THEORY OF MULTIPLE SCATTERING

Yu. N. GNEDIN and A. Z. DOLGINOV

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

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A theory is proposed for multiple scattering of particles passing through a body of finite dimensions. The quantum mechanical equations for multiple scattering are employed. It is assumed that the force centers are randomly distributed and that $N_0 a^2 \lambda \ll 1$, where λ is the wavelength of the incident particle, a is the scattering amplitude, and N_0 is the concentration of the force centers. An explicit expression is obtained for the intensity of scattered particles inside and outside the body.

INTRODUCTION

A. We consider here the multiple scattering and absorption of particles passing through a body of finite dimensions (plate, sphere, etc.). Such problems are usually solved by means of the transport equation [1,2] with given boundary conditions. Strictly speaking, multiple scattering must be described by means of the quantum equation for the density matrix. In a series of researches [3,4], a derivation of the quantum kinetic equation has been proposed for nonstationary processes, for which the boundary conditions do not play a significant role. In these researches, in which stationary processes were considered, the conditions of applicability of the kinetic equation were not obtained [5] or were incorrectly estimated. [6]

We shall start out from the equations of quantum mechanics, [7] and shall show under what conditions they reduce to an equation which can be called the kinetic equation for multiple scattering; we shall give the conditions for its applicability and a method for finding corrections to it, and shall make clear the character of the dependence of the solution on the dimensions of the scattering system. Only that case will be considered in detail for which the scattering amplitude a is much less than the distance between the force centers $N_0^{-1/3}$, while the wavelength of the incident particles λ is such that $N_0 |a|^2 \lambda \ll 1$. Moreover, we shall assume that λ is much less than the dimension of the system R.

B. In spite of the abundance of literature devoted to the solution of the transport equation for bodies of finite dimensions, the solution of the equation in closed form for an isotropic scattering law (let alone for the anisotropic case) is still lacking at the present time. The fundamental methods for its solution consist either in the numerical integration of the given equation, [1,2] or in the reduction of it to another equation that is suitable for numerical solution. [8] Meanwhile, a solution in closed form is of considerable interest both for neutron physics [1] as well as for astrophysics. [2] As examples, we consider the following problems: multiple scattering of neutrons in bodies of arbitrary dimension; light scattering in clouds of gas or dust; transmission of radiation or particles through the atmospheres of stars and planets, etc.

In the present work, the resultant equation is analyzed, and a method of its solution is pointed out. An approximate solution is obtained in closed form for the isotropic case.

1. STATEMENT OF THE PROBLEM

The equations for the wave function of a particle whose interaction with the surrounding medium is determined by the potential

$$V = \sum_n V_n, \qquad V_n \equiv V_n(\mathbf{r} - \rho_n), \tag{1}$$

have the form

$$\Psi(\mathbf{p}) = \delta(\mathbf{p} - \mathbf{p}_0) + G_0(p)\, T(\mathbf{p}\mathbf{p}_0), \tag{2}$$

$$G_0(p) = (2\pi^2)^{-1} (p^2 - p_0^2 - i\eta)^{-1}; \qquad \eta \to 0; \tag{3}$$

$$\sum_n T_n(\mathbf{p}_1\mathbf{p}_0) = T(\mathbf{p}_1\mathbf{p}_0), \tag{4}$$

$$T_n(\mathbf{p}_1\mathbf{p}_0) = t_n(\mathbf{p}_1\mathbf{p}_0) + \int d\mathbf{p} G_0(p)\, t_n(\mathbf{p}_1\mathbf{p}) \sum_{m \neq n} T_m(\mathbf{p}\mathbf{p}_0); \tag{4'}$$

$$t_n(\mathbf{p}_1\mathbf{p}_0) = -4\pi^2 m \left[V_n(\mathbf{p}_1 - \mathbf{p}_0) + \int d\mathbf{p} G_0(p)\, V_n(\mathbf{p}_1 - \mathbf{p})\, t_n(\mathbf{p}\mathbf{p}_0) \right]. \tag{5}$$

Here $V_n(\mathbf{p})$ is the Fourier transform of $V_n(\mathbf{r} - \rho_n)$; $\mathbf{p}$ is the momentum of the particle; m is its mass;

ρ_n is the coordinate of the n-th force center; units are used in which $\hbar = 1$; $T(\mathbf{p_1 p_0})$ is the amplitude of transition of the particle from the state $\mathbf{p_0}$ to the state $\mathbf{p_1}$ as the result of multiple scattering; $t_n(\mathbf{p_1 p_0})$ is the amplitude of single scattering by the n-th center. Assuming that all the scatterers are identical and immovable, and taking (5) into account, we get

$$t_n(\mathbf{p_1 p_0}) = a(\mathbf{p_1 p_0}) \exp[-i(\mathbf{p_1} - \mathbf{p_0})\rho_n], \quad (6)$$

where $a(\mathbf{p_1 p_0})$ no longer depends on n. All inelastic processes will be considered as absorption. We shall regard the scattering as purely elastic, i.e., $\mathbf{p_1} = p_0 \mathbf{n_1}$; $\mathbf{p_0} = p_0 \mathbf{n_0}$.

We shall assume that the scattering body (the target) contains N force centers. The intensity of flow of particles scattered by the target, at large distances from it, is given by

$$I = \sum_{n=1}^{N} \sum_{k=1}^{N} \langle T_n(\mathbf{p_1 p_0}) T_k^+(\mathbf{p_1 p_0}) \rangle. \quad (7)$$

Here $\langle\ \rangle$ denotes averaging over the locations of the force centers. Inasmuch as the latter are randomly located, averaging over the position of the n-th center reduces to integration over ρ_n and division by the volume of the target Ω. It is convenient to introduce the function

$$F(\mathbf{q} - \mathbf{p}) = \langle \exp(-i(\mathbf{q} - \mathbf{p})\rho) \rangle = \frac{1}{\Omega} \int_{\Omega} d\rho \exp(-i(\mathbf{q} - \mathbf{p})\rho). \quad (8)$$

For all macroscopic bodies, (8) is a spike function with a height equal to unity and width $q_x - p_x \sim L_x^{-1}$, etc., where L_x are the dimensions of the body along the x axis.

Values of ρ_n identical with ρ_k can occur in Eq. (7) under the averaging sign. Each such coincidence decreases the number of integrations over ρ_0 by one. Those terms for which all ρ_n and ρ_k are different describe the coherent scattering; the remainder describe the noncoherent.

$$I = I_{\text{coh}} + I_{\text{inc}}, \qquad I_{\text{coh}} = \sum_{nk} \langle T_n(\mathbf{p_1 p_0}) \rangle \langle T_k^+(\mathbf{p_1 p_0}) \rangle. \quad (9)$$

Averaging removes the dependence of $T_n(\mathbf{p_1 p_0})$ on the index n; therefore, the sum (9) reduces to multiplication by N^2.

