ELECTRON SCATTERING BY LIGHT NUCLEI IN THE UNIFIED MODEL

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Submitted to JETP editor May 18, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1940-1942 (December, 1963)

The scattering of electrons by light nuclei whose charge density can be calculated according to the unified model is considered. From the expressions given in the paper one can easily obtain the form factors which determine the cross sections for elastic and inelastic electron scattering with excitation of rotational levels of nuclei with $Z \leq 20$. The calculations are compared with the experimental data on the elastic and inelastic scattering of electrons by Mg^{24} with excitation of the 1.37 MeV level.

THE scattering of high energy electrons on nuclei yields valuable information on the structure of nuclei, the positions and characteristics of the levels, and other features. The success of the unified model in the description of light nuclei [1,2] makes it desirable to analyze the scattering of electrons on the basis of this model. To this end we derive, in the present paper, expressions from which one can easily obtain the nuclear form factors defining the elastic and inelastic cross sections for electron scattering with excitation of rotational levels of nuclei with $A \leq 40$.

We shall assume that the nucleons move independently of one another in an axially symmetric potential. It is clear that the nuclear form factors will have the form of a sum of form factors for the individual protons of the nucleus. We shall only consider the elastic scattering and the scattering with excitation of rotational levels, i.e., we shall assume that the intrinsic state of the nucleus is unaltered. Then the cross section for a process in which the electron is scattered into the direction **n** and the nucleus makes a transition from a state with spin I_0 to a state with spin I has the form [3]

$$\sigma'_I(\mathbf{n}) = \sigma_C(\theta)(2I+1)\sum_L (I_0 IK - K \mid L0)^2 \left| Z^{-1}\sum_k F_k^{(L)}(q)\right|^2, \qquad (1)$$

where σ_C is the cross section for a point nucleus with charge Z and q is the momentum transfer.

The form factor $F_k^{(L)}$ is related to the charge distribution of the proton in the k-th state:

$$F_k^{(L)}(q) = 4\pi \int_0^1 P_L(x)\,dx \int_0^\infty \rho_k(r, x)\, j_L(qr)\, r^2\, dr, \qquad (2)$$

where $x = \cos\vartheta$. The quantities ρ_k found with the help of the wave functions calculated earlier [4] are given in [5].

The calculation of $F_k^{(0)}$ and $F_k^{(2)}$ for $k \leq 10$ (which is sufficient in the region $A \leq 40$) gives

$$F_1^{(0)} = J, \qquad F_2^{(0)} = (1-\beta)J + \tfrac{1}{2}\varphi,$$
$$F_{3,4}^{(0)} = C^2_{010-}F_2^{(0)} + C^2_{001+}(\chi - \varphi),$$
$$F_5^{(0)} = (1 - 2\beta + \beta^2/2)J + (1 - \tfrac{1}{2}\beta)\varphi + \tfrac{1}{4}\beta P,$$
$$F_{6,7}^{(0)} = C^2_{020-}F_5^{(0)} + C^2_{011+}\{(\beta - 2)\varphi + \chi - \beta P\},$$
$$F_{8,9,10}^{(0)} = C^2_{011-}f^{6,7}_{011+} + C^2_{110+}(F_1^{(0)} - 2F_2^{(0)} + 2F_5^{(0)})$$
$$+ C^2_{002+}\{\tfrac{1}{2}F_1^0 + \varphi + (\tfrac{1}{2} - \alpha^2\beta - 2\beta)\chi + \beta P\}$$
$$- \sqrt{2}C_{110+}C_{002+}(F_1^{(0)} - F_2^{(0)} - f^{3,4}_{001+} + f^{6,7}_{011+}),$$
$$F_1^{(2)} = \frac{6\omega_x\sqrt{\omega_z}}{q^3}\int_0^1 (1 - 3x^2)\left(\frac{\sqrt{\pi}}{2}\Phi(z) - ze^{-z^2} - \tfrac{2}{3}z^3e^{-z}\right)dx; \qquad (3)$$
$$F_2^{(2)} = \tfrac{1}{2}\{(\beta - 1)J + (3/2\beta - 2)\varphi + \tfrac{3}{2}P\},$$
$$F_{3,4}^{(2)} = C^2_{010-}F_2^{(2)} + \tfrac{1}{2}C^2_{001+}(\varphi - 3P - 2\alpha^2 P),$$
$$F_5^{(2)} = \tfrac{1}{4}\{(4\beta - \beta^2 - 2)J + (\tfrac{5}{2}\beta + 3/\beta - 8 - 9/4\alpha^2)\varphi$$
$$+ (6 - 2\beta + 15/4\alpha^2)P\},$$
$$F_{6,7}^{(2)} = C^2_{020-}F_5^{(2)} + C^2_{011+}\{(\tfrac{5}{2} - \beta/2 + 3/\alpha^2)\varphi$$
$$- \beta\chi/2\alpha^2 - (\alpha^2 + \tfrac{9}{2} + 15/4\alpha^2)P\},$$
$$F_{8,9,10}^{(2)} = C^2_{011-}f^{6,7}_{011+} + C^2_{110+}(F_1^{(2)} - 2F_2^{(2)} + 2F_5^{(2)})$$
$$+ C^2_{002+}\{\tfrac{1}{2}F_1^{(2)} - f^{3,4}_{001+} - 3\varphi/4\alpha^2$$
$$+ (\beta/\alpha^2 - 2\beta - \alpha^2\beta)\chi + (3 + 15/4\alpha^2)P\}$$
$$- \sqrt{2}C_{110+}C_{002+}(F_1^{(2)} - F_2^{(2)} - f^{3,4}_{001+} + f^{6,7}_{011+}), \qquad (4)$$

