# ON THE MECHANISM OF TOTAL NUCLEAR DISINTEGRATION

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Total angular momenta for each star of the reaction $C^{12} + C^{12} \rightarrow 6\alpha$, which has been studied previously [1], are determined on the basis of the statistical model of direct nuclear decay. A mechanism is indicated which differs from that of direct nuclear decay and which also leads to total disintegration of carbon atoms into $\alpha$ particles. Corrections to the statistical model of direct nuclear decay are discussed.

## 1. INTRODUCTION

As is well known, the properties of a whole series of nuclei agree well with the assumption that they contain groups of strongly bound nucleons of the $\alpha$-particle type. The total disintegration into $\alpha$ particles is observed experimentally when heavy nuclei collide, and is well described by the statistical model of the direct nuclear disintegration [1]. The statistical model of direct nuclear decay is based on the assumption that when two nuclei collide their kinetic energy is instantaneously released in a small effective volume surrounding these nuclei, and is then statistically distributed among the $\alpha$ particles of which the initial nuclei consist, and not among the individual nucleons. This model is constructed in analogy with the Fermi model for multiple production of pions [2].

The intermediate system of $\alpha$ particles usually has a large angular momentum (frontal collisions of the nuclei have low probability) and can be described in the c.m.s. by a Boltzmann distribution

$$dn = C(2\pi\hbar)^{-3} e^{-\alpha w + \beta m_x} d\mathbf{p}\, d\mathbf{r}, \tag{1}$$

where the colliding nuclei move in the direction of the z axis, the total angular momentum of the system is directed along the x axis, C is a normalization factor, w and $m_x$ are the energy and x-component of the angular momentum of the $\alpha$ particle, and $d\mathbf{p}\,d\mathbf{r}$ is the element of phase volume. The constants C, $\alpha$, and $\beta$ are chosen so as to cause the total number of particles N, the total energy E, and the total angular momentum M to have the obtained experimental values.

The Boltzmann distribution (1) makes it possible to obtain the angular and energy distributions of the $\alpha$ particles which are produced upon complete disintegration of two nuclei, if the total energy E and the total angular momentum M of the intermediate system are known.

Since the total angular momentum M is not known directly from experiment, and furthermore M differs for different stars, and since the collisions between two nuclei occur every time with different impact parameters, one can choose for M the mean value of the total angular momentum. The mean value of the total momentum can be estimated, for example, from geometrical considerations which do not depend on the model of the direct nuclear disintegration [1].

## 2. SEPARATION OF THE TOTAL NUCLEAR DISINTEGRATION MECHANISMS

The total disintegration of nuclei into $\alpha$ particles is observed experimentally in emulsions, in the form of multiprong stars. The calculation of the kinematics of such stars makes it possible to determine practically all the principal characteristics of the reactions involved in total disintegration. These characteristics include the total number of particles N produced as a result of the reaction, the total c.m.s. energy E of the colliding nuclei, the energy w and momentum p of each emitted particle, the polar and azimuthal angles $\vartheta$ and $\chi$ of each particle. However, the kinematic calculation of the star does not make it possible to determine the total angular momentum of the reaction M without making certain assumptions concerning the mechanism of the nuclear reaction.

The Boltzmann distribution (1) makes it possible to find the total angular momentum of the star M as a function of N, E, and $u = m\beta^2R^2/2\alpha$ (m —mass of emitted $\alpha$ particle, R —radius of the intermediate volume):

$$M = (4mNER^2)^{1/4}[\Gamma(3/2, u)/\Gamma(1/2, u)]^{1/2} \times [1 - \Gamma(5/2, u)/u\Gamma(3/2, u)], \quad (2)$$

where $\Gamma(a, b)$ is the incomplete $\Gamma$ function.