2. COHERENT SCATTERING

We write out the right side of Eq. (4′) in the form of a series of successive approximations and average it. We get

$$\langle T_n(\mathbf{p_1 p_0}) \rangle = \langle t_n(\mathbf{p_1 p_0}) \rangle + \int d\mathbf{p}\, G_0(p) \times \langle t_n(\mathbf{p_1 p}) \sum_{m \neq n} [t_m(\mathbf{p p_0}) + \int d\mathbf{q}\, G_0(q)\, t_m(\mathbf{p q}) \sum_{l \neq m} t_l(\mathbf{q p_0}) + \ldots] \rangle \quad (10)$$

On the right side of (10), $m \neq n$ and $l \neq m$, but, possibly, $n = l$, etc.

Let us group separately those terms which do not contain indices of different n. In this group one can average $t_n(\mathbf{pq})$ independently of all the rest. Terms containing but one pair of identical indices will be joined by the dashed (connective) line. Using (6) and (8), we can represent (10) in the form

$$\langle T_n(\mathbf{p_1 p_0}) \rangle = a(\mathbf{p_1 p_0}) F(\mathbf{p_1} - \mathbf{p_0}) + N \int d\mathbf{p}\, G_0(p)\, a(\mathbf{p_1 p}) \times F(\mathbf{p_1} - \mathbf{p}) \langle T_m(\mathbf{p p_0}) \rangle + \int d\mathbf{p}\, G_0(p) \sum_{m \neq n} \langle t_n(\mathbf{p_1 p}) T_m(\mathbf{p p_0}) \rangle. \quad (11)$$

We shall show that the last term in the expression (11) is small and can be neglected. For this purpose, we again replace $T_m(\mathbf{p p_0})$ by a series of successive approximations. The first nonvanishing term with a connective will be

$$N \int d\mathbf{p} G_0(p) \int d\mathbf{q} G_0(q)\, a(\mathbf{p_1 p})\, a(\mathbf{p q})\, a(\mathbf{q p_0}) \times F(\mathbf{p} - \mathbf{q})\, F(\mathbf{p_1} - \mathbf{p} + \mathbf{q} - \mathbf{p_0}). \quad (12)$$

Equation (12) (and many other integrals with which we shall have to deal) contains the amplitudes $a(\mathbf{pq})$. We denote the cross section of elastic scattering by $\sigma(\mathbf{p_1 p_0}) = |a(\mathbf{p_1 p_0})|^2$, and the total cross section, including all inelastic processes, by $\sigma_0 = 4\pi p_0^{-1} \operatorname{Im} a(\mathbf{p_0 p_0})$.

For complex values of the energy, the amplitude $a(\mathbf{p p_0})$ can have poles of the type $(p^2 - p_r^2 - i\gamma)^{-1}$, which correspond to resonance levels. The quantities $G_0(p)$ and $F(\mathbf{p} - \mathbf{q})$, which also enter under the integral, are much steeper functions of p than are the amplitudes. By making use of this fact, we can remove the amplitude from under the integral. In what follows, we shall remove the amplitudes from under the integrals without special comment. In each such case, it has been verified by direct computation that the resultant error, which reaches a maximum value for coincidence of the energy of the particle E with the resonance E_r, does not exceed $N_0 \sigma_0 \lambda_r E_r \Gamma_r^{-1}$, where Γ_r is the width of the resonance level, while λ_r is the wavelength for the energy E_r. For example, in scattering of neutrons by the resonance level of In^{115}, for which $E_r = 1.44$ eV while $\Gamma_r = 0.08$ eV, the cor-

rection is of the order of $\sim 10^{-9}$ in comparison with unity.

The possible amplitude singularities of the branch point type do not change our result. They appreciably change only in the case in which the amplitudes have singularities that are more essential than simple poles.

Taking out the amplitudes and carrying out the integration, we get the following estimate for (12):

$$\sim N_0 a^2 \lambda a\,(\mathbf{p}_1\mathbf{p}_0)\,F\,(\mathbf{p}_1 - \mathbf{p}_0). \qquad (13)$$

The value of (13) is smaller than the first term of (11) by the factor $N_0 a^2 \lambda$. By estimating the successive approximations in exactly the same way, it is not difficult to verify that each term with the connective is smaller than the corresponding term without the connective by a factor $\sim N_0 \lambda a^2$.

We shall assume for definiteness that the target has the form of a sphere of radius R and set

$$\langle T_n\,(\mathbf{p}_1\mathbf{p}_0)\rangle = \frac{4\pi}{\Omega}\,a\,(\mathbf{p}_1\mathbf{p}_0)\sum_l (2l+1)\,K_l\,(p_0, p_0)\,P_l\,(\mathbf{n}_1\mathbf{n}_0). \qquad (14)$$

Discarding the last term in (11), we get the equation (A.7) for $K_l(p_0, p_0)$ (see Appendix), the solution of which will be (A.15):

$$\langle T_n\,(\mathbf{p}_1\mathbf{p}_0)\rangle = a\,(\mathbf{p}_1\mathbf{p}_0)\,e^{i\mu R}F\,(\mathbf{p}_1 - \mathbf{p}_0 - \mu\mathbf{n}_0),$$

$$\mu = 2\pi N_0 \lambda a\,(\mathbf{p}_0\mathbf{p}_0), \qquad 2\,\mathrm{Im}\,\mu = N_0\sigma_0 = \alpha. \qquad (15)$$

The expression (15) has a sharp maximum in the $\mathbf{n}_0$ direction:

$$I_{\mathrm{coh}}\,(\mathbf{p}_0\mathbf{p}_0) = N^2\,|\,a\,(\mathbf{p}_0\mathbf{p}_0)\,|^2\,e^{-\alpha R}\,|\,F\,(-\mu\mathbf{n}_0)\,|^2$$
$$= 16\pi^2 R^2 N_0^2\,\sigma\,(\mathbf{p}_0\mathbf{p}_0)\,e^{-\alpha R}\left|\frac{\cos\mu R}{\mu^2} - \frac{\sin\mu R}{\mu^3 R}\right|^2. \qquad (16)$$

If $R \to 0$, we get the well known expression $N^2\sigma(\mathbf{p}_0\mathbf{p}_0)$. For very large R, the intensity of the coherent scattering per unit area becomes a constant.

3. INCOHERENT SCATTERING

The intensity of incoherent scattering is determined by the density matrix

$$U\,(\mathbf{p}_1\mathbf{p}_0\mathbf{p}_3\mathbf{p}_2) = \sum_{nm}\langle \overbrace{T_n\,(\mathbf{p}_1\mathbf{p}_0)\,T_m^+\,(\mathbf{p}_3\mathbf{p}_2)}\rangle \qquad (17)$$

for $\mathbf{p}_2 = \mathbf{p}_0$, $\mathbf{p}_3 = \mathbf{p}_1$. In order to find the equation for (17), we represent the expression under the average sign in the form of a series of successive approximations, and then average and sum this series.

