RESONANCE PROPERTIES OF INHOMOGENEOUS PLASMA STRUCTURES

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We consider oscillations excited by ac electric fields in plasma structures with diffuse boundaries. The thickness of the transition layer of the plasma is assumed to be much greater than the Debye radius. The resonance properties of an inhomogeneous plane layer are investigated; the radially inhomogeneous sphere and cylinder (with piecewise linear distribution of electron density) are also treated. It is shown that strong damping of plasma waves in regions where the dielectric constant $\epsilon \sim 1$ reduces appreciably the quality factor (Q) of the fundamental dipole resonance (the frequency of this resonance is determined by the geometry of the structure) and causes the complete disappearance of the characteristic plasma resonances ($\epsilon \approx 0$) for the cylinder and sphere.

INTRODUCTION

A number of authors [1-6] have investigated the effect of the thermal motion of electrons on the resonance properties of a plasma layer bounded by ideally reflecting planes. In particular, [6] it has been noted that peculiar resonance effects can arise in the region of the most probable frequency ω_T of the characteristic oscillations of the electrons between the layer boundaries. It has also been shown [1-5] that the experimentally observed frequency splitting of the dipole resonance in bounded plasma structures [7-10] can be explained (at least qualitatively) by the thermal motion of the electrons. The appropriate expressions for the frequency ω_k and width γ_k^0 of a resonance line (for a Maxwellian velocity distribution and $\omega \gg \omega_T$) are [4,5]

$$\omega_k^2 = \omega_p^2 + \tfrac{3}{2}(2k-1)^2\omega_T^2,$$
$$\gamma_k^0 = \frac{2\sqrt{\pi}\,\omega_k^4}{\omega_T^3(2k-1)^3}\exp\left[-\frac{\omega_k^2}{\omega_T^2(2k-1)^2}\right], \qquad (1)$$

where $k = 1, 2, 3, \ldots$; $\omega_p^2 = 4\pi ne^2/m$; $\omega_T^2 = \pi^2\kappa T/2mL^2$; e, m, n, and T are respectively the charge, mass, density, and temperature of the electrons; κ is Boltzmann's constant; 2L is the thickness of the layer.[1)] Similar expressions have been obtained for the plasma oscillation frequencies of uniform cylinders and spheres with sharp boundaries. [3,12]

[1)]We note that (1) can be obtained directly from the solution of the familiar problem of propagation and damping of longitudinal waves in an infinite uniform plasma; for this purpose one introduces standing plasma waves whose nodal planes correspond with the boundaries of the layer (cf.[11]).

These results derive primarily from the assumption of specular reflection of the electrons at the boundaries of the plasma; when applied to real plasma structures, they are strictly speaking subject to the requirement of a sharp drop in plasma density in distances much less than the Debye radius.

It is the purpose of the present work to investigate the resonance properties of a bounded plasma in the other limiting case, i.e., the case in which the thickness of the boundary layer is much greater than the Debye radius. We shall consider the oscillations of a plane layer with a smoothly varying density as well as a cylinder and sphere with non-idealized boundaries. Oscillations of an inhomogeneous plasma layer were considered earlier by Wolff; [13] however, this analysis was not completely correct because the hydrodynamic equations were used to describe oscillatory processes in regions with relatively large values of the Debye radius and because the energy losses due to damping of the plasma waves were not taken into account. It is shown below that in inhomogeneous structures these losses can become so important that the plasma resonance lines for the plane layer become wider than γ_k^0; in the cylinder and sphere cases the characteristic plasma resonances ($\omega \approx \omega_p$) can, in general, be quenched altogether.

1. OSCILLATIONS OF A PLANE INHOMOGENEOUS LAYER

We consider oscillations excited by a specified external field $\mathbf{E}_0 = \mathbf{x}_0 E_0 e^{i\omega t}$ in a symmetric plasma layer with piecewise linear equilibrium density distribution n(x) (the x axis is perpendicular to the boundaries of the layer):

$$n(x) = \begin{cases} n_0, & |x| \leqslant L \\ n_0[1+(L-|x|)/l], & L \leqslant |x| \leqslant L+l. \\ 0, & L+l \leqslant |x| \end{cases} \quad (2)$$

It is assumed that the dimensions of the homogeneous and inhomogeneous regions of the layer (L and l) as well as the characteristic scale size of the field inhomogeneity $\lambda\!\!\!^{-}$ in the region $\omega_p(x) \approx \omega$ are much greater than the Debye radius D $= \kappa T/8\pi ne^2 = \kappa T/2m\omega_p^2$, so that the problem can be solved in the hydrodynamic approximation; where necessary this treatment is supplemented by well known results of the kinetic analysis.

