LONGITUDINAL POLARIZATION OF ELECTRONS IN ALLOWED BETA TRANSITIONS

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Formulas are derived for the longitudinal polarization of β particles in allowed β transitions. In the derivation account is taken of corrections due to relativistic matrix elements, including weak magnetism, and due to the Coulomb field of a nucleus of finite size. Numerical calculations of the longitudinal polarization for the β^- decay of In^{114} show that the indicated corrections decrease the longitudinal polarization of the electrons by a few percent relative to $P_\beta = -v/c$.

1. INTRODUCTION

THE experiments of Spivakov et al[1] on the measurement of the longitudinal polarization (P_β) of β particles of 300 keV energy in the case of allowed transitions showed deviations from the law $P_\beta = -v/c$ (v — electron velocity). Thus $P_\beta = (1.02 \pm 0.033)v/c$ for P^{32}, but for In^{114} it is given by $(-0.93 \pm 0.033)v/c$. This raises the question: is it possible to account for this discrepancy within the framework of the universal V-A weak interactions theory by considering corrections to the main matrix elements?

Among the theoretical papers devoted to this question in the past the most interesting is the one by Berestetsky et al.[2] In it the Coulomb effects of a nucleus of finite size are taken into account as well as corrections due to the finiteness of the de Broglie wavelength ($\sim R_0/\lambda\!\!\!^{-}$, where R_0 is the nuclear radius). It is shown that for allowed β transitions the terms of order $R_0/\lambda\!\!\!^{-}$ are negligible, and the formula for P_β in the case of allowed transitions is given with the main matrix elements only taken into account. For the β decay of In^{114} this formula gives a deviation of P_β from $-v/c$ by 1.5—1.8%.

Morita[3,4] has given a formula for the longitudinal polarization of β particles in the case of allowed transitions, taking into account corrections from relativistic matrix elements of order v_N/c without weak magnetism; the Coulomb field of a point nucleus is taken into account, and $R_0/\lambda\!\!\!^{-}$ is assumed to be finite. However, for In^{114} this formula leads to a deviation of P_β from $-v/c$ by less than one percent.

This paper is devoted to the calculation of the longitudinal polarization of β particles in the case of allowed transitions. In the derivation of the formula for the longitudinal polarization of the β particles we have taken into account corrections to the main matrix elements due to the relativistic matrix elements of order v_N/c, including the "weak magnetism" ones, and due to the Coulomb field of a nucleus of finite size with the charge distributed uniformly through its volume; $R_0/\lambda\!\!\!^{-}$ is taken to be finite.

From these corrections, satisfying the selection rules in allowed transitions, there remain matrix elements of order

$$\alpha Z/MR_0, \quad v_N \alpha Z/c, \quad v_N R_0/c\lambda\!\!\!^{-},$$

where $\alpha = 1/137$; Z is the nuclear charge, M and v_N are the mass and velocity of the nucleon; units are used such that $e = \hbar = m_{el} = 1$. The factor $\alpha Z/MR_0$ appears when the Coulomb field of a finite size nucleus is taken into account, the factor $v_N \alpha Z/c$ —when relativistic corrections of order v_N/c and the Coulomb field are taken into account, the factor $v_N R_0/c\lambda\!\!\!^{-}$ —when relativistic corrections of order v_N/c and the finiteness of the de Broglie wavelength $R_0/\lambda\!\!\!^{-}$ are taken into account.

The main matrix elements remain practically unchanged when the finiteness of the de Broglie wavelength is taken into account; the corrections of order $R_0 v_N/\lambda\!\!\!^{-}c$ are also small and are of the order of 10^{-3} times the main matrix elements.

The effect of the Coulomb field of the finite size nucleus reduces P_β by 1.5—2% due to a change in the main matrix elements. Relativistic corrections of order v_N/c, together with corrections due to weak magnetism, with the Coulomb field of the finite size nucleus taken into account lead to an additional reduction of P_β by 1.5% (in the case of the β decay of In^{114}).