In order to carry out this program, it is convenient to introduce diagrams. We shall represent $\langle T(\mathbf{p}_1\mathbf{p}_0)\rangle$ and $\langle T^+(\mathbf{p}_3\mathbf{p}_2)\rangle$, respectively, by the

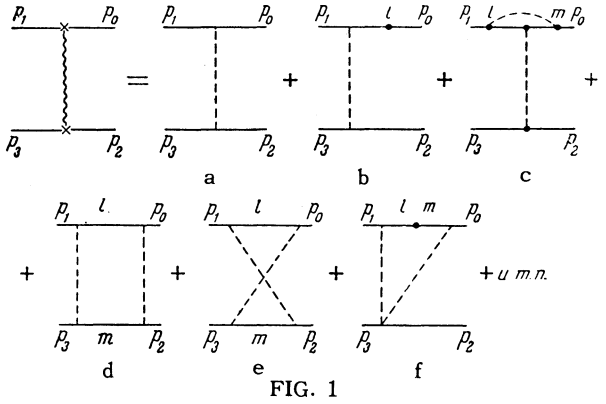

FIG. 1

upper and lower lines with a cross in the middle. We denote the quantity $\langle t(\mathbf{p}_1\mathbf{p}_0)\rangle$ by a line with a dot in the middle. To each part of the line is ascribed the momentum $\mathbf{p}_0, \mathbf{p}_1$, etc. The quantity $G_0(p)$ is associated with the interior segments

•—p—• , •—p—× or ×—p—× (18)

and integration is carried out over p. The connection of two points on different lines (see Fig. 1a) corresponds to

$$\langle \overbrace{t_n\,(\mathbf{p}_1\mathbf{p}_0)\,t_m^+(\mathbf{p}_3\mathbf{p}_2)}\rangle$$
$$= a\,(\mathbf{p}_1\mathbf{p}_0)a^+\,(\mathbf{p}_3\mathbf{p}_2)\,F\,(\mathbf{p}_1 - \mathbf{p}_0 - \mathbf{p}_3 + \mathbf{p}_2). \qquad (19)$$

The connection of a point with a cross or of two crosses has similar meaning. The set of possible connections of two lines is indicated by a wavy line. We represent the quantity

$$S\,(\mathbf{p}_1\mathbf{p}_0) = G_0^{-1}\,(p_1)\,\delta\,(\mathbf{p}_1 - \mathbf{p}_0) + N\,\langle T\,(\mathbf{p}_1\mathbf{p}_0)\rangle \qquad (20)$$

in the form of a segment with a cross in the middle. By means of these designations, Eq. (11) can be represented in the form

—×— = —•— + —•—×— + —•—×— , (21)

where the last term on the right is small in comparison with the sum of the first two. The expression (17) is the sum of diagrams (see Fig. 1) with an arbitrary number of points on each of the lines and all possible connections. In each of the diagrams there can be only a single connection between lines.

Diagrams with connections of points on one of the lines contain integrals of the type (12). In all cases, they are smaller than similar diagrams without connections by the factor $N_0|\,a\,|^2\lambda$ (see Fig. 1c and Fig. 1a). We shall neglect all diagrams of such a type. The same can be said relative to diagrams with intersecting connections (the types of Fig. 1e, f, etc.). For example, let us evaluate Fig. 1e. To it corresponds

$$N^2\int dl\int dm G_0(l)G_0^+(m)\,a(p_1l)\,a(lp_0)\,a^+(p_3m)\,a^+(mp_2)$$
$$\times F(p_1 - l + p_2 - m)$$
$$\times F(l - p_0 + m - p_3) \sim Na(p_1p_0)\,a^+(p_3p_2)$$
$$\times F(p_1 - p_0 - p_3 + p_2)$$
$$\times N_0|a|^2(|p_3 + p_0|)^{-1}\tan^{-1}(|p_3 + p_0|R). \qquad (22)$$

We see that (22) differs from (19) by a small factor of the order of $N_0|a|^2\lambda$. This is incorrect only for a narrow range of angles $\sim\lambda/R$ near $p_3 = -p_0$. By limiting the accuracy to within less than λ/R, and neglecting the corresponding diagrams, we need average in the estimates over the range of angles $\sim\lambda/R$. Then the smallness of (22) becomes obvious.

The expression for the diagram of Fig. 1d, whose connections do not intersect, differs from (22) by the argument of the F function, namely, $F(p_1 - l + m - p_3)\,F(l - p_0 + p_2 - m)$. Therefore, instead of (22), we get an expression which differs from (22) only by the substitution of $|p_0 - p_2|$ for $|p_3 + p_0|$. The value of this quantity is of the order of $N_0|a|^2R$, which is not small in comparison with unity.

By writing down the expressions for the different diagrams, it is not difficult to establish general rules which allow us to estimate without calculation. Let n_0 be the number of nonintersecting connections between lines and n_1 the number of intersecting connections. If the connection passes through more than two points, but is not interrupted (see, for example, Fig. 1e), it may be regarded as a single connection, but classified as intersecting.

The intersecting connections can be broken up into groups such that no one of the connections of one group intersects the connections of the others. The number of such groups is denoted by n_2. The numbers n_3 and n_3^+ are the numbers of points which the intersecting connections approach, n_4 and n_4^+ are the numbers of free points between intersecting connections (for example, the points between l and m in Fig. 1e). The numbers n_5 and n_5^+ are the numbers of remaining free points. The superscript + denotes that the point is located on the lower line. For example, for Fig. 1a, $n_0 = 1$ is different from zero; for Fig. 1b, $n_0 = n_5 = 1$; for Fig. 1c, $n_1 = 2$, $n_3 = 3$, $n_2 = n_3^+ = 1$; for Fig. 1d, $n_0 = 2$; for Fig. 1e, $n_1 - n_3 = n_3^+ = 2$, $n_2 = 1$; for Fig. 1f, $n_1 = n_2 = n_3^+ = n_4 = 1$, $n_3 = 2$, etc.

