SCREENING IN PAIR CONVERSION

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The effect of screening on the pair conversion coefficient is taken into account in first order perturbation theory.

THE existing calculations of the coefficient of pair conversion (CPC) have been carried out without taking into account the effect of the screening by the atomic electrons. However, one might think that the screening has a significant effect on the magnitude of the CPC for small transition energies. In the present paper we take account of the effect of the screening by an approximate method using the Thomas-Fermi potential. [1]

Let us represent the potential of the electrons of the shell as a sum of Yukawa potentials: [1]

$$V_1 = -\alpha Z \gamma_l V_{\lambda_l} = -\frac{\alpha Z}{r} \sum_{l=1}^{4} a_l e^{-\lambda_l r},$$

$$\lambda_l = b_l \nu, \quad \nu = \frac{m Z^{1/3}}{121}, \quad \sum_{l=1}^{4} a_l = 0. \qquad (1)$$

The values of a_l and b_l are given in a paper of one of the authors. [1] It is easily seen that (1) is proportional to λ and is hence of order ν.

The wave function of the electron in the field of the atom can, in first approximation in the potential (1), be written in the form

$$|\psi_p\rangle = |\varphi_p\rangle + G_p V_1 |\varphi_p\rangle + O(V_1 V, V_1^2), \qquad (2)$$

where $|\varphi_p\rangle$ is the exact wave function in the Coulomb field of the nucleus V, and G_p is the Green's function for the free Dirac equation.

The amplitude for the pair conversion with function (2) is written in the form

$$W = \langle \psi_{p_-} | \hat{B}_{Lm} | \psi_{-p_+} \rangle = \langle \varphi_{p_-} | \hat{B}_{Lm} | \varphi_{-p_+} \rangle$$

$$- \alpha Z N_+ N_- \gamma_l \{\langle p_- | B_{Lm} G_{p_+} V_{\lambda_l} | -p_+ \rangle$$

$$+ \langle p_- | V_{\lambda_l} G_{p_-} \hat{B}_{Lm} | -p_+ \rangle\} + O(\alpha^2 Z^2 \lambda), \qquad (3)$$

$$N_\pm = 2\pi\xi_\pm / (1 - e^{-2\pi\xi_\pm}), \quad \xi_\pm = \mp \alpha Z E_\pm / p_\pm. \qquad (4)$$

Here B_{Lm} denotes the potential of the multipole, and $E_\pm$ and $p_\pm$ are the energies and momenta of the electron and the positron. The first term in (3) represents the matrix element for the CPC in the nuclear Coulomb field. The second term takes account of the effect of the screening; we have expanded the functions $|\varphi_{\mp p_\pm}\rangle$ in this term in powers of αZ, keeping, however, the exact normalization factors $N_\pm$ which depend on the quantities $\xi_\pm$, which are large for small $p_\pm$.

The matrix elements in the second term of (3) were calculated for arbitrary λ. [2] Expanding these expressions in the small parameter λ and keeping only the first term, we obtain the following expressions for the differential CPC of the electric and magnetic types:

$$\beta^{ML}_{E_+\vartheta} = \beta^{V}_{E_+\vartheta} + N_+^2 N_-^2 \alpha Z (\gamma_l \lambda_l) L (E_+/p_+^2 - E_-/p_-^2) \beta^0_{E_+\vartheta}$$

$$+ \alpha Z (\gamma_l \lambda_l) \frac{2\alpha}{\pi} \frac{p_+ p_- \cos\vartheta}{q} N_+^2 N_-^2 \left\{\left[\frac{q^{2L+1}}{\omega^{2L+1}} \left(\frac{E_+}{\Delta^2} + \frac{E_+ - E_-}{2p_-^2 \Delta}\right)\right.\right.$$

$$+ \frac{q^{2L-1}}{\omega^{2L+1}} \frac{E_-}{p_-^2} \left(\frac{E_-\omega}{\Delta} - \frac{2E_-\omega (p_-^2 + p_+^2)}{\Delta^2} - \frac{4E_- m^2 \omega^3}{\Delta^3}\right)$$

$$+ \frac{q^{2L-3}}{\omega^{2L+1}} \frac{E_-}{p_-^2} \left(-\frac{L\omega (E_+ - E_-)}{4} + \frac{L\omega (E_+ - E_-)(p_-^2 + p_+^2)}{2\Delta}\right.$$

$$\left.\left.\left. + \frac{L m^2 \omega^3 (E_+ - E_-)}{\Delta^2}\right)\right] - (2 \rightleftarrows 1)\right\}, \qquad (5)$$

$$\beta^{EL}_{E_+\vartheta} = \beta^{V}_{E_+\vartheta} + N_+^2 N_-^2 \alpha Z (\gamma_l \lambda_l) \left(\frac{E_+}{p^2_+} - \frac{E_-}{p^2_-}\right) \beta^0_{E_+\vartheta}$$

$$+ N_+^2 N_-^2 \alpha Z (\gamma_l \lambda_l) \frac{2\alpha}{\pi} \frac{p_+ p_- \cos\vartheta}{q}$$

$$\times \left\{\left[\frac{q^{2L+1}}{\omega^{2L+1}} \frac{2Lm^2 E_+}{p_-^2 \Delta^2} - \frac{q^{2L+1}}{\omega^{2L}} \left(\frac{2L}{\Delta^2} + \frac{L}{2p_-^2 \Delta}\right)\right.\right.$$

