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ANGULAR CORRELATIONS NEAR THE THRESHOLD OF FORMATION OF AN UNSTABLE PARTICLE

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The X(a, b)Y* reaction is considered. The Y* particle is unstable and decays during a time $\tau \sim \Gamma^{-1}$ into two particles. The angular dependence of the reaction cross section near the threshold for Y* production is obtained. The spin and parity of Y* can be determined by comparing the results with experimental data.

1. INTRODUCTION

BAZ'[1] investigated the energy dependence of the cross section of the reaction X(a, b)Y near the threshold of production of an unstable particle Y, which decays into two particles c and d within a time $\tau \sim \Gamma^{-1}$ (Γ —width of the excited level Y). It is shown that the width Γ can be determined from the shape of the energy dependence of the reaction cross section near threshold.

It will be shown in the present article that additional information concerning the unstable particle can be obtained from an analysis of the angular dependence of the same reaction

$$a + X \nearrow^{b+Y} \searrow \longrightarrow b + c + d \tag{1}$$

near the threshold of production of Y. It turns out that in the simplest cases measurement of the angular correlations makes it possible to determine the spin and the parity of the unstable particle. In the general case it is possible to obtain the intervals of the possible values of these characteristics for Y. The calculations are carried out on the basis of the theory of Migdal[2] and Watson[3], which describes well the cross section of the three-particle reaction only in that part of the energy spectrum of the particles b, c, and d where an appreciable role is played by the interaction in the final state of any pair of these particles.

However, in order to carry out our calculations we must know the cross sections over the entire energy interval. We therefore assume that the cross section of the reaction (1) has in the entire energy region the same energy dependence as in the region of interaction in the final state. It turns out that such an extension over the entire energy region is valid provided two conditions are satisfied: (a) the unstable particle has only one resonant excited state from which the decay $Y \rightarrow c + d$ takes place; (b) the width of this excited state is $\Gamma \rightarrow 0$. In the case of a finite width Γ, approximate formulas are obtained for the angular dependence of the cross section of the reaction (1). The results are qualitatively the same as when $\Gamma \rightarrow 0$.

2. ANGULAR DEPENDENCE OF THE CROSS SECTION

An examination of reaction (1), in the intermediate state of which an unstable particle is produced, suggests that in the final state, a Breit-Wigner resonant interaction should appear at a definite relative energy of the particles c and d, if the energy of the collision of the particles a + X lies above the production threshold of Y. In the region of resonant interaction it is possible to use directly the formulas obtained by Migdal[2] and Watson[3] for the cross section. On the other hand, a collision energy a + X much larger than the threshold energy should not be considered, for this leads to a broadening of the non-resonant region and decreases the calculation accuracy. For these two reasons we shall consider in this paper a + X collision energies exceeding the threshold by an amount on the order of the Breit-Wigner resonance width.

We carry out the calculations in the center-of-mass system. The energy of relative motion of the particles c and d will be denoted by ϵ, and the energy of the particle b relative to the center of gravity of the particle c + d will be denoted by ϵ_b. The direction of motion of particle c (or d) is specified by means of an angle θ between the momentum of the particle b and the momentum of the relative motion of particles c and d.

In the region of energies ϵ where the influence of the interaction in the final state is significant, the cross section of the reaction (1), accurate to factors which depend little on the energies ϵ and ϵ_b, is of the form[2,3]

$$d\sigma(\varepsilon, \theta) \sim |\psi_\varepsilon \varphi_{\varepsilon_b}|^2_{R=R_0} (\varepsilon\varepsilon_b)^{1/2} d\varepsilon \, d\cos\theta. \quad (2)$$

The wave function ψ_ϵ, which describes the relative motion of particles c and d, is well known near resonance[4]

$$\psi_\varepsilon \sim \varepsilon^{-1/2} (\varepsilon - \varepsilon_0 + i\Gamma)^{-1} P_{lm}(\theta), \quad (3)$$

where ϵ_0 and Γ are the energy and level width of the unstable particle, and P_{lm} is the associated Legendre polynomial. The function φ_{ϵ_b} which describes the motion of the particle b, can be regarded as a plane wave, since we neglect the interaction in the final state between the particle b and particles c and d near the resonance $\epsilon \approx \epsilon_0$. The index $R = R_0$ shows that the values of the wave functions ψ_ϵ and φ_{ϵ_b} must be taken on the surface of the interaction volume in which the reaction takes place.