A diagram of general form is equal to $NN_0^{\alpha_1}a^{\alpha_2}(a^+)^{\alpha_3}\lambda^{\alpha_4}R^{\alpha_5}$ in order of magnitude, where

$$\alpha_1 = n_0 + n_1 + n_4 + n_4^+ + n_5 + n_5^+ - 1,$$
$$\alpha_2 = n_0 + n_3 + n_5,\ \alpha_3 = n_0 + n_3^+ + n_5^+,$$
$$\alpha_4 = 3(n_1 + n_4 + n_4^+) + n_5 + n_5^+ - n_3 - n_3^+ - n_2,$$
$$\alpha_5 = n_0 + n_2 + n_5 + n_5^+ - 1.$$

A similar method of averaging over the locations of the force centers for the Green's function of electrons in a superconductor, with account of scattering by impurity atoms, was proposed by Abrikosov and Gor'kov.[9] It should be noted that in our case, one must consider a rather broad class of diagrams, many of which are unimportant in the theory of superconducting alloys.[9]

We now proceed to the summation of the diagrams, which give the fundamental contribution. The following equality holds (Fig. 2a):

$$\sum_{nm}\langle t_n(p_1p_0)\,T_m^+(p_3p_2)\rangle = Na(p_1p_0)\int dp\,G_0(p)\int df\,G_0^+(f)$$
$$\times S^+(p_3p)\,a^+(pf)\,F(p_1 - p_0 - p + f)\,S^+(fp_2). \qquad (23)$$

For proof, we substitute a series of successive approximations for $T_m(p_3p_2)$ on the right side of (23) and make use of the fact that a dependence on the signs of n is lacking in the averaging. Making use then of (20), we verify the validity of (23). We can prove the equality indicated in Fig. 2b in an entirely similar way.

Taking Fig. 2b and Fig. 1 into account, we get the following equation for (17) (see Fig. 3):

$$U(p_1p_0p_3p_2) = \int dp\,G_0(p)S(p_1p)\int df\,G_0^+(f)\,S^+(p_3f)\,P(pp_0fp_2), \qquad (24)$$

$$P(pp_0fp_2) = P_0(pp_0fp_2) + \int dm\int dm'\,G_0(m)\,G_0^+(m')$$
$$\times P_0(pmfm')\,P(mp_0m'p_2), \qquad (25)$$

$$P_0(pp_0fp_2) = N\int dl\int dl'\,G_0(l)\,G_0^+(l')\,S(lp_0)\,S^+(l'p_2)$$
$$\times F(p - l - f + l')\,a(pl)\,a^+(fl'). \qquad (26)$$

The expression (25) is represented in Fig. 3a by a rectangle, while the first diagram in Fig. 3b is equal to (26). We substitute (20), (25) and (26) in (24). The resultant expression will contain integrals of the form

$$\int dp\,G_0(p)\,a(pl)\,\langle T(qp)\rangle\,F(p - s). \qquad (27)$$

In order to calculate them, use is made of the steepness of $F(p - s)$. By replacing $a(pl)$ by $a(ps)\,a(sl)\,[a(ss)]^{-1}$, we can take $a(sl)\,[a(ss)]^{-1}$ from under the integral. We are left with an integral equal to

(28)

which follows from the equation for $\langle T(p_1p_0)\rangle$ and

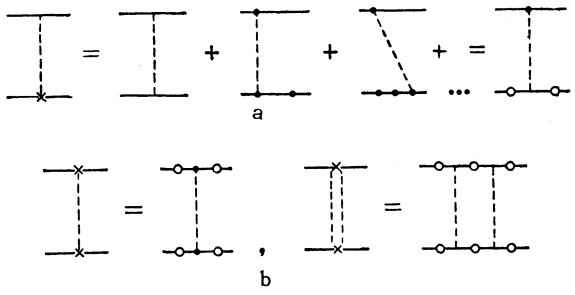

FIG. 2

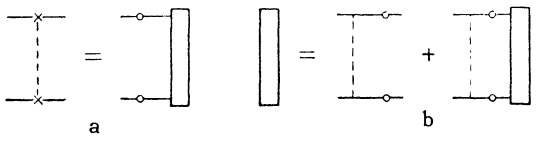

FIG. 3

the relations (15) and (A.14). Again collecting the terms, we obtain

$$U(\mathbf{p}_1\mathbf{p}_0\mathbf{p}_1\mathbf{p}_0) = e^{-\alpha R} P((p_0+\mu)\,\mathbf{n}_1\mathbf{p}_0,\ (p_0+\mu^*)\,\mathbf{n}_1\mathbf{p}_0). \quad (29)$$

We have chosen $\mathbf{p}_2 = \mathbf{p}_0$, $\mathbf{p}_1 = \mathbf{p}_3 = \mathbf{p}_0\mathbf{n}_1$, $\alpha = N_0\sigma_0$, since precisely this case is needed for the calculation of the intensity.

We now transform Eq. (25). Substituting the quantity $P(\mathbf{mp}_0\mathbf{m'p}_2)$ in the form of (25) and (26) on the right side of (25), we get an expression which contains the integral

$$J(\mathbf{g}_1\mathbf{g}_2) = \int d\mathbf{l}\int d\mathbf{m}\, a(\mathbf{pl})\, a(\mathbf{ms})\, G_0(l)\, G_0(m)\, S(\mathbf{lm}) \times F(\mathbf{g}_1 - \mathbf{l})\, F(\mathbf{m} - \mathbf{g}_2),$$

$$\mathbf{g}_1 = \mathbf{p} - \mathbf{f} + \mathbf{l}', \qquad \mathbf{g}_2 = \mathbf{m}' + \mathbf{s} - \mathbf{s}', \quad (30)$$

where $\mathbf{l}'$, $\mathbf{m}'$, $\mathbf{s}$ and $\mathbf{s}'$ are variables along which the integration is carried out in what follows. Taking out the amplitudes and substituting (20) in (30), we get two components, one of which contains $\langle T(\mathbf{l},\mathbf{m})\rangle$. Rewriting $\langle T(\mathbf{l},\mathbf{m})\rangle$ in the form of (11), integrating, and discarding terms of the order $N_0|a|^2\lambda$, we get

$$J(\mathbf{g}_1\mathbf{g}_2) = \int d\mathbf{x}\, G_1(x)\, F(\mathbf{g}_1 - \mathbf{x})\, F(\mathbf{x} - \mathbf{g}_2)\, a(\mathbf{pg}_1)\ a(\mathbf{g}_2\mathbf{s}),$$

$$G_1(x) = (2\pi^2)^{-1}(x^2 - p_0^2 - 2p_0\mu)^{-1}. \quad (31)$$

Subsequent integration over $\mathbf{l}'$ and $\mathbf{m}'$ is carried out in a fashion similar to that of (30).

