MASER WITH TWO RESONATORS IN SERIES

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The amplitude and frequency characteristics of a maser with two resonators in series were investigated experimentally. The results are interpreted theoretically. An effect of electric and magnetic fields on the beam of active molecules outside the resonator that produces a change in the frequency and amplitude of the maser was discovered and studied experimentally. A theoretical interpretation of these experiments is presented.

1. INTRODUCTION

THE investigation of masers with resonators in series has been the subject of several papers,[1-5] in which the most important characteristics of the two-resonator system have been studied, such as the frequency, spectrum, and intensity of the oscillations in the second resonator. As a result, a number of facts have been established which are quite remarkable at first sight. It turned out that the frequency of oscillation in the second resonator ω_{II} was precisely the same as the frequency of oscillation in the first resonator ω_I, in which the usual self-oscillation condition exists. No dependence of the frequency of oscillation in the second resonator on its resonant frequency was found. On the contrary, ω_{II} was observed to follow accurately the change in ω_I caused by tuning the first resonator, such that ω_{II} always remained equal to ω_I. This clearly indicates a mode in the second resonator different from the usual self-oscillating maser mode on which the process of induced emission is based. At the same time, it would seem impossible to associate it with spontaneous emission. In fact, spontaneous emission causes a frequency distribution of intensity with a maximum at the molecular transition frequency ω_0, and not at the frequency ω_I, which can be quite different from ω_0. Besides, Strakhovskiĭ and Tatarenkov[5] have demonstrated the high monochromaticity of the emission in the second resonator.

A detailed interpretation of these effects has been given in[4]. It was shown that molecules situated initially in an excited state make a transition to a super-radiating state (with non-zero polarization) while emitting in the first resonator and are capable of radiating significant energy during their flight time in the second resonator. Each molecule, during the time of flight τ_{II} through the second resonator, emits a pulse of spontaneous radiation with spectral width $\Delta\omega \sim \tau_{II}^{-1}$ and peaked at ω_0. However, the polarization phase of each molecule varies continuously with the frequency ω_I, and the phases of the pulses that follow continuously one after the other in the second resonator also vary with this frequency. Hence, in the entire frequency spectrum of the pulse, only oscillations with frequency ω_I will be in phase. Their amplitudes add up, forming an intense monochromatic response in the second resonator. Oscillations at other frequencies, on the other hand, form only additive noise of considerably less intensity, since they are incoherent.

It has also been shown in[1,4,5] that with a small intensity of the field in the first resonator in the second resonator there can arise, along with the oscillations at ω_I, the usual self-oscillations at a frequency ω that depends on the tuning of the resonator. This condition can be provided either by reducing the Q of the first resonator or by perturbing it relative to the characteristic frequency of the molecular transition. Since, generally speaking, $\omega \neq \omega_I$, it is possible to observe a beat with frequency $\Omega = |\omega - \omega_I|$ in the second resonator. The oscillations at ω_I can in this case be considered as an external force acting on the self-oscillations. For rather small perturbations Ω, synchronization (locking-in) of the self-oscillations by the external force can set in; the beats disappear and only oscillations at frequency ω_I remain, in spite of the fact that the conditions for self-oscillation in the second resonator are still fulfilled.

Because of these properties a two-resonator maser is an extremely interesting system both from the practical point of view and for investi-

gating radiation processes. Therefore a detailed study of the characteristics of such systems is extremely valuable.

2. CHARACTERISTICS OF A TWO-RESONATOR MASER

We investigated an ammonia maser and spectroscope (line J = K = 3) with three resonators in series through which the molecular beam passed. A schematic diagram of the apparatus is shown in Fig. 1.

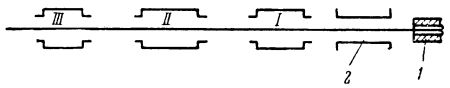

FIG. 1. Schematic diagram of a maser with three resonators in series: 1—source of the beam, a 1.2 mm channel of length 10 mm, 2—state selector (hexapole capacitor), I—first resonator, II—second resonator, III—third resonator.

The resonators (E_{010} mode) were tuned to the line J = K = 3 ($\nu = \omega/2\pi$ = 23870 Mc). The Q of the resonators was 6000—7000. Microwaves were fed in and out each cavity with waveguides. The microwave output of each resonator was connected to its own superheterodyne receiver, which was independent of the others. For frequency measurements, we used as a standard another maser with relative stability not lower than 10^{-10}.[6]

The characteristics of the emission of the molecular beam during its passage through the second and third resonators were investigated as a function of the tuning of the first resonator, the pressure at the beam source, and the voltage applied to the state separator.