The value of the momentum M can be determined if one knows the value of u from experiment. The value of u can be related with the aid of the Boltzmann distribution (1) to the mean square of the cosine of the azimuth angle $\chi$ by the relation

$$\overline{\cos^2\chi} = \frac{e^{-u}}{\Gamma(3/2, u)} \int_0^{\sqrt{u}} (e^{t^2} - 1)\, dt, \quad (3)$$

where the azimuth angle $\chi$ is reckoned from the direction of the total angular momentum M.

The direction of the total angular momentum M can be determined from the condition that the quantity

$$\sum_i (p_i \sin \vartheta_i \cos \chi_i)^2 \quad (4)$$

be minimal in the plane perpendicular to the direction of motion of the colliding nuclei; the summation is over all the emitted $\alpha$ particles. Thus, by finding $\overline{\cos^2 \chi}$ from experiment, we can determine with the aid of expressions (2) and (3) the total angular momentum M for each star separately. Obviously, the momentum M cannot exceed $M_{max}$.

The maximum momentum $M_{max}$ can be estimated from the energy conservation law, $M_{max} = \sqrt{2IE}$, where I is the moment of inertia of the nucleus. This value of $M_{max}$ is more exact than that previously obtained [1] from geometrical considerations, where the nuclei were regarded as spheres of radius R. The moment of inertia I of the nucleus can be estimated independently. If formulas (2) and (3) lead to values $M > M_{max}$, it must be assumed that such a star occurred, not as a direct nuclear disintegration, but as a result of some other mechanism, to which the description with the aid of the distribution (1) cannot be applied.

We have analyzed from this point of view the experimental data on the total disintegration of carbon nuclei into $\alpha$ particles, $C^{12} + C^{12} \rightarrow 6\alpha$, which were published previously (see [1]). This reaction was observed in the form of six-prong stars in emulsions bombarded in the heavy-ion linear acceleration of the Khar'kov Physico-technical Institute with 115-MeV carbon ions.

We have calculated 70 stars and found the polar and azimuth angles. For each star we found the total angular momentum M. In the total disintegration of the carbon nuclei into $\alpha$ particles with an incident ion energy of 100 MeV in the laboratory system (l.s.), the maximum possible value of the total angular momentum is $M_{max} = 20\hbar$, if the moment of inertia I of the nucleus is assumed equal to that of a rigid sphere $I_{rig} = 2mNR^2/5$. Actually the moment of inertia of the nucleus is smaller than $I_{rig}$, and $M_{max}$ is somewhat smaller than $20\hbar$.

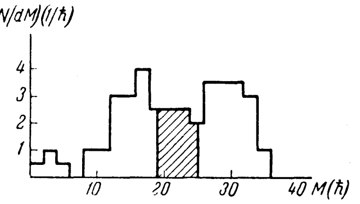


FIG. 1. Distribution of stars with respect to the magnitude of the total angular momentum, on the basis of the statistical model of direct nuclear disintegration.

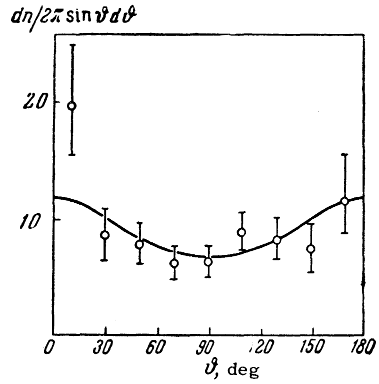


FIG. 2. Angular distribution of the $\alpha$ particles produced as a result of the direct nuclear disintegration ($M < 19\hbar$ in Fig. 1).

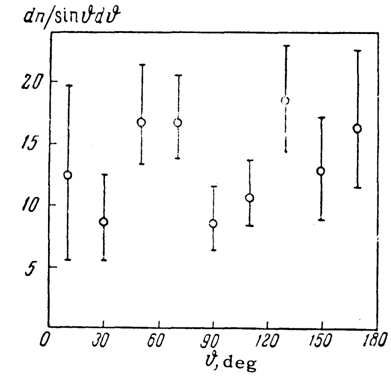


FIG. 3. Angular distribution of $\alpha$ particles for $M > 25\hbar$ in Fig. 1.