where $f^{3,4}_{001+}$ denotes the part of the form factor $F_{3,4}$ which multiplies the quantity C^2_{001+}, and we have introduced the notation

$$J = \frac{\sqrt{\pi}}{2}\frac{\Phi(\alpha\sqrt{\beta})}{\alpha\sqrt{\beta}}e^{-\beta},$$

$$\chi = e^{-(1+\alpha^2)\beta}, \quad \varphi = \alpha^{-2}(J - \chi), \quad P = \alpha^{-2}(3\varphi/2\beta - \chi),$$

$$\alpha^2 = \gamma/(1-\gamma) = (\omega_x - \omega_z)/\omega_x, \ \beta = q^2/4\omega_x,$$
$$z = q/2\,[\omega_x(1-\gamma x^2)]^{1/2}, \qquad (5)$$

[$\Phi(x)$ is the error integral].

The connection between the frequencies ω_i and the equilibrium deformation parameter δ and the values of the quantities C as functions of this parameter are given in [4]. The value of the equilibrium deformation parameter has been calculated in [1] for a large number of nuclei.

In Figs. 1 and 2 we compare the calculations with the experimental data of Helm [6] on the elastic and inelastic scattering of electrons with excitation of the 1.37-MeV level of Mg^{24} (the energy of the electrons is E = 187 MeV). The values of the equilibrium deformation parameter and the r.m.s. radius of the nuclear charge distribution needed for the calculation are taken from [1] and the review article of Hofstadter, [7] respectively ($\delta = 0.36$, a = 2.98 f).

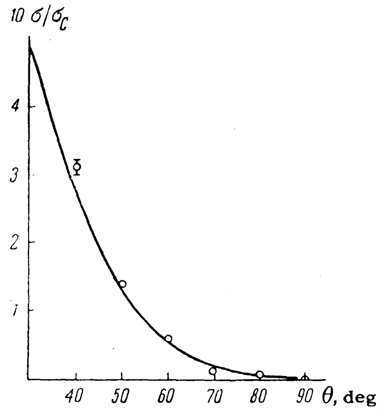

FIG. 1. Elastic scattering of electrons on Mg^{24}.

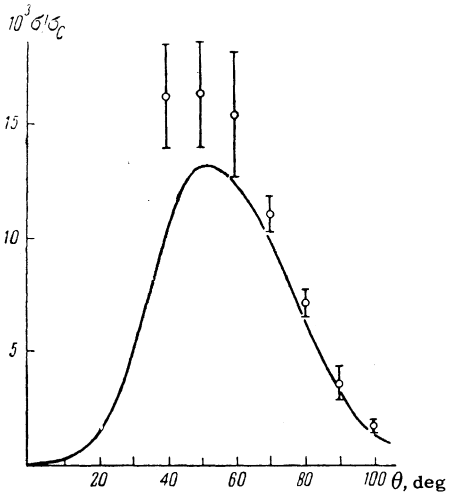

FIG. 2. Inelastic scattering of electrons on Mg^{24} with excitation of the 1.37-MeV level.

We see that the calculated curves describe well the elastic scattering of the electrons on Mg^{24} and reproduce correctly the behavior of the angular distribution of the inelastic scattering with excitation of the 1.37-MeV level. The theoretical curve for the inelastic scattering lies somewhat below the experimental values, but the discrepancy between the theoretical and experimental data is not very large. Moreover, there are reasons to expect that the agreement between theory and experiment can be improved by changing the parameters δ and a, which have not been varied in this paper. Thus it is seen that the unified model describes adequately the available experimental data on the scattering of electrons on Mg^{24}. It is desirable, however, to carry out a more careful analysis of the experimental data on Mg^{24} as well as other light nuclei on the basis of the unified model.

It is my pleasant duty to express my deep gratitude to E. V. Inopin and A. A. Kresnin for their interest in this work and valuable comments.

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Translated by R. Lipperheide
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