When conditions for self-excitation were fulfilled in the first resonator, a microwave field was also generated in the second. The power of this field was approximately an order of magnitude less than the power of the oscillations in the first resonator.

When the frequency of oscillation in the second resonator was compared with the frequency of another maser of the usual type, it was found that the oscillations in the second resonator were monochromatic to better than 10^{-12}. When the resonant frequency of the second resonator was changed, we did not observe any changes in the frequency of radiation from the molecules within it due to that change. The amplitude and phase of the oscillations did change. The frequencies of the oscillations in the first and second resonators were found to be the same. In this it was observed that when the second resonator is being tuned, changes occur in the phase of the oscillations relative to the oscillations in the first resonator as well as in their amplitude.

When the first resonator is detuned (the second being tuned to the line), the frequency in the second "tracks" the frequency in the first over a tuning range of about $\Delta\nu \approx \pm 4$ Mc. When the first resonator is detuned by more than ± 4 Mc, oscillations at two frequencies arise in the second; one of these is the same as the frequency in the first, whereas the other is determined by the tuning of the second and is self-generated. The range in which this mode of generation persists is ~ 0.5 Mc of detuning of the first resonator. The region from 4.5 to 5 Mc and from -4.5 to -5 Mc, indicated by the lines with diagonal strokes in Fig. 2, shows the range of existence of beats with frequency $\Omega = |\omega - \omega_I|$. Upon further detuning of the first resonator, the oscillations within it disappear, and in the second there remains the self-generation at a frequency determined by the second resonator. The system reverts to single resonator operation,[7] the first resonator being turned off, as it were.

The dependence of the power W_2 of the oscillations in the second resonator on the following parameters was investigated:

1) detuning of the first resonator $\Delta\omega = |\omega_0 - \omega_1|$;
2) voltage on the state separator V;
3) pressure at the source of the molecular beam p.

The dependence of the power W_2 on $\Delta\omega_1$ for various voltages on the state separator V for a constant value of the pressure at the beam source p is shown in Fig. 2.

From this figure it can be seen that the character of the dependence $W(\Delta\omega_1)$ changes with the magnitude of the voltage V. Whereas for small V there the frequency of the second resonator is determined by the first resonator and depends on $\Delta\omega_1$ monotonically over the entire detuning range of the first resonator $\Delta\nu_1 \approx 10$ Mc, then when V is increased and at large detunings of the order

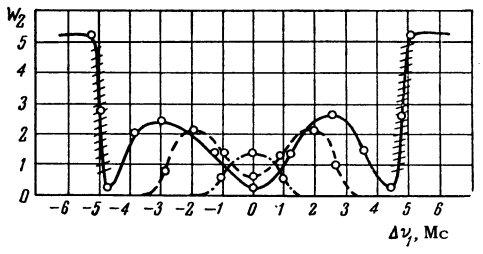

FIG. 2. Dependence of the power of the oscillations in the second resonator on $\Delta\nu_1$ for various voltages V. Solid line—V = 18 kV, dashed line—V = 14 kV, dot-dashed line—V = 9 kV.

$\Delta\nu_1 \approx 4.5$ Mc, a new mode of operation appears—two-resonator, and for still larger detunings (~5 Mc) the system reverts to one-resonator operation. In addition, for large values of V the power W_2 depends on $\Delta\omega_1$ monotonically. The maximum in W_2 is shifted from the point $\Delta\omega_1 = 0$ to a point where $\Delta\nu_1 \approx \pm 2$ Mc. For small V no beats are observed in the second resonator. The absence of this mode for small V can be explained by the fact that the conditions for self-generation in the second resonator are fulfilled for larger values of V than in the first because of the large distance of the second resonator from the state selector.

The dependence of W_2 on V for $\Delta\nu_1 = 0$ and $\Delta\nu_1 \approx \pm 2$ Mc, where the power is a maximum, is interesting and is given in Fig. 3. It can be seen that W_2 at $\Delta\nu_1 \approx \pm 2$ Mc depends monotonically on V, just as the power of the radiation in the first resonator depends on V. The dependence of $W_2(V)$ for $\Delta\omega_1 = 0$, however, is altogether different. At first it increases, then it begins to fall and becomes zero for some value V_K, and then it begins to increase anew. Beginning at V = 22 kV the growth slows down.