A comparison of the cross section (2) with the experimental data is best carried out in the case when the cross section is expressed in terms of the angle ϑ between the momenta of the particles c and d. From the kinematic equations we obtain the connection between the angles θ and ϑ:

$$F(\vartheta, \xi) = \cos\theta = (1 + \eta_2^2 \operatorname{tg}^2\vartheta)^{-1} \times \{-\eta_2 \xi \operatorname{tg}^2\vartheta + [1 + \operatorname{tg}^2\vartheta\, (\eta_2^2 - \xi^2)]^{1/2}\}. \quad (4)^*$$

We have introduced here the following notation:

$$\xi = m_b\chi/M - \eta_1/\chi, \quad \chi = [\eta_1(E-\varepsilon)/\varepsilon]^{1/2},$$
$$\eta_1 = \mu M/m_b m, \quad \eta_2 = (m_c - m_d)/m,$$
$$m = m_c + m_d, \quad M = m + m_b, \quad \mu = m_c m_d/m, \quad (5)$$

where E is the kinetic energy of the particles b + c + d. In (5) we make use of the energy conservation law $E = \epsilon + \epsilon_b$.

The Jacobian of the transformation from θ to ϑ is

$$J(\vartheta, \xi) = \frac{d\cos\theta}{d\cos\vartheta} = f(\vartheta)\left\{2\eta_2\xi + \frac{\xi^2(1-\eta_2^2\operatorname{tg}^2\vartheta) + \eta_2^4\operatorname{tg}^2\vartheta}{[1+\operatorname{tg}^2\vartheta\,(\eta_2^2-\xi^2)]^{1/2}}\right\}, \quad (6)$$

$$f(\vartheta) = (\cos\vartheta)^{-3}(1 + \eta_2^2\operatorname{tg}^2\vartheta)^{-2}. \quad (7)$$

In order to find the differential cross section $d\sigma(\vartheta)$, which describes the angular correlations between particles c and d, we must integrate the cross section, expressed in the new variables using (3) and (7),

$$\frac{d\sigma(\varepsilon, \vartheta)}{d\cos\vartheta} \sim \frac{|P_{lm}[F(\vartheta, \xi)]|^2}{[(\varepsilon-\varepsilon_0)^2 + \Gamma^2]} \left(\frac{E-\varepsilon}{\varepsilon}\right)^{1/2} J(\vartheta, \xi)\, d\varepsilon \quad (8)$$

with respect to ϵ from zero to E. In the integration it is assumed that the energy dependence of the cross section (8) holds true for all values of ϵ, although this dependence is actually not known away from resonance. Such an assumption is fully justified if $\Gamma \to 0$.

Indeed

$$\frac{1}{\pi}\lim_{\Gamma\to 0}\frac{\Gamma}{(\varepsilon-\varepsilon_0)^2+\Gamma^2} = \delta(\varepsilon - \varepsilon_0), \quad (9)$$

and it is immaterial in the integration of (8) which form is assumed by the energy dependence away from resonance. In this limiting case we obtain immediately the angular dependence of the cross section of reaction (1)

$$\frac{d\sigma_{lm}(\vartheta)}{d\cos\vartheta} \sim \left(\frac{E-\varepsilon_0}{\varepsilon_0}\right)^{1/2} |P_{lm}[F(\vartheta, \xi_0)]|^2 J(\vartheta, \xi_0), \quad (10)$$

where

$$\xi_0 = m_b\chi_0/M - \eta_1/\chi_0. \quad \chi_0 = [\eta_1(E-\varepsilon_0)/\varepsilon_0]^{1/2}.$$

If we now start increasing Γ, then the main contribution to the value of the integral will be made as before by the resonant region, although the contribution of the non-resonant region should, generally speaking, increase together with increasing Γ. The integration of the cross section (8) with respect to ϵ in the case of a finite width Γ can be carried out only if $\tan^2\vartheta \ll 1$ (see the appendix).

In view of the unwieldiness of the formulas, we give the angular dependences of the cross section $d\sigma/d\cos\vartheta$ only for the first three values of the orbital angular momentum l and its projections m on the direction of motion of the particle b accurate to within terms of order $\tan^2\vartheta$:

$$\frac{d\sigma_{00}}{d\cos\vartheta} \sim \frac{1}{2}\frac{1}{\cos^3\vartheta}\operatorname{Re}[\alpha(\xi_1, \vartheta) - \alpha(\xi_2, \vartheta)],$$

$$\frac{d\sigma_{10}}{d\cos\vartheta} \sim \frac{3}{2}\frac{1}{\cos^3\vartheta}\operatorname{Re}[\alpha(\xi_1, \vartheta)\beta(\xi_1, \vartheta) - \alpha(\xi_2, \vartheta)\beta(\xi_2, \vartheta)],$$

