqrt[4]{nn'}\sqrt{x_0}\left(1 - \frac{x}{x_0}\right)^{3/2}\right\} \quad (28)$$

with $x_0 = (\sqrt{n} - \sqrt{n'})^2$. We shall use these approximations to calculate the radiation intensity.

3. INVESTIGATION OF THE RADIATION INTENSITY IN THE CASE OF LONGITUDINAL POLARIZATION OF THE ELECTRON SPIN

The form of the coefficients C_n and $C_{n'}$ in the matrix elements is determined by formula (11). It is convenient to carry out the entire calculation in the form of a series expansion in the quantity $1-\beta^2$, since for the ultrarelativistic motion, which is of particular interest, we have $1-\beta^2 \ll 1$. Eliminating intermediate steps, we present now the expressions for the spectral composition of the radiation, obtained as a result of the integration of the intensity over all the photon emission angles. Then the intensity of the radiation can be reduced to the form

$$W_i = \frac{3\sqrt{3}}{8\pi}\frac{e_0^2 c}{R^2\varepsilon_0^2}\int_0^\infty \frac{y\,dy}{(1+\xi y)^4}F_i(y). \quad (29)$$

In this formula a transition is made to the continuous spectrum: the summation over the number ν of the harmonic is replaced by an integral after introducing the variable

$$y = \frac{2}{3}\frac{\nu'}{1-\nu'/2n}\varepsilon_0^{3/2}, \quad \varepsilon_0 = 1 - \beta^2, \quad (30)$$

and ξ is a characteristic parameter:

$$\xi = \frac{3}{2}\frac{\hbar}{m_0 cR}\left(\frac{E}{m_0c^2}\right)^2. \quad (31)$$

The components $F_i(y)$ are of the following form:

a) linearly-polarized radiation without spin flip $\tilde{\zeta}' = \tilde{\zeta}\,(\rightrightarrows)$ and with spin flip $\tilde{\zeta}' = -\tilde{\zeta}\,(\rightleftarrows)$:

$$F_{\sigma\pi}^{\rightrightarrows} = (1+\xi y)\left[\int_y^\infty K_{5/3}(x)\,dx \pm K_{2/3}(x)\right] + \frac{1}{2}\xi^2y^2\int_y^\infty K_{5/3}(x)\,dx. \quad (32)$$

(the upper sign pertains to the σ component and the lower to the π component),

$$F_\sigma^{\rightleftarrows} = F_\pi^{\rightleftarrows} = \frac{1}{2}\xi^2y^2\int_y^\infty K_{1/3}(x)\,dx; \quad (33)$$

b) circular polarization

$$F_l^{\rightrightarrows} = [1 + (1 + l\tilde{\zeta})(\xi y + \tfrac{1}{2}\xi^2y^2)]\int_y^\infty K_{5/3}(x)\,dx, \quad (34)$$

$$F_l^{\rightleftarrows} = (1 + l\tilde{\zeta})\,\frac{1}{2}\xi^2y^2\int_y^\infty K_{1/3}(x)\,dx. \quad (35)$$

All the formulas are equally applicable for arbitrary values of the parameter ξ.

We now consider the integrated radiation intensity in two limiting cases, respectively

$$\xi \ll 1, \text{ or } E \ll E_{1/2} = m_0c^2(m_0cR/\hbar)^{1/3} \quad (36)$$

and

$$\xi \gg 1, \text{ or } E \gg E_{1/2}. \quad (37)$$

The integration for $\xi \ll 1$ can be carried out by expanding all the expressions in powers of ξ, and then using the well known integral

$$\int_0^\infty x^{q-1}K_p(x)\,dx = 2^{q-2}\Gamma\left(\frac{q-p}{2}\right)\Gamma\left(\frac{q+p}{2}\right) \quad (38)$$

to obtain

$$W_{\sigma}^{\substack{\rightarrow\\ \rightarrow}} = W^{\mathrm{cl}}\left\{\frac{7}{8} - \frac{25\sqrt{3}}{12}\xi + \frac{163}{9}\xi^2 - \ldots\right\}, \tag{39}$$

$$W_{\pi}^{\substack{\rightarrow\\ \rightarrow}} = W^{\mathrm{cl}}\left\{\frac{1}{8} - \frac{5\sqrt{3}}{24}\xi + \frac{19}{9}\xi^2 - \ldots\right\}, \tag{40}$$

$$W_{\sigma}^{\substack{\rightarrow\\ \leftarrow}} = W_{\pi}^{\substack{\rightarrow\\ \leftarrow}} = \frac{5}{9}\xi^2 W^{\mathrm{cl}}; \quad W^{\mathrm{cl}} = \frac{2}{3}\frac{e_0^2 c}{R^2}\left(\frac{E}{m_0 c^2}\right)^4. \tag{41}$$

It follows from these formulas that the intensity of radiation of linearly polarized σ and π components without spin flip does not depend on the initial state of the electron polarization, and the spin flip influences only the terms that are proportional to the square of Planck's constant $\hbar^2$, and the probability of the change of spin projection on the direction of motion is likewise independent of the initial spin orientation.