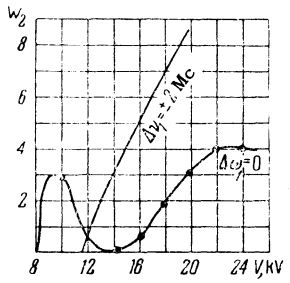

FIG. 3. Dependence of the power W_2 on the voltage V on the state selector for different $\Delta\omega_1$.

All the curves presented (Figs. 2 and 3) were taken at a source pressure of 0.6 mm Hg. For other values of the pressure p the character of $W_2(p)$ does not change.

The dependence of W_2 on pressure at the source of the beam p for $\Delta\omega_1 = 0$ and $\Delta\omega_2 = 0$ for different V is presented in Fig. 4. For moderate V (~9 kV) this dependence is the same as the dependence of the generated power on the pressure in a one-resonator system. For larger V this resemblance vanishes. Dips appear in $W_2(p)$ in which W_2 goes to zero.

The question of the energy states of the beam at the outputs of the first and second resonators was investigated. In order to do this for the NH_3 beam passing through the first and second resonators, a third resonator was added on the same axis (see Fig. 1).

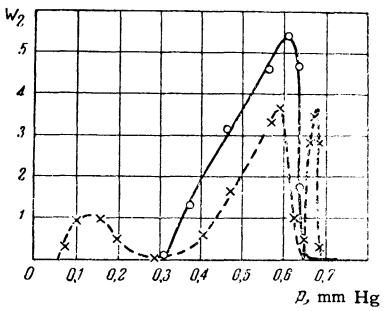

FIG. 4. Dependence of the power W_2 on the pressure p at the beam source. Solid line—V = 9 kV, broken line—V = 16 kV.

The stimulated emission or absorption line could be observed by the usual radiospectroscope using as an "illuminating" signal the power from a quartz amplifier modulated in frequency.

By this means it was found that the beam in the third resonator, after passage through the first and second resonators, had strong microwave absorption when $\Delta\omega_1 \approx 0$ and $\Delta\omega_2 \approx 0$. The absorption increased with increasing voltage on the selector system.

The expression "strong absorption" here means that the absorption in the third resonator when $W_2 \neq 0$ and $W_1 \neq 0$ is much greater than the absorption in an unsorted beam (when V = 0) in the same resonator.

Similar investigations were carried out on the beam in the second resonator after it had passed through the first. It turned out that the beam in it also absorbed strongly, but only for certain values of the voltage on the selector and pressure at the beam source. For these values of the voltage and pressure, V_K and p_K, W_2 goes through zero. It is interesting that for sufficiently marked deviations from p_K and V_K the beam absorbs, but W_2 in the second resonator has a rather significant magnitude.

3. INTERACTION WITH WEAK ELECTRIC AND MAGNETIC FIELDS

A phenomenon which seems strange at first was observed in working with the ammonia beam maser (line J = K = 3): a weak electric (hundreds of V/cm) and magnetic (~1 Oe/cm) field, acting on the beam of active molecules outside the resonator, is found to affect strongly the state of the beam of active molecules. This effect is observed by the change in induced emission, power, and frequency of the maser.

Measurements were carried out in the apparatus depicted schematically in Fig. 1. The fields were applied in the space between the selector and the

first resonator, and between the first resonator and the second. The electric field was created in a capacitor (length along beam, 3 cm, separation between plates, 8 mm), between the plates of which passed the beam. The magnetic field was produced in the gap of an electromagnet, of dimensions 1 × 1 cm. Penetration of the fields into the resonators was prevented by screening, which was verified also by increasing the distance between the capacitor (or magnet) and the resonator to 15 cm.

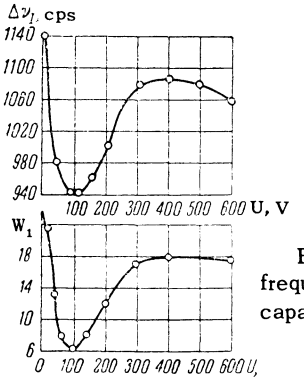

FIG. 5. Dependence of power W_1 and frequency ω on the voltage U on the capacitor.