The scale size of the inhomogeneity of the external field E_0 is assumed to be large compared with $L+l$ so that at distances appreciably greater than $L+l$ (in the plane x = const) it can be treated as a uniform field and the system analyzed as a uniform system. Under these conditions the resulting field E and current density j in the layer have only x components and are described by the following equations in the linear approximation (particle collisions and the ion motion are neglected):

$$i\omega n\dot{x} = n\frac{e}{m}E + n_1\frac{1}{m}F_0 - \frac{3\varkappa T}{m}\frac{dn_1}{dx},$$
$$j = ne\dot{x}, \quad \frac{dE}{x} = 4\pi e n_1, \quad j = -\frac{i\omega}{4\pi}(E - E_0),$$

where $\dot{x}$ and n_1 are the amplitudes of the velocity and the ac part of the electron density and $F_0(x)$ is a static force field due to the specified equilibrium density distribution (2); this force is given by the stationary-state equation

$$nF_0 = \varkappa T dn/dx. \quad (4)$$

Substituting (4) in (3) and eliminating $\dot{x}$, n_1 and j we have

$$D_0^2\frac{d^2E}{dx^2} - \frac{1}{3}D_0^2\frac{1}{n}\frac{dn}{dx}\frac{dE}{dx} + \varepsilon(x)E = E_0, \quad (5)$$

where

$$D_0^2 = 3\varkappa T/m\omega^2 \quad (D_0 \ll L, l), \qquad \varepsilon(x) = 1 - \omega_p^2(x)/\omega^2.$$

The expression in (5) will describe the field distribution E(x) properly only in regions in which $\lambda\!\!\!^{-} \gg$ D. According to (5)

$$\lambda\!\!\!^{-} \begin{cases} D_0/|\varepsilon|^{1/2} & \text{for } |\varepsilon|^{3/2} \gg D_0|d\varepsilon/dx| \\ D_0(l/(1-\varepsilon_0)D_0)^{1/3} & \text{for } |\varepsilon|^{3/2} \leqslant D_0|d\varepsilon/dx|, \end{cases} \quad (6)$$

and this inequality can be satisfied only when $\omega_p(x) \approx \omega$, that is to say, in regions in which $|\epsilon|^{1/2} \ll 1$ (where it is especially necessary to take account of the thermal motion and the associated spatial dispersion). However, in regions where $|\epsilon| \gtrsim 1$ spatial dispersion is generally not important and the field is determined by the simple relation $\epsilon E = E_0$ (cf. below).

It will be assumed that the absolute value of the dielectric constant in the uniform portion of the layer $\epsilon_0 = 1 - 4\pi n_0 e^2/m\omega^2$ is not large compared with unity ($|\epsilon_0| \ll l/D_0$). Under these conditions $|\epsilon|^{1/2} \ll 1$, the second term on the left side of (5), is $l/\lambda\!\!\!^{-}$ times smaller than the first; since $l/\lambda\!\!\!^{-}_{max} \approx (l/D_0)^{2/3} \gg 1$ [as follows from (6)], this term can be neglected and reasonable accuracy[2)] can still be obtained. The final expression for the field E is then

$$D_0^2 d^2E/dx^2 + \varepsilon(x)E = E_0. \quad (7)$$

Since this layer is symmetric with respect to the plane x = 0 it is sufficient to solve (7) for $x \leq 0$ requiring that the derivative dE/dx vanish at x = 0.

We now consider two cases: $|\epsilon_0| \ll 1$ and $|\epsilon_0| \sim 1$.

1. $|\epsilon_0| \ll 1$. When $-L \leq x \leq 0$ the solution of (7) that satisfies the condition dE/dx = 0 at the point x = 0 is

$$E = E_0/\varepsilon_0 + C_1\cos h_0 x, \quad (8)$$

where $h_0^2 = \epsilon_0/D_0^2$.

