HALL COEFFICIENT AND ELECTRICAL RESISTANCE OF FERROMAGNETS

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It is shown that the value of the ferromagnetic Hall coefficient is related by a simple expression to the magnitude of the "magnetic resistance." The change of the resistance of several alloys in a magnetic field (in the paramagnetic region) is also shown to be proportional to the magnetic resistance.

THE problem of the correlation between the Hall coefficient of ferromagnets and their electrical resistance has been studied intensively in recent years. The start was made by Karplus and Luttinger ^[1] who established the following slightly surprising relationship between the ferromagnetic Hall coefficient R_F and the electrical resistance of a substance ρ :

$$R_F = A \rho^2. \tag{1}$$

We recall that in ferromagnets the Hall emf is the sum of the 'classical' part, proportional to the magnetic induction B, and the considerably greater ferromagnetic part, proportional to the magnetization J, i.e., the Hall emf is given by the equation

$$E = (R_F J + R_0 H)i/d, \qquad (2)$$

where R_F is the ferromagnetic Hall coefficient, R_0 is the so-called ordinary Hall coefficient, J is the volume magnetization, H is the magnetic field intensity, i is the current in the sample, and d is the thickness of the sample; for most ferromagnets $R_F \gg R_0$.

The meaning of Eq. (1) is that the temperature dependence of the ferromagnetic Hall coefficient should be the same as the temperature dependence of the square of the electrical resistance.

Several experimental investigations have been carried out to check Eq. (1) but have failed to confirm it. [2-5]

It seems to us that, from the physics point of view, we are more justified in seeking a relationship between R_F and that part of the electrical resistance of the sample (which we shall call the magnetic resistance ρ_M) which is due to the spontaneous magnetization, and not between R_F and the total resistance ρ . The magnetic resistance ρ_M is due to the scattering of electrons by elementary magnetic moments (spins), which are responsible for the spontaneous magnetization. The measured resistance of a ferromagnet can be represented by the sum

$$\rho = \rho_0 + \rho_T + \rho_M, \tag{3}$$

where ρ_0 is the residual resistance (independent of temperature), and ρ_T is the resistance due to the lattice vibrations (caused by the scattering of electrons by phonons).

The magnetic resistance accounts for the anomalous temperature dependence of the resistance of ferromagnets, the nature of which can be seen in Fig. 1. The characteristic kink of curve I represents the transition through the Curie point. If there were no magnetic resistance, the temperature dependence of the electrical resistance would be represented by curve II, which is merely the well-known Grüneisen function, which describes quite satisfactorily the temperature dependence of the resistance of ordinary metals. It is evident that the segment ρ_{M} (Fig. 1) represents the magnetic resistance of the ferromagnet at a given temperature. In the case of metals whose resistance above the Curie point varies linearly with temperature and for which $\rho_0 = 0$, it is not difficult to separate out the magnetic resistance (the rectilinear part of the dashed curve II passes through the origin of coordinates parallel to the linear portion of the curve $\rho(T)$ above the Curie point).

Analysis of the experimental data shows that



the ferromagnetic Hall coefficient R_F and the magnetic resistance are related by the following simple expression valid for all temperatures both above and below the Curie point (except, perhaps, the lowest temperatures):

$$R_F - R_{F0} = a \rho_M, \qquad (4)$$

where R_{F0} is the value of R_F at 0°K. This means that the temperature-dependent ferromagnetic Hall coefficient is proportional to the magnetic resistance of the sample ρ_M . This can be easily proved by examining the curves in Fig. 2 where the abscissa gives the magnetic resistance in relative units, and the ordinate the value of the ferromagnetic Hall coefficient (in arbitrary units) for several substances.