Figure 5 shows the dependence of the power and frequency of the oscillations in the first resonator on the magnitude of the voltage U applied to the capacitor plates. The polarity of U had no effect on this dependence. The power falls sharply at first, then attains a maximum and begins to drop again with further increase in U. A periodicity with increasing period can be clearly discerned in these changes. In order to clarify the behavior of the maser at large voltages, a short selector system was put in place of the capacitor described above. For small voltages (up to 1000 V) the dependence was periodic, approximately like that of Fig. 5. But with U ~ 1 kV a monotonic growth with increasing U began, as with the usual selector.

The dependence of the induced emission on U was studied. The dependence turned out to be similar to the dependence of W_1 on U shown in Fig. 5. No significant effect of U on the line width of induced emission was observed.

If an ac voltage $U = U_0 \cos \Omega t$ is applied to the capacitor instead of dc, the power and frequency of the maser become modulated at a frequency 2Ω (in the absence of a constant bias) or Ω (for $U_{bias} \geq U_0$). The depth of modulation depends on U_0 and the frequency Ω. Up to a frequency of the order of $\Omega/2\pi = 1$ kcs, the depth of modulation does not depend on Ω. On further increase in Ω it decreases and becomes equal to zero at $\Omega/2\pi \approx 5.5$ kcs. At frequencies greater than this, the power of the maser depends on $U = U_0 \cos \Omega t$, just as with a steady voltage. The disappearance of modulation at large Ω is obviously connected with the establishment time of the oscillations in the maser.

The dependence of the power W_2 on the voltage U applied to a capacitor mounted in the space between the resonators is shown in Fig. 6. Figure 6a shows the changes in $W_2(V)$ for various U. The dependence of W_2 on U for various V is shown in Fig. 6b. It is characteristic that the power W_2 is already down to zero at small values of $U \approx 60$ V and does not appear again at U > 60 V. The polarization of the beam, governed by the first resonator, is obviously destroyed under the influence of the voltage U and therefore the power associated with it W_2 vanishes. The dependence of the power on H is stronger both in the first and in the second resonator (Fig. 7). The periodicity here is clearly expressed. If the generator is weakly excited (small beam or low voltage on the selector), then turning on the magnetic field (always a few oersteds) between the selector and the resonator increases the power generated in it by approximately an order of magnitude (curve 1 in Fig. 7). The power in the second resonator, on the other hand, decreases by an order of magnitude when the same field is applied between the resonators.

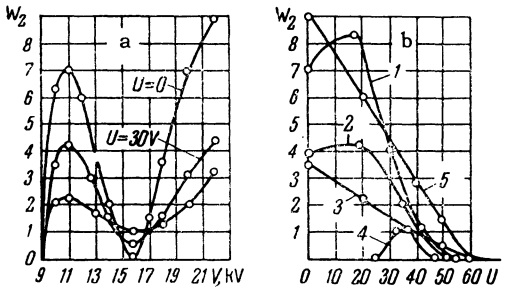

FIG. 6. Dependence of power W_2 on U for different V. In Fig. 6b, curve 1 is for V = 10.6 kV; 2, V = 13 kV; 3, V = 18 kV; 4, V = 15.6 kV; 5, V = 22 kV.

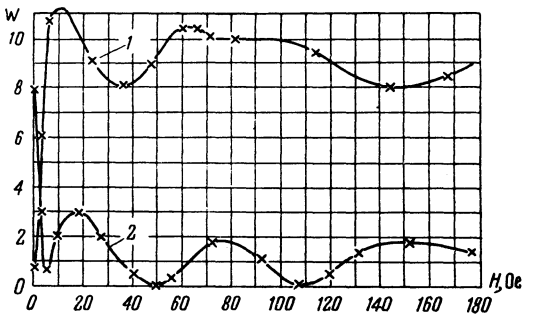

FIG. 7. Dependence of the powers W_1 and W_2 on the magnetic field H.

Investigations were also carried out of the effect of fields on the frequency of oscillation both in the first and in the second resonators. Electric and magnetic fields placed before the first resonator can change the frequency in it by a few hundred cps. In this it was observed that the character of the dependence of frequency of the maser on U and H and the maximum change in frequency depend on what frequency the resonator is tuned to. There exists only one frequency of the resonator at which electric and magnetic fields have the least effect on frequency. We are presently investigating the problem of what determines this resonator frequency and how much it differs from the frequency at the peak of the stimulated emission line. In Fig. 5 is shown the dependence of the maser frequency on U for a rather large detuning of the resonator (~ 2 Mc) relative to the frequency of the resonator for which the dependence on U and H is a minimum. A strong effect of fields acting on the beam between resonators on the frequency of radiation in the second resonator was not found, in contradiction to the assertion of Reder and Bickart.[2]

4. INTERPRETATION OF THE EXPERIMENTAL RESULTS

The experimental results given in Sec. 2 fit well the ideas briefly presented in the introduction.