In the region $x \leq -L$ we have $\epsilon(x) = \epsilon_0 - (1-\epsilon_0) \times (L+x)/l$; introducing the new variable

$$z = -\alpha\varepsilon(x), \quad \alpha = [l/(1-\varepsilon_0)D_0]^{2/3},$$

we write (7) in the form

$$d^2E/dz^2 - zE = \alpha E_0. \quad (9)$$

The solution of this equation is expressed in terms of the Airy functions u(z) and v(z):[15]

$$E = C_2[u(z) - iv(z)] + \alpha E_0\left[u(z)\int_{-\infty}^{z} v(t)\,dt - v(z)\int_{-\infty}^{z} u(t)\,dt\right]. \quad (10)$$

The limits of integration and the relations between the coefficients of u(z) and v(z) are chosen in such a way that when $-z \gg 1$ (the quantity $\epsilon = -z/\alpha$ can still be small compared with unity since $\alpha \sim (l/D_0)^{2/3} \gg 1$) the second term becomes the field E_0/ϵ obtained with spatial dispersion

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2)Taking account of this term in the present case would only yield a small relative correction (of order $|\epsilon|$) to the amplitude of the plasma wave (11). It should be noted, however, that in the general case the term containing the first derivative of the field can be neglected only if certain additional conditions are satisfied (for example if one introduces a small but finite collision frequency; cf.[14]).

neglected while the first gives the geometric optics approximation for a plasma wave propagating in the $-x$ direction (toward the edge of the layer):[3)]

$$E_p = C_2(-z)^{-1/4} \times \exp\left[-i\frac{2}{3}(-z)^{3/2} - i\frac{\pi}{4}\right] \sim \varepsilon^{-1/4}\exp\left\{i\int h(x)\,dx\right\}, \quad h^2(x) = \varepsilon(x)/D_0^2. \tag{11}$$

The plasma wave traveling in the opposite direction vanishes by virtue of the following considerations. Using a well-known result of linear kinetic theory we can replace the expression for the plasma wave (11) by a more exact expression

$$E_p' = E_p \exp\left\{\int \gamma(x)\,dx\right\} \tag{12}$$

(γ is the Landau damping constant) which holds in the region in which the inequality $|\epsilon|^{1/2} \ll 1$ is not satisfied. The wavelength is of the same order as the Debye radius in this region and γ increases sharply so that the wave is damped rapidly and never reaches the edge of the layer. Hence the plasma wave in the $+x$ direction, which can be excited only as a result of reflection of the wave in (12) from the edge of the layer, is quenched. We note that the introduction of this wave damping is necessary only for relating the constants of integration in (9); in what follows the difference between (11) and (12) is unimportant.

The continuity conditions on the field E and its derivative dE/dx at $x = L$ (these boundary conditions are completely reasonable physically and can be obtained from (7) or (5) by taking the limit from the smooth function $\epsilon(x)$ to the piecewise linear function) give the following expressions for the constants C_1 and C_2:

$$C_1 = \frac{E_0}{\varepsilon_0}\,\frac{w'(z_0) - z_0\int\limits_{-\infty}^{z_0} w(t)\,dt}{-w'(z_0)\cos(M\sqrt{-z_0}) + \sqrt{-z_0}\,w(z_0)\sin(M\sqrt{-z_0})},$$

$$C_2 = \frac{E_0}{\varepsilon_0}\,\frac{\sqrt{-z_0}\sin(M\sqrt{-z_0})\,[1 + z_0\psi(z_0)] - z_0\psi'(z_0)\cos(M\sqrt{-z_0})}{-w'(z_0)\cos(M\sqrt{-z_0}) + \sqrt{-z_0}\,w(z_0)\sin(M\sqrt{-z_0})}. \tag{13}$$

Here,

$$z_0 = -\alpha\varepsilon_0, \quad M = \alpha(1-\varepsilon_0)L/l, \quad w(z) = u(z) - iv(z),$$

$$\psi(z) = u(z)\int\limits_{-\infty}^{z} v(t)\,dt - v(z)\int\limits_{-\infty}^{z} u(t)\,dt;$$

the primes denote differentiation with respect to z.

