CONDITIONS FOR HEATING UP OF A PLASMA BY THE RADIATION FROM AN OPTICAL GENERATOR

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Various processes accompanying rapid pulsed heating of small volumes of a plasma by radiation from a laser are considered. The power and duration of the radiation pulse required for heating a hydrogen plasma to temperatures of the order of 10^6-10^7 degrees are estimated.

THE rapid progress in the technology of generating optical radiation and particularly progress in the development of powerful generators raises the hope that unusually large energy flux densities will be obtained by focusing coherent light in small volumes. In the present article we discuss the possibility of using radiation from a laser to heat small volumes of dense plasma to high temperatures $^{1)}$. We do not discuss the specific systems in which the plasma heating would be realized most effectively, and consider only various physical processes occurring in a high-temperature plasma, separating those which play the most important role in the determination of the parameters of the radiation pulse necessary to attain definite temperatures.

The main process limiting the growth of plasma temperature is the large thermal loss connected with the radiation and thermal conductivity of the plasma^[2], so that the attainable temperatures will be determined principally by the limiting radiation power of the laser.

The estimates that follow show that at present there are still no lasers with parameters sufficient to heat deuterium plasma to temperatures at which neutrons resulting from thermonuclear reactions could be observed, but it is of undoubted interest even now to investigate the kinetics of the heating of a plasma by lasers of high energy and power^[3-5]. In considering the heating problem we shall have in mind a deuterium plasma with an ion density on the order of 10×3^{21} cm⁻³ and a final temperature close to 10^7 deg, when the thermonuclear reaction rates reach the appreciable value ~ 10^{-22} cm³/sec.

1. OPTICAL RADIATION ABSORPTION COEF-FICIENT

The plasma is heated when the electrons absorb radiation in the ion field and subsequently transfer the energy to the ions. The radiation absorption coefficient k and the limiting plasma density n can be determined from the dielectric constant $\epsilon(\omega)$ of the plasma^[6,7]:

$$\varepsilon(\omega) = 1 - (\omega_0^2/\omega^2) (1 - i\nu/\omega), \qquad (1)$$

where the Langmuir frequency is $\omega_0 = (4\pi ne^2/m)^{1/2}$ and the collision frequency is $\nu = 4\sqrt{2\pi} e^4 nL/3(kT)^{3/2}m^{1/2}$; $L = ln(\lambda/\rho)$ is the Coulomb logarithm and $\lambda = (kT/4\pi e^{2n})^{1/2}$ is the Debye length; ρ is the minimal impact parameter: $\rho \approx e^2/kT$ for $e^2/\hbar v \gg 1$ (v — electron velocity) or $\rho \approx \hbar/mv$ for $e^2/\hbar v \ll 1$.

The limiting value of the density is determined by the condition $\omega_0 = \omega$ and amounts to $n \approx 3 \times 10^{21}$ cm⁻³ for ruby emission ($\omega = 3 \times 10^{15} \text{ sec}^{-1}$). In the case when Re $\epsilon \gg \text{Im } \epsilon$, the absorption coefficient $k = 2(\omega/c) \text{Im } \sqrt{\epsilon}$ is equal to

$$k = \frac{16\pi \sqrt{2\pi}e^6 n^2 L}{3 (mkT)^{3/2} (\text{Re } \epsilon)^{1/2} \omega^2 c} .$$
 (2)

It is seen from (2) that if $n \leq 3 \times 10^{21}$, so that Re $\epsilon \approx 1$, and if T $\approx 10^7$ deg, the order of magnitude of the absorption coefficient can reach 10^2 cm⁻¹, from which it follows that the radiation absorption is quite effective up to high temperatures, $\sim 10^7$ deg.

2. ENERGY LOSS IN PLASMA

In the initial stage of the process, when the temperature is sufficiently high, we can use for the plasma the equation of state of an ideal gas. At $T = 10^7 \text{ deg}$ and $n = 3 \times 10^{21} \text{ cm}^{-3}$ we have for the energy density u, assuming practically total

¹⁾This was noted in papers delivered by N. G. Basov at the Presidium of the Academy of Sciences USSR (March 1961) and by P. Franken at the Congress of the American Optical Society (see [¹]).

ionization, $u = 1.4 \times 10^{13} \text{ erg/cm}^3$, from which it follows that the radiation energy is of the order of 10 J and is sufficient in principle to heat a volume $V \approx 10^5 \text{ cm}^3$ of a rather dense plasma to 10^7 deg. The relative correction to the equation of state, due to the Coulomb interaction, is of the order of $e^2/\lambda kT$, and under these conditions it amounts to 10^{-3} .