Therefore the change in the electron spin polarization will occur with equal probability, and will not depend on whether the initial spin direction is with or against the motion. Thus, the spin effects in the radiation of linearly polarized components can come into play only in terms proportional to $\hbar^2$ (see also [9]).

Considering in the same approximation the circular polarization

$$W_{l}^{\substack{\rightarrow\\ \rightarrow}} = \frac{1}{2}W^{\mathrm{cl}}\left\{1 - \frac{55\sqrt{3}}{24}\left(1 - \frac{1}{3}l\tilde{\zeta}\right)\xi^2\right.$$
$$\left. + \frac{182}{9}\left(1 - \frac{7}{13}l\tilde{\zeta}\right)\xi^2 - \cdots\right), \tag{42}$$

$$W_{l}^{\substack{\rightarrow\\ \leftarrow}} = \frac{1}{2}W^{\mathrm{cl}}\left\{\frac{10}{9}\left(1 + l\tilde{\zeta}\right)\xi^2\right\}, \tag{43}$$

we see that a correlation exists between the electron and photon spins, and that the radiation intensity has a correlation term already in the first order in Planck's constant $\hbar$. In the classical approximation the circular polarization vanishes as $\xi \to 0$ (see [1]).

In the other limiting case when $\xi \gg 1$, the radiation intensity differs greatly from the classical value and, as is well known, in place of W^{cl} we obtain a different quantity (see [8]):

$$W^{\mathrm{glob}} = \frac{8}{27}\frac{e_0^2 c}{R^2}\frac{1}{\varepsilon_0^2}\frac{2^{2/3}\Gamma(2/3)}{\xi^{4/3}}. \tag{44}$$

To obtain the main terms of the expansion in ξ^{-1} we can use the asymptotic behavior of the function K_μ at small values of the argument (we recall that at large arguments K_μ decreases exponentially):

$$K_\mu(x) = 2^{\mu-1}\Gamma(\mu)/y^\mu. \tag{45}$$

Then integration yields

$$W_{\sigma}^{\substack{\rightarrow\\ \rightarrow}} = \frac{41}{64}W^{\mathrm{glob}}; \quad W_{\pi}^{\substack{\rightarrow\\ \rightarrow}} = \frac{23}{64}W^{\mathrm{glob}}; \tag{46}$$

$$W_{\sigma}^{\substack{\rightarrow\\ \leftarrow}} = W_{\pi}^{\substack{\rightarrow\\ \leftarrow}} = \frac{81}{256}\frac{2^{1/3}}{\Gamma(2/3)}\frac{\ln\xi}{\xi^{2/3}}W^{\mathrm{glob}}; \tag{47}$$

$$W_{l}^{\substack{\rightarrow\\ \rightarrow}} = \frac{1}{2}W^{\mathrm{glob}}\left[1 + \frac{11}{16}l\tilde{\zeta}\right], \tag{48}$$

$$W_{l}^{\substack{\rightarrow\\ \leftarrow}} = \frac{81}{256}\frac{2^{1/3}}{\Gamma(2/3)}\frac{\ln\xi}{\xi^{2/3}}\left(1 + l\tilde{\zeta}\right)W^{\mathrm{glob}}. \tag{49}$$

Thus, in the region of very large values of the energy $E \gg E_{1/2}$ the general character of the spin effects remains the same in the sense of the influence on the radiation, but the role of the spin in the radiation of photons with circular polarization increases sharply: the correlation term $l\tilde{\zeta}$ does not have the character of the small correction, but the character of the principal expression for the intensity. The radiation of an electron with longitudinal polarized spin turns out to be polarized, and the sign of the circular polarization depends on the initial spin orientation. The deduction that the photons which have circular polarization can be emitted by an electron only if the electron has a longitudinally oriented spin is analogous to the polarization correlations in the bremsstrahlung of an electron in the Coulomb field of the nucleus (see [10]).

We note also that longitudinal polarization of the electron spin has a unique stability: the probability of quantum transitions with changes in spin orientation remains small in both cases both when $E \ll E_{1/2}$ and when $E \gg E_{1/2}$ [see formulas (41)–(43) and (47)–(49)].

4. INVESTIGATION OF THE INTENSITY OF RADIATION IN THE CASE OF POLARIZATION OF THE ELECTRON SPIN ALONG A MAGNETIC FIELD

In this case the coefficients C_n and C_n' in the formulas for the matrix elements of the Dirac matrices are defined by (17). Repeating the method described above, we obtain for the radiation intensity the following expression:

$$W_i = \frac{3\sqrt{3}}{8\pi}\frac{e_0^2 c}{R^2}\frac{1}{\varepsilon_0^2}\int_0^\infty \frac{y\,dy}{(1+\xi y)^4}F_i(y), \tag{50}$$

where $F_i(y)$ characterizes the spectral composition of the radiation only for the linear polarization components—there is no circular polarization, since the corresponding components of the circular polarization vanish upon integration with respect to the angle θ. .