The frequency dependence of the constants C_1 and C_2 and the polarizability of the layer (per unit area) expressed in terms of the constants

$$\chi = \frac{1}{E_0}\int\limits_{-L-l}^{L+l} en_1 x\,dx = \frac{L+l}{2\pi} - \frac{1}{2\pi E_0}\int\limits_{-L-l}^{0} E\,dx \tag{14}$$

can be obtained simply (without using numerical methods) only in the particular limiting cases that we consider below.

When $|z_0| \gg 1$ we have:

a) $z_0 < 0$ ($\epsilon_0 > 0$); $|C_1|$, $|C_2| \ll E_0/\epsilon_0$; the amplitude of the plasma wave is negligibly small and to a high degree of accuracy the field is given by E_0/ϵ in the entire layer;

b) $z_0 > 0$ ($\epsilon_0 < 0$);

$$|C_1| \ll E_0/\varepsilon_0, \quad C_2 = -\alpha\int\limits_{-\infty}^{+\infty} v(t)\,dt E_0 = -\sqrt{\pi}\alpha E_0;$$

the plasma wave is characterized by a large amplitude ($\alpha \gg 1$) and a finite power flux

$$\Pi = \frac{1}{2}\,\mathrm{Re}\,(3\varkappa T n_1 \dot{x}^*) = \frac{\omega l}{8(1-\varepsilon_0)}E_0^2, \tag{15}$$

which is independent of temperature and inversely proportional to the slope of the plasma density distribution. The polarizability is a complex quantity in this case:

$$\chi = \chi_r + i\chi_i;$$

$$\chi_i = -\frac{C_2 l}{2\pi\alpha(1-\varepsilon_0)E_0}\int\limits_{-\infty}^{+\infty} v(t)\,dt = -\frac{l}{2(1-\varepsilon_0)}. \tag{16}$$

The real part χ_r for $L \gtrsim l$ is determined primarily by the polarizability of the uniform portion:

$$\chi_r = -L/2\pi\varepsilon_0. \tag{17}$$

From the point of view of resonance properties greatest interest attaches to the other limiting case: $|z_0| \ll 1$. In (13) we replace the functions u, v, u' and v' by their values at $z = 0$, obtaining the resonance condition in the form

$$\sqrt{-z_0}\,\tan(M\sqrt{-z_0}) = \frac{w'(0)}{w(0)} = a + ib; \quad a = 0{,}367, \quad b = 0.635. \tag{18}$$

When $M \gg 1$ ($L \gtrsim l$) we obtain the following expressions for the resonance frequencies ω_k and the damping factors (line widths) γ_k;

$$\omega_k^2 = \omega_p^2\left[1 + \frac{\pi^2}{4}(2k-1)^2\left(\frac{D_0}{L}\right)^2\right],$$

$$\gamma_k = \frac{\pi^2 b\omega_k(2k-1)^2}{4(a^2+b^2)}\left(\frac{l}{L}\right)^{1/3}\left(\frac{D_0}{L}\right)^{4/3}, \quad k = 1, 2, 3, \ldots \tag{19}$$

Comparing (19) with the analogous expression (1) that characterizes the characteristic oscillation spectrum of a layer with sharp boundaries, in which the electrons experience specular reflection, we see

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[3)]A similar solution has been obtained by Denisov[16] who investigated the interaction between the electromagnetic wave and the plasma wave in a linear plasma layer.

that the real part of the characteristic frequency is the same in both cases (this holds only when $|z_0| \ll 1$); on the other hand, the line widths are considerably different: when $T \to 0$ the width γ_k^0 is an exponentially small quantity while $\gamma_k \sim T^{4/3}$ and is appreciably greater than γ_k^0.