FIG. 2

The derived relationship can be justified in the following way. As established earlier, ^[6] the ferromagnetic Hall coefficient R_F is given by the equation

$$R_F - R_{F0} = b \left(J_0^2 - J_s^2 \right), \tag{5}$$

where $J_{\rm S}$ and J_0 are the spontaneous magnetizations at a given temperature and at 0°K, respectively. The same relationship was obtained theoretically by Yu. M. Kagan and L. A. Maksimov.¹⁾ It follows from Eq. (5) that the temperature dependence of $R_{\rm F}$ is due solely to the temperature dependence of $J_{\rm S}$. Then for $\rho_{\rm M}$, we obtain the expression

$$\rho_M = c \left(J_0^2 - J_s^2 \right), \tag{6}$$

which agrees with the empirical data of Borelius [7] and the theoretical calculations of Kasuya [8] and

Mannari.^[9] It is of interest to note that the value of the coefficient a in Eq. (4) for the group of ferromagnets Fe, CrTe and MnSb is numerically equal to the value of $\rho_{\rm M}$ at the Curie point.

On the basis of the results obtained, we can suggest that other galvanomagnetic effects in ferromagnets-in particular, the magnetoresistance (the even galvanomagnetic effect)-should also be governed by the value of the magnetic resistance. It is best to check this hypothesis by measurements above the Curie point (in the paramagnetic region), where the value of the magnetic resistance may be found sufficiently accurately by the method proposed above (Fig. 1). Unfortunately, the values of the relative magnetoresistance of ferromagnets above the Curie point are very small and, even in fields of the order of 25,000 Oe, amount to less than 0.01%. Therefore, sufficiently accurate measurements on substances with high Curie temperatures entail great difficulties.

In view of this, we carried out a study of the magnetoresistance of several nickel—copper alloys containing from 23.4 to 46.2 at.% copper (and, correspondingly, from 76.6 to 53.8 at. % nickel). The Curie temperatures of these alloys are sufficiently low and therefore measurements of the temperature dependence of the even gal-vanomagnetic effect in the paramagnetic region give sufficiently accurate values.

The experimental data obtained are presented in Fig. 3, where the abscissa represents, in dimensionless units the square of the magnetization $(\kappa H/J_0)^2$ (κ is the volume susceptibility of a sample, H is the magnetic field intensity, and J_0 is the spontaneous magnetization of a given sample at $T = 0^{\circ}K$), and the ordinate is the ratio of the change of the resistance $\Delta \rho = \rho_{\rm H} - \rho$ to the magnetic resistance ρ_M . From this figure, it is evident that, in spite of the considerable differences between the values of ρ_M for the investigated alloys ($\rho_{\rm M} = 18.9 \times 10^{-6} \Omega$. cm for the alloy with 23.4 at. % copper and $\rho_{\rm M}$ = 7.1 × 10⁻⁶ $\Omega.\ cm$ for the alloy with 46.2 at. $\%\ copper$), the value of $\Delta \rho / \rho_{\rm M}$ is given by the general relationship

$$-\Delta \rho / \rho_{M} = B (\varkappa H / J_{0})^{2}, \qquad (7)$$

where B lies between 0.53 and 0.47, i.e., it is practically independent of the copper concentration in the sample. If the change of the resistance $\Delta \rho$ is divided by the total resistance ρ , and not the magnetic resistance $\rho_{\rm M}$, then the dependence of $\Delta \rho / \rho$ on $(\kappa H/J_0)^2$ remains linear but the coefficient B varies from 0.23 to 0.064 on increase

¹⁾This paper will be published soon.





FIG. 3. Dependence of the magnetoresistance on the magnetization for nickel-copper alloys with the following copper concentrations (at.%): 0-23.4; +-28.7; $\bullet-31.6$; $\oplus-36.8$; $\triangle-42.4$; $\times-46.2$; \square -represents CrTe alloy.

of the copper concentration in the alloy.

To check the generality of the relationship (7), we also measured the even galvanomagnetic effect above the Curie point using chromium-tellurium samples. It was found that for this alloy (whose Curie point was 54°C and $\rho_{\rm M} = 280 \times 10^{-6}$ Ω . cm) Eq. (7) is valid, too, as shown in Fig. 3.

Thus the available experimental data show that

it is the magnetic resistance ρ_M which is the quantity with which both the odd and even galvano-magnetic effects should be compared.

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