The heating of the plasma, as indicated above, occurs as a result of absorption of radiation by the electrons. On the other hand, the ion gas is heated by electron-ion collisions. The rate of electron and ion temperature equalization has an order of magnitude $\nu m/M$, where M is the ion mass. The corresponding relaxation time^[6] is equal to

$$\tau = 3M \, (kT)^{3/2} / \, 16 \sqrt{2\pi m} \, e^4 n L. \tag{3}$$

If the heating time t exceeds τ , then the plasma heating can be regarded as isothermal; in the opposite case the ions are heated inefficiently. When $T = 10^7 \text{ deg}$ and $n = 3 \times 10^{21} \text{ cm}^{-1}$, the relaxation time is $\tau = 1.5 \times 10^{-10} \text{ sec}$, i.e., quite short.

The degree of plasma ionization can be determined from the known Saha formula^[8], which, however, is valid under conditions of thermodynamic equilibrium. Inasmuch as under our conditions the plasma is practically completely transparent to the hard ("thermal") radiation (see below), there is no equilibrium with the radiation. However, if the process of ionization by electron impact is more efficient than the process of photoionization under equilibrium conditions, then Saha's formula can be used, but the temperature in this formula is that of the electrons. The photoionization probability is $w_{ph} \approx Q_r n_0 kT$, where n_0 is the thermodynamic-equilibrium density of the neutral atoms and Q_r is the recombination radiation power, given by (7). The probability of ionization by electron impact is $w_e = n_e \overline{\sigma_i v}$, where σ_i is the ionization cross section and ne the electron density. Under the conditions of interest to us $(T = 10^7 \text{ deg}, n = 3 \times 10^{21} \text{ cm}^{-3}, \sigma_i \approx 10^{-18} \text{ cm}^2)$ we have $w_e \gg w_{ph}$ so that Saha's formula is applicable.

The plasma energy loss is due to the electronic and radiative thermal conductivities. The coefficient of electronic thermal conductivity $K_{\rm e}$ is, in accordance with $^{[2,6]}$

$$K_{e} = 40\sqrt{2} k^{\frac{3}{2}} T^{\frac{5}{2}} \delta / \pi^{\frac{3}{2}} m^{\frac{1}{2}} e^{4} L = 1.24 \cdot 10^{-6} T^{\frac{5}{2}} \quad (L=15), (4)$$

where the coefficient δ corrects for the electronelectron collisions^[6] and for the secondary electric field. The coefficient of radiative thermal conductivity K_{rad} is^[8]

$$K_{\rm rad} = \frac{16}{3} \, \mathrm{o} l T^3, \tag{5}$$

where l is the photon range as averaged by Rosseland^[8], and σ is the Stefan-Boltzmann constant. At densities on the order of 3×10^{21} cm⁻³ in a hydrogen plasma, l becomes comparable with the dimensions of the system (~ 10^{-2} cm) at temperatures lower than approximately 10^5 deg, and reaches ~ 10^2 at T = 10^7 deg.

Thus, at $T = 10^7$ deg the plasma is practically transparent and the radiation energy loss is determined by the bremsstrahlung and recombination radiation, ^[6,9,10], the powers of which per unit volume are

$$Q_{\rm b} = 1.57 \cdot 10^{-27} \ n^2 T^{1/2},\tag{6}$$

$$Q_{\rm r} = 1.08 \cdot 10^{-21} n^2 T^{-1/2}.$$
 (7)

The recombination radiation becomes appreciable at $T \leq 10^5$ deg; at higher temperatures, formula (7) cannot be used, but the contribution of recombination radiation is small in this region.

On the basis of (6), the power loss of a plasma with $n = 3 \times 10^{21} \mbox{ cm}^{-3}$ and $V = 10^{-5} \mbox{ cm}^{3}$ is Q_{rad} = $Q_bV = 1.4 \times 10^{11} \mbox{ T}^{1/2}$ erg/sec. The maximum energy loss due to thermal conductivity is $^{[2]}$ $Q_e \approx 8\pi r K_e T/7 \approx 3.5 \times 10^{-8} \mbox{ T}^{7/2}$ erg/sec. At $T = 1.6 \times 10^6$ deg we have $Q_e = Q_{rad}$. At $T = 10^7$ the losses due to thermal conductivity prevail and amount to $Q_e \approx 1.3 \times 10^{17}$ erg/sec ($Q_{rad} \approx 4.5 \times 10^{14} \mbox{ erg/sec}$). The question of possible thermal insulation of the plasma thus becomes important here. The dependence of the maximum attainable temperature, determined by the thermal conductivity, on the laser power Q is of the form $T \sim Q^{2/7}$.