The polarizability near resonance is given by

$$\chi = -\frac{2L\omega_k}{\pi^3 (2k-1)^2 (\omega - \omega_k - i\gamma_k)}. \qquad (20)$$

2. When $|\epsilon_0| \sim 1$ we obtain the same results as in the preceding case for $|z_0| \gg 1$: when $\epsilon_0 > 0$ we have $E = E_0/\epsilon$; if $\epsilon_0 < 0$ for $|z_0| \gg 1$ we have

$$E\,(z > 0) = E_0/\varepsilon,$$

$$E\,(z < 0) = (E_0/\varepsilon) - \sqrt{\pi}\,\alpha E_0 (-z)^{-1/4} \times \exp\left[-i\frac{2}{3}(-z)^{3/2} - i\frac{\pi}{4}\right],$$

while the energy dissipated in the plasma wave and the imaginary part of the polarizability χ_i are determined by the same expressions (15) and (16). It is interesting to note that Π and χ, (15)—(17), are respectively the Joule loss and the polarizability computed for the same layer neglecting spatial dispersion ($T = 0$) but keeping a finite collision frequency $\nu \ll \omega$.

2. OSCILLATIONS OF A SPHERE OR CYLINDER WITH DIFFUSE BOUNDARIES

The results obtained in the previous section can be used for investigating the resonance properties of two and three-dimensional structures with diffuse boundaries if the widths of the transition regions are small compared with the radius of curvature. As an example we consider the oscillations of a spherically symmetric plasmoid in a uniform field $\mathbf{E}_0 = \mathbf{x}_0 E_0 e^{i\omega t}$. The dependence of equilibrium density n on radius r is chosen in the same form as the piecewise linear function (2) ($|x| \to r$) and it is assumed that $L \gg l \gg D_0$ (L is the radius of the region with uniform density). It is also assumed that the wavelength of the electromagnetic field, both in free space and in the plasma, is large compared with L. We shall consider the solution of the problem in the quasistatic approximation (taking $\mathbf{E} = -\nabla\varphi$).

Generalization of (3) and (4) to the three-dimensional case gives

$$\operatorname{div}\left(\varepsilon\mathbf{E} + D_0^2 \operatorname{grad}\operatorname{div}\mathbf{E} - \frac{1}{3} D_0^2 \frac{\operatorname{grad} n}{n}\operatorname{div}\mathbf{E}\right) = 0. \qquad (21)$$

In the uniform region ($r \le L$) this equation (which holds only when $|\epsilon|^{1/2} \ll 1$) splits up into an equation for the scalar potential of the irrotational field ($\operatorname{div}\mathbf{E}_S = 0$) and the plasma field ($\operatorname{div}\mathbf{E}_p \neq 0$) ($\mathbf{E} = \mathbf{E}_S + \mathbf{E}_p = -\nabla\varphi_S - \nabla\varphi_p$):

$$\Delta\varphi_s = 0, \quad \Delta\varphi_p + h_0^2\varphi_p = 0. \qquad (22)$$

The solutions of these equations with the same dependence on polar angle θ (with respect to the x axis) as the potential of the external field $\varphi_0 = -E_0 r\cos\theta$ are

$$\varphi_{s,p} = \Phi_{s,p}(r)\cos\theta,$$

$$\Phi_s = A_s r, \qquad \Phi_p = A_p j_1(h_0 r), \qquad (23)$$

where $j_1(h_0 r)$ is the spherical Bessel function of first order.

In the region $L \le r \le L + l$ a separation procedure analogous to that used in obtaining (22) cannot be used and the complete potential is described by the equation (terms proportional to ∇n are ignored)

$$\varepsilon\Delta\varphi + \nabla\varepsilon\nabla\varphi + D_0^2\Delta(\Delta\varphi) = 0. \qquad (24)$$

We substitute everywhere $\varphi = \Phi(r)\cos\theta$ and neglect terms of higher order in l/L, obtaining

$$D_0^2 d^3\Phi/dr^3 + \varepsilon(r)\,d\Phi/dr = \text{const} = A_1, \qquad (25)$$

which coincides with (7) for the uniform problem. As before we introduce the new variable $z = -\alpha\epsilon(r)$ and write (25) in the form [cf. (10)]

$$\Phi(r) = -\frac{l}{\alpha(1-\varepsilon_0)}\int_{z_0}^{z} [A_2 w(t) + \alpha A_1 \psi(t)]\,dt + A_3. \qquad (26)$$

Outside the plasma region ($r \ge L + l$) the total potential is made up of potential associated with the external field φ_0 and the potential of a point dipole

$$\varphi = -E_0 r\cos\theta + r^{-2} p\cos\theta \qquad (27)$$

(p is the dipole moment of the plasmoid).