For the components of the linear polarization without spin flip ($\uparrow\uparrow$) and with spin flip ($\uparrow\downarrow$) we obtain

$$F_\sigma^{\uparrow\uparrow} = (1+\tfrac{1}{2}\xi y)^2\Big[\int_y^\infty K_{5/3}(x)\,dx + K_{2/3}(y)\Big]$$

$$+\tfrac{1}{2}\xi^2y^2\int_y^\infty K_{1/3}(x)\,dx - \zeta(2+\xi y)\,\xi y K_{1/3}(y), \tag{51}$$

$$F_\sigma^{\uparrow\downarrow} = \tfrac{1}{4}\xi^2y^2\Big[\int_y^\infty K_{5/3}(x)\,dx - K_{2/3}(y)\Big], \tag{52}$$

$$F_\pi^{\uparrow\uparrow} = (1+\tfrac{1}{2}\xi y)^2\Big[\int_y^\infty K_{5/3}(x)\,dx - K_{2/3}(y)\Big], \tag{53}$$

$$F_\pi^{\uparrow\downarrow} = \tfrac{1}{4}\xi^2y^2\Big[\int_y^\infty K_{5/3}(x)\,dx + K_{2/3}(y) + 2\int_y^\infty K_{1/3}(x)\,dx$$

$$+ 4\zeta K_{1/3}(y)\Big]. \tag{54}$$

Let us consider the integral radiation in analogy with the preceding case. Then for energies $E \ll E_{1/2}$ ($\xi \ll 1$) we obtain

$$W_\sigma^{\uparrow\uparrow} = W^{\mathrm{cl}}\Big\{\frac{7}{8} - \Big(\frac{25\sqrt{3}}{12}+\zeta\Big)\xi$$

$$+\Big(\frac{335}{18}+\frac{245\sqrt{3}}{48}\zeta\Big)\xi^2 - \ldots\Big\}; \tag{55}$$

$$W_\sigma^{\uparrow\downarrow} = W^{\mathrm{cl}}\frac{1}{18}\xi^2; \tag{56}$$

$$W_\pi^{\uparrow\uparrow} = W^{\mathrm{cl}}\Big\{\frac{1}{8} - \frac{5\sqrt{3}}{24}\xi + \frac{52}{18}\xi^2 - \ldots\Big\}; \tag{57}$$

$$W_\pi^{\uparrow\downarrow} = W^{\mathrm{cl}}\frac{23}{18}\Big\{1+\frac{105\sqrt{3}}{184}\zeta\Big\}\xi^2. \tag{58}$$

It follows from these formulas that in the case where the electron spin is polarized along the field W_σ the radiation component depends on the initial spin orientation, and this dependence is contained in the terms proportional to the first power of Planck's constant $\hbar$. Spin flip is manifest, as before, in the terms proportional to $\hbar^2$, but the spin-flip probability now depends on the initial spin orientation. The change in the polarization of electron spin during radiation occurs in such a way that the spin strives to orient itself against the field. Because of this, the spin of an electron beam which is not polarized at the initial instant should acquire with time a preferred orientation against the field (see [11]). Estimates of the "lifetime" of an electron with spin polarization along the field, which can be readily obtained on the basis of (58), show that for electrons with energy 1 BeV and a magnetic field ~ 10^4 Oe it amounts to a quantity of the order of 1 hour.

In conclusion let us find the integral value of the radiation intensity in the other limiting case $\xi \gg 1$. We get

$$W_\sigma^{\uparrow\uparrow} = \frac{75}{128}W^{\mathrm{glob}},\quad W_\pi^{\uparrow\uparrow} = \frac{25}{128}W^{\mathrm{glob}}; \tag{59}$$

$$W_\sigma^{\uparrow\downarrow} = \frac{7}{128}W^{\mathrm{glob}},\quad W_\pi^{\uparrow\downarrow} = \Big\{\frac{21}{128}+\frac{5}{8}\zeta\frac{\Gamma(1/3)}{\Gamma(2/3)\,2^{1/3}\xi^{1/3}}\Big\}W^{\mathrm{glob}}. \tag{60}$$

Thus, at large values of the energy $E \gg E_{1/2}$, transitions with reversal of the spin orientation make a contribution to the main term of the intensity. In this lies the essential difference from longitudinal polarization, which is more stable with respect to changes in the spin orientation. We note also that in the case when $E \gg E_{1/2}$ the directivity in the change of the spin orientation decreases and, as can be seen from (60), when $\xi \gg 1$ the radiation intensity depends little on the initial spin polarization.

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