Then, collecting all the terms, we get

$$P(\mathbf{pp}_0\mathbf{fp}_2) = P_0(\mathbf{pp}_0\mathbf{fp}_2) + N\int d\mathbf{x}\int d\mathbf{y}\, G_1(x)\, G_1^+(y) \times a(\mathbf{px})\, a^+(\mathbf{fy})\, F(\mathbf{p} - \mathbf{f} - \mathbf{x} + \mathbf{y})\, P(\mathbf{xp}_0\mathbf{yp}_2). \quad (32)$$

We shall compute Eq. (26) by substituting (20) therein, and making use of (15);

$$P_0(\mathbf{pp}_0\mathbf{fp}_2) = Ne^{-\alpha R}\, a(\mathbf{pp}_0)\, a^+(\mathbf{fp}_0)\, F(\mathbf{p} - \mathbf{f} - (p_0+\mu)\,\mathbf{n}_0 + (p_0+\mu^*)\,\mathbf{n}_0). \quad (33)$$

In (32), we replace $\mathbf{f}$, $\mathbf{p}_2$ and $\mathbf{y}$ by the new variables $\mathbf{u} = \mathbf{p} - \mathbf{f}$, $\mathbf{z} = \mathbf{p}_0 - \mathbf{p}_2$ and $\mathbf{w} = \mathbf{x} - \mathbf{y}$. We denote the quantity $P(\mathbf{pp}_0\mathbf{fp}_2)$ in the new variables by $K(\mathbf{pp}_0, \mathbf{uz})e^{-\alpha R}$. Its dependence on $\mathbf{u}$ and $\mathbf{z}$ is steep, since $\mathbf{u}$ and $\mathbf{z}$ are contained in the F-functions, while its dependence on $\mathbf{p}$ and $\mathbf{p}_0$ is comparatively smooth, since $\mathbf{p}$ and $\mathbf{p}_0$ only enter in the amplitude.

As above, we shall assume that the amplitudes do not have steeper singularities than resonance poles. In the integration over $\mathbf{x}$, one can evaluate $a(\mathbf{px})\, a^+(\mathbf{p} - \mathbf{u}, \mathbf{x} - \mathbf{w})\, K(\mathbf{xp}_0, \mathbf{wz})$ at the point $|\mathbf{x}| = p_0$. There is then left

$$\int G_1(x)\, G_1^+(|\mathbf{x} - \mathbf{w}|)\, x^2\, dx = G(\mathbf{wn}_x) = (\alpha + i\mathbf{wn}_x)^{-1}. \quad (34)$$

In the case in which

$$\mathbf{p} = (p_0+\mu)\,\mathbf{n}_1, \qquad \mathbf{f} = (p_0+\mu^*)\,\mathbf{n}_1,$$

$$\sigma(\mathbf{n}_1\mathbf{n}_0) = |a(p_0\mathbf{n}_1, p_0\mathbf{n}_0)|^2,$$

we get

$$K(\mathbf{p}_1\mathbf{p}_0, \mathbf{uz}) = \sigma(\mathbf{n}_1\mathbf{n}_0) F(\mathbf{u} - \mathbf{z}) + (2\pi)^{-3} N\int d\Omega_x\, \sigma(\mathbf{n}_1\mathbf{n}_x) \times \int d\mathbf{w}\, F(\mathbf{u} - \mathbf{w})\, G(\mathbf{wn}_x)\, K(\mathbf{xp}_0, \mathbf{wz}). \quad (35)$$

In order to find $K(\mathbf{p}_1\mathbf{p}_0, \mathbf{uz})$ it is necessary to know the angular dependence of $\sigma(\mathbf{n}_1\mathbf{n}_0)$. According to (28), in the important case for which the transverse cross section is isotropic (for example, for slow neutrons), the intensity of multiple scattering has the form

$$I(\vartheta) = N\sigma_s e^{-2\alpha R} K(\vartheta)/4\pi, \quad (36)$$

where

$$\sigma_s K(\vartheta) = 4\pi K(\mathbf{p}_1\mathbf{p}_0, \mathbf{uz}), \qquad \mathbf{u} = i\alpha\mathbf{n}_1, \qquad \mathbf{z} = i\alpha\mathbf{n}_0,$$
$$\mathbf{p}_1 = p_0\mathbf{n}_1, \qquad \mathbf{p}_0 = p_0\mathbf{n}_0, \qquad \cos\vartheta = \mathbf{n}_1\mathbf{n}_0.$$

Equation (35) is identical with (A.3), if we set

$$\varepsilon = \sigma_s\sigma_0^{-1}, \qquad \sigma_s = \int \sigma(\vartheta)\, d\Omega$$

in that expression.

The function $K(\mathbf{p}_1\mathbf{p}_0, \mathbf{uz})$ is found in the Appendix with an accuracy to within $\sim \frac{1}{2}(\epsilon/2)^2 \ln 2$ in comparison with unity. It is written as $K(\mathbf{uz})$ there. If $[\epsilon c/4\beta^2(\alpha+\beta)] \ll 1$, which is well satisfied at $\epsilon \leq 0.7$, then, according to (A.5) and (A.18), we get

$$K(\vartheta) \sim F(i\alpha\mathbf{n}_1 - i\alpha\mathbf{n}_0) \times \frac{\alpha^2(1-\varepsilon)+\beta^2}{\alpha^2(1-\varepsilon)-\beta^2} + \frac{3}{R^3}\sum_l (2l+1)\, P_l(\cos\vartheta) \times \left\{\frac{2\beta^3(\alpha^2-\beta^2)}{\alpha^2(1-\varepsilon)-\beta^2} B_l(i\alpha, i\beta)\, A_l(i\beta, i\alpha) + \varepsilon\alpha\int_\alpha^\infty dx\, \vartheta(x)\left[\frac{A_l(i\alpha, ix)}{x^2-\alpha^2} - xB_l(i\alpha, ix)\, A_l(ix, i\alpha)\right]\right\}. \quad (37)$$

All the notation entering into (37) is given in the Appendix.

Equation (37) is a converging series in l so that it suffices in practice to sum up to $l = \alpha R$. With accuracy up to $(\alpha-\beta)\alpha^{-1}$, the value of $B_l(i\alpha, i\beta)$ does not depend on l, which makes it possible to sum (37) completely, using (A.4). For example, for $\epsilon = 0.6$, the quantity $(\alpha-\beta)\alpha^{-1} \approx 0.1$, and decreases with decrease in ϵ. Moreover, $\theta(x)$ is small if $x < x_0 = \alpha^2\beta^{-1}$; it has a sharp maximum if $x = x_0$; and it is close to unity if $x > x_0$. This makes it possible to reduce (37) to the form

$$K(\vartheta) = F(i\alpha\,\mathbf{n}_1 - i\alpha\mathbf{n}_0) \times \left\{1 + \frac{\varepsilon}{2}[0.577 + \ln 2\alpha R - \mathrm{Ei}(-2\alpha R)]\right\} + \frac{2\beta^2}{\beta^2+\alpha^2(\varepsilon-1)}[e^{(\alpha-\beta)R}F(i\beta\mathbf{n}_1 - i\alpha\mathbf{n}_0) - F(i\alpha\mathbf{n}_1 - i\alpha\mathbf{n}_0)] + \frac{\alpha\varepsilon\gamma(\varepsilon)}{2(\alpha-\beta)}\Big[F(i\alpha\mathbf{n}_1 - i\alpha\mathbf{n}_0) - e^{-\alpha(\alpha-\beta)R/\beta} \times F\Big(i\frac{\alpha^2}{\beta}\mathbf{n}_1 - i\alpha\mathbf{n}_0\Big)\Big] - \frac{\varepsilon}{2}\Phi(\vartheta),$$

$$\gamma(\varepsilon) = \frac{1}{\alpha}\int_\alpha^\infty dx\,[\theta(x) - 1], F(\mathbf{x}) = \frac{3}{(xR)^2}\Big(\frac{\sin xR}{xR} - \cos xR\Big). \quad (38)$$