The constants $A_{s,p}$, $A_{1,2,3}$ and p must be found from the conditions of continuity on the potential and its derivatives at the boundary surfaces: at $r = L + l$ the quantities φ and $\partial\varphi/\partial r$ are continuous; at $r = L$, as follows from the general equation (21), the quantities $\partial^2\varphi/\partial r^2$ and $\partial^3\varphi/\partial r^3$ are continuous. Since the formulas giving these constants are extremely complicated we present an expression only for the most important quantity, the dipole moment p

$$p = E_0 L^3 \frac{\varepsilon_0 - G(z_0)}{\varepsilon_0 + 2G(z_0)}, \qquad (28)$$

$$G(z_0) = \frac{\sqrt{-z_0}\,w(z_0)\,j_1''(\rho) + w'(z_0)\,j_1'(\rho) - z_0\int_{-\infty}^{z_0} w(t)\,dt\,[j_1'''(\rho) + j_1'(\rho)]}{\sqrt{-z_0}\,j_1''(\rho)\,w(z_0) - j_1'''(\rho)\,w'(z_0)}$$

$$-\frac{j_1(\rho)\left[w'(z_0)-z_0\int\limits_{-\infty}^{z_0} w(t)\,dt\right]-z_0\int\limits_{-\infty}^{z_0} w(t)\,dt\left\{[1+z_0\psi(z_0)]\,j_1''(\rho)+\sqrt{-z_0}\,\psi'(z_0)\,j_1'''(\rho)\right\}}{\rho\left[\sqrt{-z_0}\,j_1''(\rho)\,w(z_0)-j_1'''(\rho)\,w'(z_0)\right]}-\frac{z_0}{M}\int\limits_{-\infty}^{z_0}\psi(t)\,dt;\quad \rho=M\sqrt{-z_0}. \tag{29}$$

This expression is simplified considerably when $z_0 \gg 1$ $(-\epsilon_0 \gg 1/\alpha)$

$$p = E_0L^3\frac{\varepsilon_0-1-i\delta}{\varepsilon_0+2+2i\delta}, \tag{30}$$

where $\delta = \pi\epsilon_0/L(1-\epsilon_0)$ is a quantity that determines the linewidth of the so-called fundamental dipole resonance $(\epsilon_0 = -2)$.

It is important to note that the feature pointed out at the end of the preceding section also appears in the present case: if the basic dissipation is due to particle collisions rather than the excitation of plasma waves the energy loss and the expression for the dipole moment (30) remain unchanged (cf. [17] and [18]).

In view of the results given in the introduction and in Sec. 1, we expect that in addition to the resonance at $\epsilon_0 = -2$ there will also be characteristic plasma resonances in the region of small positive ϵ_0. However, a detailed investigation of the function $G(z_0)$ shows that for all values of ϵ_0 satisfying the condition $|\epsilon_0 + 2| \gg \delta$ the dipole moment is given by the expression $p = E_0L^3 \times (\epsilon_0 - 1)/(\epsilon_0 + 2)$ to a high degree of accuracy and does not exhibit maxima. In particular, when $0 < -z_0 \ll 1$ the minimum value that can be assumed by $G(z_0)$ (in absolute magnitude) is approximately $\sqrt{-z_0}$, and since $\sqrt{-z_0} = \sqrt{\alpha\epsilon_0} \gg \epsilon_0$, then $p = -E_0L^3/2$. We note that the amplitude of each of the fields $\mathbf{E}_s$ and $\mathbf{E}_p$ increases strongly in this case; nonetheless, the total field distribution in the plasma $\mathbf{E}_s + \mathbf{E}_p$ is such that the dipole moment does not exhibit a resonance.

A similar analysis shows that such resonance properties are also exhibited by an infinite plasma cylinder with diffuse boundaries with the cylinder axis perpendicular to the electric field. The dipole moment per unit length of cylinder $(-\epsilon_0 \gg 1/\alpha)$ is given by

$$p = E_0L^2(\varepsilon_0 - 1 - i\delta)/(\varepsilon_0 + 1 + i\delta). \tag{31}$$

Far from resonance (i.e., when $|\epsilon_0 + 1| \gg \delta$) we have $p = E_0L^2(\epsilon_0 - 1)/(\epsilon_0 + 1)$ (the notation is similar to that used above).