$$\frac{d\sigma_{11}}{d\cos\vartheta} = \frac{d\sigma_{1,-1}}{d\cos\vartheta} \sim \frac{3}{4}\frac{\operatorname{tg}^2\vartheta}{\cos^3\vartheta}\operatorname{Re}[\alpha(\xi_1, \vartheta)(\eta_2^2 + \xi_1^2) - \alpha(\xi_2, \vartheta)(\eta_2^2 + \xi_2^2)],$$

$$\frac{d\sigma_{20}}{d\cos\vartheta} \sim \frac{5}{2}(\cos\vartheta)^{-3}\operatorname{Re}\left\{\alpha(\xi_1, \vartheta)\left[\frac{3\beta(\xi_1, \vartheta) - 1}{2}\right] - \alpha(\xi_2, \vartheta)\left[\frac{3\beta(\xi_2, \vartheta) - 1}{2}\right]\right\},$$

$$\frac{d\sigma_{21}}{d\cos\vartheta} = \frac{d\sigma_{2,-1}}{d\cos\vartheta} \sim \frac{15}{4}\frac{\operatorname{tg}^2\vartheta}{\cos^3\vartheta}\operatorname{Re}[\alpha(\xi_1, \vartheta)\beta(\xi_1, \vartheta)(\eta_2^2 + \xi_1^2) - \alpha(\xi_2, \vartheta)\beta(\xi_2, \vartheta)(\eta_2^2 + \xi_2^2)],$$

$$\frac{d\sigma_{22}}{d\cos\vartheta} = \frac{d\sigma_{2,-2}}{d\cos\vartheta} \sim \frac{15}{16}\frac{\operatorname{tg}^4\vartheta}{\cos^3\vartheta}\operatorname{Re}[\alpha(\xi_1, \vartheta)(\eta_2^2 + \xi_1^2)^2 - \alpha(\xi_2, \vartheta)(\eta_2^2 + \xi_2^2)^2]. \quad (11)$$

*tg = tan.

We have introduced here the notation

$$\alpha(\xi_j, \vartheta) = \xi_j^2 [1 + \mathrm{tg}^2 \vartheta (\eta_2^2 - \xi_j^2)]^{-1/2}, \quad j = 1, 2; \qquad (12)$$

$$\beta(\xi_j, \vartheta) = 1 + \mathrm{tg}^2 \vartheta (\eta_2^2 - \xi_j^2) + \xi_j^2 \eta_2^2 \mathrm{tg}^4 \vartheta; \qquad (13)$$

$$\xi_j = (m_b/M) \chi_j - \eta_1/\chi_j,$$

$$\chi_j = [\eta_1 (E - \varepsilon_j)/\varepsilon_j]^{1/2}, \qquad \varepsilon_j = \varepsilon_0 \pm i\Gamma. \qquad (14)$$

In formula (13) we have retained a term of order $\tan^4 \vartheta$, since the sum of the first two terms can be smaller than the third.

In comparing the corresponding partial cross sections (10) and (11), it is easy to verify that the angular dependences of the cross sections in the case when $\Gamma \to 0$ and in the case of a finite width Γ are identical. The partial cross sections have minimum values for $\tan^2 \vartheta = 0$ and decrease upon approaching the maximum value of $\tan^2 \vartheta$, determined by the kinematic condition

$$\mathrm{Re}\,[1 + \mathrm{tg}^2 \vartheta (\eta_2^2 - \xi^2)] \geqslant 0.$$

3. SPIN AND PARITY OF THE UNSTABLE PARTICLE

Formulas (10) and (11) describe the partial angular cross sections accurate to within a certain constant factor. The statistical weight of the partial cross sections for a given angular momentum l can be obtained from simple physical considerations. Indeed, if the unstable particle lives a time $\tau > 10^{-21}$ sec, i.e., if $\Gamma \leq 0.1$ MeV, then the spin S_Y of the unstable particle is a sufficiently good quantum number, and the equation for the momenta is satisfied with good degree of accuracy

$$S_Y = l + S_c + S_d, \qquad (15)$$

where S_c and S_d are the spins of the particles c and d. All the states with different projections m can be regarded as equally probable.

From (15) we see than an unambiguous determination of the spin is possible in two cases: (1) $l = 0$ and (2) $S_c + S_d = 0$. In all other cases we can indicate only an interval of values S_Y. The parity of the particle Y is determined unambiguously in the case when two out of the three particles (Y, c, d) are spinless.

Unfortunately, the lack of experimental data does not permit us to compare (11) with the experimentally measured angular correlations.

If the particle Y disintegrates within nuclear time, then Eq. (15) is incorrect and we must write in lieu of it the law for the conservation of the total angular momentum of the system. In this case, in each specific reaction (1) the relative orbital angular momentum l can assume all possible values which are allowed by the total angular momentum conservation law. The very formulation of the problem of determining the spin of the unstable particle then becomes meaningless.