The function $\epsilon\Phi(\vartheta/2) \approx 0$ in all cases, except $2\alpha R \sin(\vartheta/2) \gg 1$, when

$$\Phi(\vartheta) \cong F(i\alpha\mathbf{n}_1 - i\alpha\mathbf{n}_0) \sum_{n=1}^{2\alpha R\sin(\vartheta/2)} \frac{1}{n}\Big(\sin\frac{\vartheta}{2}\Big)^n. \quad (39)$$

4. DISTRIBUTION OF SCATTERED PARTICLES INSIDE THE BODY

If we are interested in the number of particles then it is necessary to find the wavefunction. Using the expression (2), and the explicit form of (15) and (17), we get

$$\langle|\Psi(\mathbf{r})|^2\rangle = |\langle\Psi(\mathbf{r})\rangle|^2 + N\int d\mathbf{p}\,G_0(p)\int d\mathbf{f}\,G_0^+(f) \times \exp[i(\mathbf{p}-\mathbf{f})\mathbf{r}]\,U(\mathbf{pp_0fp_0}), \quad (40)$$

$$\langle\Psi(\mathbf{r})\rangle = e^{i\mu R}\exp[i(p_0+\mu)\mathbf{n}_0\mathbf{r}]. \quad (41)$$

Then, substituting (24), and carrying out transformations similar to (30)—(32), we get an equation which differs from (40) only by the replacement

$$G_0(p)\,G_0^+(f) \to G_1(p)\,G_1^+(f), \qquad U(\mathbf{pp_0fp_0}) \to P(\mathbf{pp_0fp_0}).$$

Transforming then to the coordinate representation

$$P(\mathbf{pp_0fp_0}) = \Omega^{-1}\int d\rho\,\exp[i(\mathbf{f}-\mathbf{p})\rho]\,P(\mathbf{pf},\rho) \quad (42)$$

and integrating (40) over $\mathbf{p}$ and $\mathbf{f}$, we get

$$\langle|\Psi(\mathbf{r})|^2\rangle = \exp[-\alpha(R+\mathbf{n}_0\mathbf{r})] + N_0\int_\Omega d\rho\,\frac{\exp[\alpha|\mathbf{r}-\rho|]}{|\mathbf{r}-\rho|}P(\mathbf{pf},\rho). \quad (43)$$

If the amplitudes do not depend on the angles, then it follows from (32), (42), and (43) that $P(\mathbf{pf},\rho) \equiv \sigma\langle|\Psi(\rho)|^2\rangle$, and Eq. (43) takes the form of (A.1). The solution of it is

$$\langle|\Psi(\mathbf{r})|^2\rangle = e^{-\alpha R}\Big\{\exp[-\alpha\mathbf{n}_0\mathbf{r}] + \frac{1}{(2\pi)^3}\varepsilon\Omega\int d\mathbf{q}\,F(\mathbf{q} - i\alpha\mathbf{n}_0) \times \int d\mathbf{p}\,e^{i\mathbf{p}\mathbf{r}}G_2(p)\,K(\mathbf{pq}). \quad (44)$$

Substituting (A.18) and (A.10) in (44) and integrating, we get

$$\langle|\Psi(\mathbf{r})|^2\rangle = e^{-\alpha R}\Big\{\exp[-\alpha\mathbf{n}_0\mathbf{r}]\frac{\alpha^2(1-\varepsilon)+\beta^2}{\alpha^2(1-\varepsilon)-\beta^2} + \varepsilon\alpha\int_\alpha^\infty dx\,\theta(x)\frac{\exp(-\alpha\mathbf{n}_0\mathbf{r})}{x^2-\alpha^2} + \sum_l i^l(2l+1)P_l(\cos\vartheta)\Big[\frac{2\beta^3(\alpha^2-\beta^2)}{\alpha^2(1-\varepsilon)-\beta^2}B_l(i\alpha, i\beta)\,j_l(i\beta r) - \varepsilon\alpha\int_\alpha^\infty dx\,x\,\theta(x)\,B_l(i\alpha, ix)\,j_l(ixr)\Big]\Big\}. \quad (45)$$

In the same approximation as for the case (38), it follows from (44) that, for $R - r > \alpha^{-1}$,

$$\langle|\Psi(\mathbf{r})|^2\rangle = e^{-\alpha R}\Big\{\exp[-\alpha\mathbf{n}_0\mathbf{r}]\Big[1 + \frac{\varepsilon}{2}\Big(0.577 + \ln 2\alpha(R+\mathbf{n}_0\mathbf{r}) - \mathrm{Ei}\big[-2\alpha(R-\mathbf{n}_0\mathbf{r})\big]\Big] + \frac{2\beta^2}{\beta^2+\alpha^2(\varepsilon-1)} + (\exp[(\alpha-\beta)R]\exp[-\beta\mathbf{n}_0\mathbf{r}] - \exp[-\alpha\mathbf{n}_0\mathbf{r}]) + \frac{\varepsilon\alpha\gamma(\varepsilon)e^{\alpha R}}{2(\alpha-\beta)}\Big(\exp[-\alpha(R+\mathbf{n}_0\mathbf{r})] - \exp\Big[-\frac{\alpha^2}{\beta}(R+\mathbf{n}_0\mathbf{r})\Big]\Big)\Big\}. \quad (46)$$

5. DISCUSSION OF THE RESULTS

A. Let us consider the physical meaning of the diagrams whose sum corresponds to the solution of the transport equation (Fig. 2b and Fig. 3). Each such diagram describes scattered radiation of a definite multiplicity. A diagram with a single dashed line describes single scattering, one with two such lines, double scattering, etc. Each of the diagrams depends on the ratio of the dimension of the system R to the value of the total path length of the particle in the medium, $(N_0\sigma_0)^{-1}$; it grows with increase in R and tends to zero as $R \to 0$.