The discrepancy between the results obtained in various experiments [7-10] in which a series of resonances have been observed in the region $\omega \sim \omega_p$ can be explained by the fact that the condition $l \gg D$ has not been satisfied in these experiments. These results were obtained with discharge tubes with radius small compared with the mean free path of the electrons in the gas. Under these conditions the direct loss of electrons to the walls of the tube is inhibited by the separated charges which form a double layer near the walls; the layer thickness (approximately equal to the Debye radius) is the characteristic dimension of the inhomogeneity in the electron density l. A qualitative description of the resonance properties of plasma structures characterized by $l \approx D$ can be obtained from the results obtained under the assumption that $l \ll D$ (1) which predict the presence of many resonances. The discrepancy between these results and the experimental data with respect to the relative position of the resonance lines [4)] is removed [19] if one takes account of the smooth increase in density in the direction from the walls to the axis in addition to the sudden drop in plasma density close to the walls of the tube.

If the plasma is not in contact with the walls of a container (meteor trails, plasmoids in laboratory experiments) the characteristic scale size of the inhomogeneity is generally greater than the Debye radius and the resonance effects associated with the excitation of longitudinal (plasma) waves must disappear. As far as the fundamental dipole resonance frequency determined by structure geometry is concerned, we note that under conditions of strong diffuseness $(l \approx L)$ its quality factor (as follows from Eqs. (30) and (31) which apply qualitatively in the present case) is also very low.

CONCLUSION

In the linear approximation the effectiveness of the resonance interaction of a bounded plasma with a field depends on the diffuseness of the plasma boundary. If the characteristic scale size of the boundary region is of the order of the dimensions of the object itself $(l \approx L \gg D)$ all resonance effects are very weak and the maximum value of the dipole moment is rather small $(p \approx L^3E_0)$. We note that in the final analysis this circumstance is responsible for the component of electric field parallel to $\nabla\epsilon$ which leads to a strong growth of the field near the point $\epsilon = 0$; when $T \to 0$ (and $\nu \to 0$) the field becomes infinite at this point and

[4)]According to (1) the spacing between neighboring lines is increased as the number k increases; experimentally, however, one observes a convergence in the sequence of resonance peaks.

the power loss becomes finite. In the absence of components $\mathbf{E} \parallel \nabla\epsilon$ (for a radially inhomogeneous sphere this is the case for magnetic modes) there is no such singularity in the field at $\epsilon = 0$; in this case the energy loss at low temperatures and collision frequencies is due only to radiation associated with the corresponding magnetic multipole. Thus the highest efficiency for interaction between a plasmoid and field is evidently achieved for resonance excitation of a magnetic mode (for example, the magnetic dipole resonance). In the case of an isotropic dielectric structure whose characteristic scale size L is small compared with the wavelength in free space λ the frequency of the magnetic resonance is given by the condition $\sqrt{\epsilon}\,L \approx \lambda$, which can be satisfied only for large positive ϵ (this is impossible in an isotropic plasma). Hence, it would be of interest to investigate the resonance properties of plasmoids consisting of magnetoactive plasma, which can exhibit the properties of a medium with $\epsilon > 1$ for certain definite relations between the components of the dielectric tensor.

It is of interest to consider the possible roles of various nonlinear effects, primarily those related to resonance interactions. In particular, the plasma oscillation spectrum can be modified if the high-frequency field changes the electron density distribution in the resonance region $\epsilon \approx 0$ (where the average high-frequency potential[20] has a sharp peak) or smooths the electron velocity distribution function in the region of the points $\dot{x} = v_\phi = h/\omega$.[21] The first of these effects has not been studied at the present time and its role is as yet unclear. As is well known, the second can lead to a strong reduction in the attenuation factor γ of the plasma wave, owing to which there may appear a plasma wave reflected from the boundaries of the structure and a sharp reduction in the dissipation of energy. In this case all of these resonance effects become more pronounced, as in the case of a plasma with sharp boundaries. Estimates show that at the relatively low levels of radio-frequency power customarily used for diagnostic measurements the indicated nonlinear effects are still very weak. However, at the high powers used, for example, in the acceleration of plasmoids, the quality factor of the resonances should increase markedly because of the nonlinear reduction in damping.

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