$$+ \frac{q^{2L-1}}{\omega^{2L}} \left(\frac{L}{2p_-^2} - \frac{2LE_-^2}{p_-^2 \Delta} - \frac{4E_-^2}{p_-^2} \left(\frac{3L+1}{4} \omega^2 - 2LE_+E_-\right)\right.$$

$$\left. \times \frac{1}{\Delta^2} + \frac{8L\omega^3 E_+ E_-^2}{p_-^2 \Delta^3}\right) + \frac{q^{2L-1} E_-}{\omega^{2L-1} p_-^2} \left(\frac{2Lm^2}{\Delta^2} - \frac{L(E_+ - E_-)}{2E_-\Delta}\right)$$

$$+ \frac{q^{2L-1}}{\omega^{2L-2}} \frac{E_-}{p_-^2} \left(\frac{(L+1) m^2}{E_-\Delta^2} + \frac{2LE_-}{\Delta^2} - \frac{L}{E_-\Delta}\right)$$

$$+ \frac{q^{2L-3}}{\omega^{2L}} \frac{E_-}{p_-^2} \left(\frac{L^2}{2} + L\left(\frac{3L+1}{2} \omega^2 - 2LE_+E_-\right) \frac{1}{\Delta}\right.$$

$$- \frac{2L\omega^2 E_+ E_-}{\Delta^2}\Big)(E_+ - E_-) + \frac{q^{2L-3}}{\omega^{2L-2}} \frac{E_-}{p_-^2} \Big(- \frac{(3L+1)E_-}{\Delta}$$

$$+ \frac{4E_-[(L+1)E_+E_- + L\omega^2 + (L+1)m^2]}{\Delta^2}$$

$$- \frac{4E_-\omega^2[2LE_+E_- + (L+1)m^2]}{\Delta^3} + \frac{q^{2L-5}}{\omega^{2L-2}} \frac{E_-}{p_-^2} \Big(\frac{3L+1)L}{4}$$

$$- \frac{L[(L+1)E_+E_- + L\omega^2 + (L+1)m^2]}{\Delta}$$

$$+ \frac{\omega^2 L[2LE_+E_- + (L+1)m^2]}{\Delta^2}\Big)$$

$$\times (E_+ - E_-)] - (2 \rightleftarrows 1)\Big\}, \qquad (6)$$

where

$$\Delta = \omega^2 - q^2, \ \omega = E_+ + E_-, \ \mathbf{q} = \mathbf{p}_+ + \mathbf{p}_-,$$

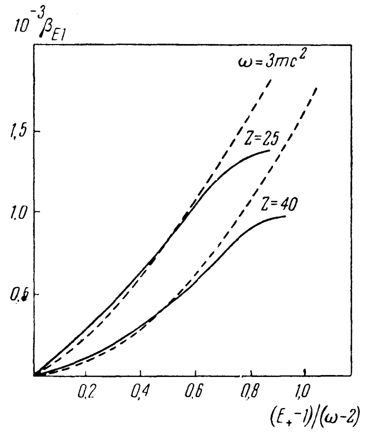
$$\cos\vartheta = \mathbf{p}_+\mathbf{p}_-/p_+p_-,$$

and $\beta^{V}_{E+\vartheta}$ and $\beta^{0}_{E+\vartheta}$ denote the CPC calculated with account of the nuclear Coulomb field V and in the first Born approximation, respectively. In accordance with (1), the symbol $(\gamma_l \lambda_l)$ denotes

$$(\gamma_l \lambda_l) = \sum_{l=1}^{4} a_l \lambda_l = 1.35\,\nu. \qquad (7)$$

It is easily seen from (3) and (4) that the effective expansion parameter in the expansion λ is the quantity $\delta = \alpha Z E \lambda / p^2$. Therefore, (3) and (4) are not applicable at the limits of the spectrum ($p_\pm \to 0$). However, when $p_\pm \to 0$, the behavior of the CPC is determined by the factor N_+ [Eq. (4)], which "subdues" all divergent terms.[1)] Thus, only the behavior of the CPC near $p_- \to 0$ remains indeterminate.

[1)]The terms with δ in (3) and (4), which diverge as $p_\pm \to 0$, come from an expansion of a finite factor $F(\delta)$, which can be obtained by summing over the remaining terms in (3). This summation does not, however, affect the factor $N_\pm$, and the CPC will, as before, go to zero as $p_\pm \to 0$.



Integrating (3) and (4) over ϑ, we can obtain the energy spectrum, which has a very complicated form in the general case of arbitrary multipolarity.

The figure shows the energy spectrum for an electric dipole transition with account of the screening (solid curves) and the nonrelativistic results for the Coulomb field of the nucleus of Rose and Uhlenbeck[3] (dotted curves). It is seen from the figure that the inclusion of the screening effect leads to a significant distortion of the spectra of the transitions. For transition energies ω > 2.5 MeV the effect of the screening is negligibly small.

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[2] V. A. Krutov and V. G. Gorshkov, JETP **39**, 591 (1960), Soviet Phys. JETP **12**, 417 (1961).

[3] M. E. Rose and G. E. Uhlenbeck, Phys. Rev. **48**, 211 (1935).

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