All the corrections to the transport equation can be divided into two groups. The first group

ϑ, deg	$_{28}Ni^{60}$, ε=0.27		$_{19}K^{39}$, ε=0.43	
	αR=1	5	1	5
0	0.120	$6.7\cdot10^{-3}$	13	0.965
90	0.144	0.183	15.6	17.6
180	0.177	1.55	18.8	135
Single value	0.747	93.5	69.4	8750

of corrections (diagrams c, d, etc. in Fig. 1) is determined by the value of the ratio of the wavelength of the incident radiation λ to the total path length of the particle. It is associated with repeated collisions with a single force center. The second group of corrections contains the scattering amplitudes but cannot be expressed in terms of cross sections.

Both the intensity of the scattering and the distribution of particles inside the body are determined by a single function K for which an integral equation is obtained.

B. In order to illustrate the application of the formulas that have been obtained, let us consider the multiple scattering of thermal neutrons by a macroscopic spherical target. This problem has independent interest for the solution of different problems in the theory of nuclear reactors, and also for an accurate account of scattering in the source and target in the study of nuclear reactions with participation of neutrons. In most cases, the transverse scattering of slow neutrons by nuclei is isotropic (S-wave) and $N_0|a|^2\lambda \ll 1$. Therefore, the conditions of applicability (36) are satisfied. All the conclusions are valid with the same accuracy with which one can neglect the energy of the recoil nucleus in comparison with the energy of the neutron.

Let us consider scattering from targets consisting of K_{19}^{39} and Ni_{28}^{60} (see the table). In all cases, for $\alpha R = 0.1$, the intensity is almost independent of the angle and is equal to 0.06 for K^{39} and 6.3×10^{-4} for Ni^{60}. We see that $I(\vartheta)$ increases with increase in the angle ϑ for any αR. For fixed ϑ, the intensity first increases with increase in αR, and then falls off. For small angles, the decrease begins earlier than for large angles. As $R \to 0$, we get the well known formula for the single scattering $N\sigma$.

If the scattering is principally elastic, $\epsilon \sim 1$, then the solution to the problem is given by (A.21). For high accuracy, one can use the method of successive approximations in (A.20).

APPENDIX

We consider the equation

$$I(\mathbf{r}) = b(\mathbf{r}) + \varepsilon \int_{\Omega} d\mathbf{r}'\, G_2(|\mathbf{r}-\mathbf{r}'|)\, I(\mathbf{r}'). \quad (A.1)$$

Equations (35) and (11) can be reduced to (A.1) in many cases. In order to get the solution of (A.1), we convert to the Fourier transform:

$$I(\mathbf{u}) = b(\mathbf{u}) + (2\pi)^{-3}\varepsilon\Omega G_2(u)\int d\mathbf{p}\, K(\mathbf{up})\, b(\mathbf{p}), \quad (A.2)$$

$$K(\mathbf{up}) = F(\mathbf{u}-\mathbf{p}) + (2\pi)^{-3}\varepsilon\Omega\int d\mathbf{q}\, G_2(q)\, F(\mathbf{u}-\mathbf{q})\, K(\mathbf{qp}). \quad (A.3)$$

We assume that the region of integration Ω is a sphere of radius R. Taking it into account that

$$F(\mathbf{u}-\mathbf{p}) = \frac{4\pi}{\Omega}\sum_l (2l+1)\, A_l(u,p)\, P_l(\cos\vartheta), \quad (A.4)$$

$$A_l(u,p) = R^2(u^2-p^2)^{-1} \times [pj_{l-1}(pR)\, j_l(uR) - uj_{l-1}(uR)\, j_l(pR)] \quad (A.5)$$

(ϑ is the angle between $\mathbf{u}$ and $\mathbf{p}$), we get

$$K(\mathbf{up}) = \frac{4\pi}{\Omega}\sum_l (2l+1)\, K_l(u,p)\, P_l(\cos\vartheta). \quad (A.6)$$

Substituting (A.4) and (A.6) in (A.3), we get

$$K_l(u,z) = A_l(u,z) + \frac{\varepsilon}{\pi}\int_{-\infty}^{+\infty} dp p^2\, G_2(p)\, B_l(u,p)\, K_l(p,z), \quad (A.7)$$

$$B_l(u,p) = R^2(u^2-p^2)^{-1}[ph_{l-1}(pR)\, j_l(uR) - uj_{l-1}(uR)\, h_l(pR)], \quad (A.8)$$

$$j_l(x) = \sqrt{\pi/2x}\, J_{l+1/2}(x), \quad h_l(x) = \sqrt{\pi/2x}\, H^{(1)}_{l+1/2}(x), \quad (A.9)$$

where $J_\nu(x)$ and $H_\nu(x)$ are the Bessel function and the Hankel function, respectively.

Equation (A.7) is a special integral equation with a kernel of the Cauchy type. We find its solution in two cases:

1. $G_2(p) = [p^2 - p_0^2 - i\eta]^{-1}$,
2. $G_2(p) = (\alpha/p)\tan^{-1}(p/\alpha)$. (A.10)

In the solution of (A.7) by the method of successive approximations, it is easy to establish the fact that the expansion parameter is not just ϵ but in fact, $\epsilon\alpha R$. We reduce the singular equation (A.7) to a Fredholm type (A.19) with a non-singular kernel. Here it happens that all the essential dependence on R is taken into account in the free term. Equation (A.19) can be solved by the method of successive approximations, where the convergence of the expansion is improved, since the parameter is seen to be ϵ while the expansion begins with terms ϵ^2.

1. Instead of $K_l(u,z)$ we substitute $A_l(u,z)$

$[1-\epsilon G_2(u)]^{-1}$ in (A.7) and transform to a contour integral, inasmuch as the integrand falls off sufficiently on the large circle. As a result of the integration, we get

$$K_l^{(0)}(u, z) = \frac{A_l(u, z)}{1-\varepsilon G_2(u)} + i\varepsilon\sqrt{p_0^2+\varepsilon}\,B_l(u, \sqrt{p_0^2+\varepsilon})\,A_l(\sqrt{p_0^2+\varepsilon}, z). \quad (A.11)$$

We construct the equation relative to $M_l(u,z) = K_l(u,z) - K_l^{(0)}(u,z)$:

$$M_l(u, z) = i\frac{\varepsilon^2}{\pi}\sqrt{p_0^2+\varepsilon}\,A_l(\sqrt{p_0^2+\varepsilon}, z)\int_{-\infty}^{+\infty}\frac{dpp^2}{p^2-p_0^2-i\eta} \times A_l(u, p) D_l(p, \sqrt{p_0^2+\varepsilon}) + \frac{\varepsilon}{\pi}\int_{-\infty}^{+\infty} dp\,\frac{p^2}{p^2-p_0^2-i\eta}B_l(u, p)\,M_l(p, z),$$

$$D_l(p, \beta) = R^2(p^2-\beta^2)^{-1}[\beta h_{l-1}(\beta R)\,h_l(pR) - ph_{l-1}(pR)\,h_l(\beta R)]. \quad (A.12)$$