3. GAS DYNAMIC EXPANSION OF A BOUNDED PLASMA

Let us estimate the role of the gas-dynamic expansion during the heating of a plasma, on the basis of the approximate model of free plasma with spherical volume $V = 4\pi r^3/3$, described by the average values of the density and temperature. The energy conservation equation (see ^[11]), integrated over the entire volume of the plasma, yields an equation that describes the time variation of the kinetic and internal energy E of the plasma:

$$\frac{d}{dt}\left(G\frac{w^2}{2}+E\right)=Q,\qquad(8)$$

where w is the average rate of gas-dynamic expansion, G the mass, and Q the power of the radiated heat. The equation of motion can be attained from (8) by putting Q = 0 and dE = -pdV, where p is the average pressure. Thus, the system of equations describing the expansion and heating of the plasma assumes the form

$$G\frac{dw}{dt} - 4\pi r^2 p = 0, \qquad \frac{d}{dt} \left(G\frac{w^2}{2} + E \right) = Q.$$
(9)

For the case of an ideal gas, when E = 3NkT/2(N is the total number of particles in the system), the solution of (9) is of the form

$$r^{2} = r_{0}^{2} + \frac{4}{G} \int_{0}^{t} dt' \int_{0}^{t'} dt'' \int_{0}^{t''} dt''' Q(t'''), \qquad (10)$$

and in the simple case when

$$Q = \begin{cases} \text{const,} & t > 0 \\ 0, & t < 0 \end{cases}$$

we have

$$r^{2} = r_{0}^{2} + \frac{2}{3} \frac{Qt^{3}}{G}, \quad T = \frac{Qt}{\frac{3}{2}Nk} \frac{Qt^{3}/6G + r_{0}^{2}}{2Qt^{3}/3G + r_{0}^{2}}.$$
 (11)

We see from (6) that as $t \rightarrow 0$ we have T $\sim 2Qt/3Nk$, which corresponds to a weak influence of the gas-dynamic expansion at the initial instant of time. It is important however, that when t $\gg (3r_0^2G/Q)^{1/3}$ the temperature also varies linearly with time: T ~ Qt/6Nk. This circumstance corresponds to the fact that asymptotically the ratio of the thermal energy of the plasma to the energy of gas-dynamic motion is constant. In this case, asymptotically, one quarter of the laser radiation energy goes to heating of the plasma and the rest goes to dynamic expansion. An asymptotic ratio exists apparently also in the real case, but the specific value of this ratio may be somewhat different.

Thus, for the same temperatures to be attained, gas-dynamic considerations lead to a need for increasing the laser power. The characteristic time parameter within which allowance for expansion becomes important is $t_0 = (3r_0^2G/Q)^{1/3}$, and amounts to $t_0 = 2 \times 10^{-9}$ for a laser power $Q = 10^9$ W, $V = 10^{-5}$ cm³, and $G = 10^{-7}$ g. This estimate shows that the gas-dynamic expansion plays an important role and calls for small time durations, on the order of the length of the radiation pulse.

From the foregoing analysis as a whole it follows that the most important role in the plasma heating process is played by the energy loss due to thermal conductivity and to gas-dynamic expansion of the hot plasma. At a laser power on the order of 10^9 W and a pulse duration of about 10^{-8} sec, it is apparently possible to heat a hydrogen plasma of density 3×10^{21} cm⁻³ to a temperature somewhat lower than 10^7 deg, although a specific method of realizing such an experiment still needs further analysis. The possibility of attaining such temperatures depends in our opinion essentially on the progress in the techniques of generating optical radiation.

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¹Electr. design. 10, 24 (1962).

² L. A. Artsimovich, Upravlyaemye termoyadernye reaktsii (Controlled Thermonuclear Reactions), Fizmatgiz, 1961.

³Basov, Zuev, and Kryukov, JETP **43**, 353 (1962), Soviet Phys. JETP **16**, 254 (1963).

 4 F. J. McClung and R. W. Hellwarth, Proc. IEEE, **51**, 46 (1963).

⁵ E. Snitzer, Intern. Conf. on Quantum Electronics, Paris, (1963).

⁶ L. Spitzer, Physics of Fully Ionized Gases (Russ. Transl.) IIL, 1962.

⁷ V. P. Silin and A. A. Rukhadze, Elektromagnitnye svoistva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Atomizdat, 1961.

⁸A. Usold, Physik der Sternatmosphären, Berlin, Springer, 1938.

⁹W. Heitler, The Quantum Theory of Radiation, Oxford, 1954.

¹⁰ V. L. Ginzburg, Trudy, Phys. Inst. Acad. Sci., **18**, 55 (1962).

¹¹ L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media), Gostekhizdat, 1959.

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