APPENDIX

We shall carry out the integration (8) with account of the assumption that a small contribution to the value of the integral is made by the resonant region. Then near the threshold of the reaction (1) we can assume $\epsilon \approx \epsilon_0$ and carry out the integration in the plane of the complex variable $z = E - \epsilon$. In the integration it is sufficient to consider the case of zero orbital angular momentum, since an account of the momentum l adds nothing that is new in principle, since all the singularities of the integrand remain the same as in the case when $l = 0$. We have

$$\frac{d\sigma_{00}}{d\cos\vartheta} \sim f(\vartheta) \int \frac{dz}{(z - z_1^0)(z - z_2^0)} \left[2\eta_2 \left(\frac{m_b}{M} \frac{z}{\varepsilon_0} - 1 \right) + \frac{(1 - \eta_2^2 \mathrm{tg}^2 \vartheta)\, \eta_1 (m_b z / M\varepsilon_0 - 1)^2 + \eta_2^4 z \varepsilon_0^{-1} \mathrm{tg}^2 \vartheta}{\sqrt{(z - z_1)(z - z_2)}\, \eta_1 m_b \,|\, \mathrm{tg}\, \vartheta \,|\, / M\varepsilon_0} \right]. \qquad (A.1)$$

The integrand function has two poles

$$z_{1,2}^0 = E - \varepsilon_0 \pm i\Gamma \qquad (A.2)$$

and two branch points on the real axis

$$z_{1,2} = A/2 \pm \sqrt{A^2/4 - B}, \qquad (A.3)$$

where

$$A = \frac{\varepsilon_0}{\eta_1} \left(\frac{M}{m_b} \right)^2 \left[\frac{1}{\mathrm{tg}^2 \vartheta} + \left(\eta_2^2 + 2 \frac{m_b}{M} \eta_1 \right) \right], \quad B = \left(\frac{M}{m_b} \varepsilon_0 \right)^2.$$

The position of the branch points z_1 and z_2 depends essentially on the angle ϑ; when $\tan^2 \vartheta \to 0$ we have $z_1 \to 0$ and $z_2 \to \infty$. With increasing ϑ, the points z_1 and z_2 come closer together, and the minimum distance between them, which is equal to $2\sqrt{A^2/4 - B}$ is obtained as $\tan^2 \vartheta \to \infty$.

We make a cut in the z plane along the real axis from z_1 to z_2. The integration contour is drawn in the following manner: we circle around the cut from z_1 to z_2 following a closed contour along the upper and lower edges. In addition, we draw a circle with large radius R centered at the origin. The integral of (A.1) along the circle of radius R does not vanish. This result is obtained after making the approximation $\epsilon \approx \epsilon_0$. In fact, the integral does vanish. In addition, the integrals on the small circles with radius $r \to 0$ around the branch points z_1 and z_2 also vanish. The values of the integrand function on the upper and lower edges of the cut differ only in sign.

It follows therefore that

$$I(z_1, z_2) = \pi i \sum_{j=1}^{2} \operatorname{Res}(z_j^0), \quad (A.4)$$

where $I(z_1, z_2)$ is the value of the integral (A.1) from z_1 to z_2. The first term in the square brackets of (A.1) makes no contribution to the value of the integral (A.4), since it has the same sign on the upper and lower edges of the cut and integration along our contour yields zero. As will be shown later, we are interested in values $\tan^2 \vartheta \ll 1$. In this case the contribution of the first term in the square brackets of (A.1) is small compared with the second, and can be neglected.

If the resonance region lies in the interval $[z_1, z_2]$, then the integral (A.4) will differ slightly from the sought integral $I(0, E)$:

$$I(0, E) \approx I(z_1, z_2) = \pi i \sum_{j=1}^{2} \operatorname{Re}(z_j^0). \quad (A.5)$$

In other words, the correctness of (A.5) follows from the condition

$$\Gamma + z_1 < \operatorname{Re}(z_j^0), \quad (A.6)$$

From this we get the region of values of $\tan^2 \vartheta$ for which (A.5) is satisfied

$$\operatorname{tg}^2\vartheta \leqslant (E - \varepsilon_0 - \Gamma)/\eta_1\varepsilon_0. \quad (A.7)$$

Account is taken here of a fact pointed out already at the beginning of the paper, namely that $E - \epsilon_0 \ll \epsilon_0$. Thus, all the calculations of the angular dependence of the cross section for a finite width Γ are valid for values $\tan^2 \vartheta \ll 1$.

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