The dependence of the amplitude of the oscillations in the first and second resonators on external (electric and magnetic) fields (Sec. 3) seems paradoxical, on the other hand, and requires additional analysis. In fact, how can a weak electric field between the resonator and selector (0—300 V/cm) strongly alter the power and frequency of the maser, whereas a strong field (10000—15000 V/cm) does not change it significantly? How does a magnetic field in the same place cause so significant an increase in power?

It might be assumed that the distribution of molecules among the energy levels is changed during their flight through electric and magnetic fields as a result of transitions from upper energy levels to lower ones. But in order for such transitions to occur with sufficient intensity, the condition of non-adiabaticity[8] must be fulfilled; this we can conveniently write in our case

$$\Delta\mathcal{H}/\tau\hbar\omega^2 \gg 1, \qquad (1)$$

where $\mathcal{H}$ is the Hamiltonian of the interaction between the molecule and the field, and τ is the time of flight through the field. The frequency of the maser transition is $\approx 10^{11}$ cps, the dimensions of the zones with inhomogeneous field—of order unity—is several cm, and for molecular speeds of 5×10^4 to 10^5 cm/sec, the condition (1) is never fulfilled. On the other hand, even in the maximum fields attained in real experiments, when $\Delta\mathcal{H} \approx \hbar\omega$, we have just the inverse relation

$$\Delta\mathcal{H}/\tau\hbar\omega^2 \ll 1. \qquad (2)$$

Therefore the distribution of molecules among the energy levels during flight through an inhomogeneous field can not change significantly in our experiments.

An explanation of our experiments can be found if account is taken of the spatial degeneracy of the energy levels of the molecule with respect to the momentum projection M. The beam of molecules passing through the selector no longer have isotropic orientation in space, since those molecules are most effectively sorted which have the maximum projection of the moment on the direction of the selector field. An additional field applied in the path of the molecular beam will change the orientation of the molecules in space by changing the proportion of molecules with different projection M. Since the intensity of radiation also depends on the mutual orientation of a molecule and the field in the resonator, it is clear that the external field will lead to a change in intensity of the radiation of the molecules in the resonator.

Let Ψ_{JM} be the wave function of an energy level with momentum J and various values of M. The wave function in a magnetic field will be a linear combination of the Ψ_{JM}:

$$\Psi_J = \sum a_M^J \Psi_{JM}.$$

The coefficients a_M^J are determined from the equation

$$i\hbar \frac{da_M^J}{dt} = -\mu_{MM}^{(z)} H_z a_M^J - \mu_{M,\,M-1}^{(x)} (H_x - iH_y)\, a_M^J$$

$$-\mu_{M,\,M+1}^{(x)} (H_x - iH_y)\, a_{M+1}^J, \qquad (3)$$

where $\hat{\mu}$ is the magnetic moment of the molecule, and H is the external magnetic field.

Considering that

$$\mu_{M,\,M}^{(z)} = \mu_0 M/J\,(J+1),$$

$$|\mu_{M,\,M\pm1}^{(x,\,y)}| = \frac{\mu_0}{J\,(J+1)} \sqrt{(J+M)\,(J-M+1)},$$

where μ_0 is a constant independent of M, the solution to the system of equations (3) can be written in the form

$$a_M^J = \sum_M A_M \exp\left(i \int_0^t \lambda_M\,(t)\,dt\right), \quad \lambda_M\,(t) = \frac{\mu_0 M}{\hbar J\,(J+1)} \mathbf{H}^2\,(t). \qquad (4)$$

We consider the simplest case, when J = 1 (M = 0, ±1). Let all molecules leaving the selector be in the state M = 1. Then, after passing through the field the probability of having the projection M = 1 is, according to Eq. (4), equal to

$$|a_1|^2 = \cos \int_0^l \lambda_1(x)\,\frac{dx}{v}, \tag{5}$$

where l is the length of the system exposed to the field, and v is the velocity of the molecule.

To have a significant change in $|a_1|^2$ it is necessary that

$$\int_0^l \lambda_1(x)\,\frac{dx}{v} \approx 1. \tag{6}$$

In ammonia the magnetic moment is of the order of a nuclear magneton, so that in passing through a field section 1 cm long, it is necessary to have a magnetic field of ~ 1 Oe in order to fulfill the condition (6); this agrees in order of magnitude with the experimental results (see the first maximum in Fig. 7).