.The effective Hamiltonian of the β interaction with four form factors taken into account was taken from the work of Goldberger and Treiman[5] and Fujii and Primakoff.[6] The induced pseudoscalar interaction does not contribute significantly to allowed transitions and was ignored in the calculations. Only those corrections were kept in the formula for P_β that amounted to no less than 0.5% of the contribution from the main matrix elements. Use was made of the electron wave function in the Coulomb field of a finite nucleus with uniform volume charge distribution given by Berestetsky et al.[2]

Formulas were obtained for the longitudinal polarization of β particles in the case of the allowed transitions $\Delta J = \pm 1$, no and $\Delta J = 0$, no; they are unwieldy and will not be given.

2. NUMERICAL CALCULATION OF THE LONGITUDINAL POLARIZATION FOR THE β^- DECAY OF In^{114}

In order to compare the formulas with experiment we performed a numerical calculation of the longitudinal polarization of electrons in the case of the β decay of In^{114}:

$$_{49}In^{114} \xrightarrow{\beta} {}_{50}Sn^{114} \quad (1^+ \to 0^+ \text{ transition}).$$

In the calculation of the matrix elements use was made of the single-particle nuclear shell model with jj coupling and without neutron-proton interaction.

For radial wave functions we have taken oscillator wave functions in an infinite well of a spherically symmetric oscillator. The parameter ω, corresponding to the oscillator frequency, was taken from the work of Balashov et al[7] and is equal to 16 MeV; although in the reference mentioned it was calculated for a p level, a change in ω by a few MeV makes practically no difference in the results; the difference in ω between p and g levels is less than 5 MeV. The scheme of neutron and proton levels in $_{49}In^{114}$ was taken from the work of others (see [8,9]).

The proton configuration has one hole in the $1g_{9/2}$ shell: $(1g_{9/2})^{-1}$. The neutron configuration is given by $(1g_{7/2})^{-1}(3s_{1/2})^2$, i.e., the $1g_{7/2}$ shell has one hole, while the following shell $3s_{1/2}$ is completely filled. The proton configuration of $_{50}Sn^{114}$ is simple: all shells are filled. The neutron configuration of $_{50}Sn^{114}$ is unknown; in this work the following scheme for filling the last two shells is used: $(1g_{7/2})^{-2}(3s_{1/2})^2$; any other conceivable neutron configuration for $_{50}Sn^{114}$ would result in a more forbidden transition than is experimentally observed. As a result of numerical calculations we obtain: $P_\beta = -0.97\,v/c$ for $E_\beta = 300$ keV.

For the sake of comparison we calculated log ft and found it equal to 3.8. In spite of the rough nature of the calculation the agreement is not bad for ft (experimentally log ft = 4.47[9]).

Comparing the theoretical value of P_β with the experimental data of Spivak et al (see the introduction) we see that the theoretical calculation gives a result that agrees with experiment within the measurement errors.

Let us briefly discuss the contributions of the various corrections to the deviation of the longitudinal polarization P_β from $-v/c$, which amounts theoretically to 3% in the case of In^{114}. The relativistic corrections, including "weak magnetism," reduce the longitudinal polarization by 1.3%; in addition P_β is reduced by 1.8% due to the effects from the Coulomb field of a finite size nucleus, so that the total change in P_β is $\approx 3\%$. In the calculation of P_β numerical values of the Coulomb amplitudes and phases were taken from the Tables of Sliv and Volchok.[10] The ratio of the axial vector to vector coupling constants is taken to be $g_A/g_V = 1.21$. The value of P_β remains practically unchanged as the parameters are varied.

Rough numerical calculations show that relativistic corrections of order v_N/c (the largest contribution to ΔP_β from this group of corrections comes from the so called weak magnetism, about 0.7%) and corrections due to the Coulomb field of a finite size nucleus can explain the order of magnitude of the deviation of the longitudinal polarization from $-v/c$. It would appear that a more precise calculation of nuclear matrix elements would lead to better agreement with experiment.

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