The resolvent of (A.12) is the function (A.7). Therefore,

$$K_l(u, z) = K_l^{(0)}(u, z) + i(\varepsilon^2/\pi)\sqrt{p_0^2+\varepsilon}\,A_l(\sqrt{p_0^2+\varepsilon}, z) \times\int_{-\infty}^{+\infty} dpp^2\,\frac{1}{p^2-p_0^2-i\eta}K_l(u, p)\,D_l(p, \sqrt{p_0^2+\varepsilon}). \quad (A.13)$$

Equation (A.13) has a degenerate kernel; therefore, its solution has the form

$$K_l(u, z) = K_l(z, u) = \frac{z^2-p_0^2}{z^2-p_0^2-\varepsilon}\left[A_l(u, z) - \frac{A_l(u, \sqrt{p_0^2+\varepsilon})\,B_l(z, p_0)}{B_l(\sqrt{p_0^2+\varepsilon}, p_0)}\right]. \quad (A.14)$$

Here well known relations between the Bessel functions have been used.[10] If $z = p_0$, then carrying out the summation over l, we get, with accuracy up to terms of order ϵ:

$$K(\mathbf{up}_0) = \exp(i\varepsilon R/2p_0)\,F(\mathbf{u} - (p_0+\varepsilon/2p_0)\,\mathbf{n}_0). \quad (A.15)$$

2. Similar contributions for a kernel with a branch point lead to the following equations

$$K_l(u, z) = W_l(u, z) + i\alpha\varepsilon A_l(i\beta, z)\int_L dx\,xK_l(u, x)\,D_l(x, i\beta), \quad (A.16)$$

$$W_l(u, z) = Z_l(u, z) + (\alpha\varepsilon)^2\int_L dx\,x\,K_l(u, x)\int_L dy\,y\theta(-iy)\,D_l(x, y)\,A_l(y, z), \quad (A.17)$$

$$Z_l(u, z) = A_l(u, z)[1-\varepsilon G_2(u)]^{-1} + i\varepsilon cB_l(u, i\beta)\,A_l(i\beta, z) + \alpha\varepsilon\int_L dx\,x\theta(-ix)\,B_l(u, x)\,A_l(x, z). \quad (A.18)$$

The integrals L are taken along the cut which extends from the point $i\alpha$ to the point $i\infty$:

$$\ln\frac{\alpha+\beta}{\alpha-\beta} = \frac{2\beta}{\alpha\varepsilon}, \qquad c = -\frac{2i\beta^3(\alpha^2-\beta^2)}{\alpha^2(1-\varepsilon)-\beta^2},$$
$$\alpha = N_0\sigma_0, \qquad \theta(x) = \left[\left(1+\frac{\alpha\varepsilon}{2x}\ln\frac{x-\alpha}{x+\alpha}\right)^2 + \left(\frac{\pi\alpha\varepsilon}{2x}\right)^2\right]^{-1}. \quad (A.19)$$

The first equation of (A.16) has a degenerate kernel. Solving it for $K_l(u, z)$, we get the following set of equations:

$$K_l(iu, iz) = E_l(iu, iz) + (\alpha\varepsilon)^2\int_\alpha^\infty dx\,xE_l(iu, ix)\,R_l(ix, iz), \quad (A.20)$$

$$R_l(ix, iz) = \Gamma_l(ix, iz) + (\alpha\varepsilon)^2\int_\alpha^\infty dy\,y\Gamma_l(iy, iz)\,R_l(ix, iy), \quad (A.21)$$

$$E_l(iu, iz) = Z_l(iu, iz) - i\varepsilon cQ_lA_l(i\beta, iz)\int_\alpha^\infty dx\,xZ_l(iu, ix)D_l(ix, i\beta), \quad (A.22)$$

$$\Gamma_l(ix, iz) = \Gamma_l^{(0)}(ix, iz) - i\varepsilon cQ_lA_l(i\beta, iz)\int_\alpha^\infty dy\,y\Gamma_l^{(0)}(ix, iy)\,D_l(iy, i\beta), \quad (A.23)$$

$$\Gamma_l^{(0)}(ix, iz) = \int_\alpha^\infty dy\,y\theta(y)\,D_l(ix, iy)\,A_l(iy, iz), \quad (A.24)$$

$$Q_l = \left[1 - i\alpha\varepsilon c\int_\alpha^\infty dy\,yA_l(i\beta, iy)\,D_l(iy, i\beta)\right]^{-1}. \quad (A.25)$$

(A.20) is an equation of the Fredholm type. An estimate of the integral term shows that it is smaller than the free term by a factor of at least $\frac{1}{2}(\epsilon/2)^2 \times \ln 2 \times [1 + (\beta/\alpha)^2\gamma(\epsilon)]$. The integral term in (A.22) can be neglected if $\epsilon \leq 0.7$. In this case, $K_l(iu, iz)$ is determined by (A.18). The formula (37) is obtained from (A.18) with the use of the Cauchy formula for the function $G_2(u)[1-\epsilon G_2(u)]^{-1}$.

By the method considered earlier, we can solve an equation of the type (A.1) with kernels that have several poles and branch points. A similar method of transformation of the integral equation can be applied also in the case of anisotropic scattering (see (35)), only in this case, it is convenient to isolate the fundamental dependence on R (term of the type of (A.22)) by the method of successive iteration (35).

[1] B. Davison, Theory of Neutron Transport (Russian translation) Atomizdat, 1960.

[2] S. Chandrasekhar, Transport of Radiant Energy (Russian translation, IIL, 1953).

[3] A. B. Migdal and N. M. Polievktov-Nikoladze, DAN **105**, 233 (1955).

[4] O. V. Konstantinov and V. I. Perel', JETP **39**, 197 (1960), Soviet Phys. JETP **12**, 142 (1961).

[5] M. Lax, Revs. Modern Phys. **23**, 287 (1951).

[6] L. L. Foldy, Phys. **67**, 107 (1945).

[7] K. Watson, Phys. Rev. **105**, 1388 (1957).

[8] V. V. Sobolev, DAN **116**, 45 (1957) and **120**, 63 (1958), Soviet Phys. Doklady **2**, 426 (1958) and **3**, 535 (1958).

[9] A. A. Abrikosov and L. P. Gor'kov, JETP **35**, 1558 (1958) and **36**, 319 (1959), Soviet Phys. JETP **8**, 1090 (1959) and **9**, 220 (1959).

[10] G. N. Watson, Theory of Bessel Functions (Russian translation, IIL, 1949).

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