The dependence of $|a_M^J|^2$ on the magnitude of the field bears a periodic character. However, averaging Eq. (5) over the velocities gives a "damping" of the oscillations with increasing period, which is also observed in experiment. Just so it can be shown that the dependence of the intensity of the radiation in the second resonator on the strength of a magnetic field placed between the resonators has a damped oscillatory character.

The interpretation of the change in intensity with the strength of a weak electric field can also be based on the change of the spatial orientation of the molecule. But this is more complicated than the magnetic field interaction. This is because in ammonia there are no matrix elements of the electric dipole moment that are diagonal in the index characterizing the inversion. Therefore the mechanism of reorientation is the following: The molecule obtains a weak polarization in the external field, and already because of this polarization spatial orientation of the molecule occurs under the influence of an inhomogeneous field. Let Ψ_M^+ be the wave function for the upper energy state and Ψ_M^- for the lower. Then, according to the theory of adiabatic perturbations, we obtain for the coefficients a_M^+, which determine the probability of a specific orientation of the molecule in space, the equation (see [9])

$$i\frac{da_M^{\pm}}{dt} = \frac{1}{\hbar\omega_{+-}}\left(\frac{\partial \mathcal{H}_{\text{int}}}{\partial t}\right)_{M,\,M-1} a_{M-1} e^{i\Phi_{M,\,M-1}} + \frac{1}{\hbar\omega_{+-}}\left(\frac{\partial \mathcal{H}_{\text{int}}}{\partial t}\right)_{M,M+1} a_{M+1} e^{i\Phi_{M,M+1}},$$

$$\Phi_{MM'} = \int_0^t \omega_{MM'}(t)\,dt, \tag{7}$$

where $\mathcal{H}_{\text{int}}$ is the Hamiltonian of the interaction between field and molecule, equal to $\hat{\mu}\,E(x) = -\hat{\mu}\,E(vt)$, where $\hat{\mu}$ is the electrical dipole moment operator; ω_{+-} is the frequency of the ammonia inversion transition, and $\hbar\omega_{MM'}$ is the difference in energy of the Stark splitting in a field E(x) at a given point x for different values of M. For a system with two energy levels

$$\omega_{M,\,M-1} = \omega_{+-}\left\{\left[1 + \frac{4\mu_0}{\hbar\omega_{+-}}\frac{K^2M^2}{J^2(J+1)^2}E^2(x)\right]^{1/2} - \left[1 + 4\mu_0\frac{K^2(M-1)^2}{J^2(J+1)^2}\right]^{1/2}\right\}, \tag{8}$$

where μ_0 is the magnitude of the dipole moment of the molecule; J and K are the rotational quantum numbers.

For small fields

$$\omega_{M,\,M-1} = 2\frac{\mu_0^2K^2}{J^2(J+1)^2}E^2(x)\,(2M-1). \tag{9}$$

It is not possible to solve Eq. (7) in general form. However, in order of magnitude the probability of transition to a state M ± 1 if the molecule is initially in state M equals

$$a_{M\pm1}|^2 = \frac{1}{\hbar\omega_{+-}}\frac{\mathcal{H}_{max}}{\tau}\int_0^{\tau} e^{i\Phi_{M,\,M-1}}\,dt, \tag{10}$$

where $\mathcal{H}_{\max}$ is the maximum value of the Hamiltonian $\mathcal{H}_{\text{int}}$.

For ammonia $\mu_0 \approx 10^{-18}$ cgs esu, so that in fields of order 100 V/cm extended over 1 cm a 100% reorientation of the molecules is possible.

The reorientation proceeds principally in weak fields, since in strong fields, where $\omega_{M,\,M\pm1}$ is comparable to ω_{+-}, the conditions for adiabaticity are fulfilled even for transitions between states with different M.

The action of fields on the molecular beam coming out of the first resonator cannot affect the frequency of oscillation in the second resonator if the field does not penetrate the latter. In fact, for a uniform field between the resonators only the phase of the polarization changes by a constant factor proportional to

$$\frac{1}{\hbar v}\int_0^l \mathcal{H}_{\text{int}}(x)\,dx,$$

because only the phase of the radiation in the second resonator is changed.

However, time-modulation of the field will produce amplitude modulation of the radiation in the second resonator.

We wish to thank A. M. Prokhorov for fruitful discussions and valuable advice.

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