Figure 1 shows the distribution of the stars relative to the total angular momentum. For $M > M_{max}$, the momenta calculated in accordance with the statistical model of the direct nuclear disintegration do not correspond to the true values and have a purely arbitrary character. The stars for which the total angular momentum M obtained with the aid of Formulas (2) and (3) exceeds $M_{max}$, cannot be described by the statistical model and apparently are the result of other mechanisms for the total disintegration of the carbon nuclei into six $\alpha$ particles. The values $M > M_{max}$ shown

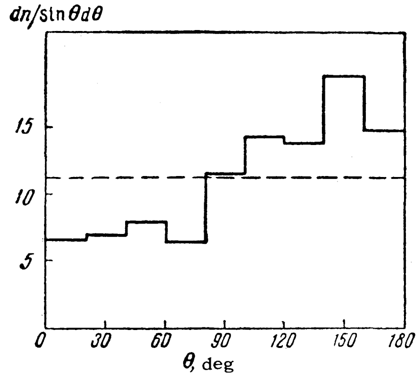


FIG. 4

FIG. 4. Angular correlation of $\alpha$ particles produced as a result of direct nuclear disintegration ($M < 19\hbar$ in Fig. 1).

FIG. 5. Angular correlation of $\alpha$ particles for $M > 25\hbar$ in Fig. 1.

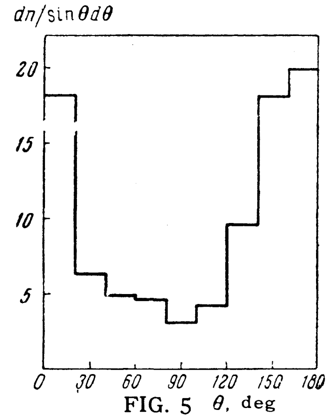


FIG. 5

in Fig. 1 undoubtedly do not coincide with the true total momenta of the corresponding stars, but these arbitrary values are essential in order to distinguish the direct nuclear disintegration from the other mechanisms of total disintegration of nuclei into $\alpha$ particles.

In separating the stars according to the direct nuclear disintegration and to other mechanisms of total nuclear disintegration, it is convenient to consider all their characteristics separately. We exclude here from consideration the intermediate region $19\hbar < M < 25\hbar$, shown shaded in Fig. 1, so as to separate more clearly the differences between the disintegration mechanisms of the two carbon nuclei into six $\alpha$ particles.

Figure 2 shows the angular distribution of the $\alpha$ particles produced in the direct nuclear disintegration ($M < 19\hbar$ in Fig. 1). The continuous curve has been calculated from the corresponding formula of the previous paper [1] (the formula was obtained from the Boltzman distribution). The angular distribution of Fig. 2 is characterized by preferred emission of the $\alpha$ particles in the forward and backward directions, something which does not occur for the almost isotropic distribution of the $\alpha$ particles from the stars produced as a result of mechanisms other than direct nuclear disintegration (see Fig. 3). The errors indicated in Figs. 2 and 3 are statistical.

The difference in the mechanisms of the reaction $C^{12} + C^{12} \rightarrow 6\alpha$ is even more pronounced if the distributions of the $\alpha$ particles with respect to the angle $\theta$ between the directions of emission of the two $\alpha$ particles are plotted—the angular correlations between the particle emission directions. For each star the number of such angles is equal to the number of combinations of prongs taken two at a time (for a six-prong star the number is 15). Such distributions are shown in Fig. 4 (region $M < 19\hbar$ of Fig. 1) and Fig. 5 (region $M > 25\hbar$). Figure 4 corresponds to the direct nuclear disintegration. The dashed curve represents the $\alpha$-particle distribution in the case of zero total angular momentum, $M = 0$. The difference between the distributions shown in Figs. 4 and 5 also shows that there exists a disintegration mechanism other than the direct one, leading to a total disintegration of the two carbon nuclei into $\alpha$ particles. This mechanism is apparently connected with the disintegration of the carbon nuclei into $\alpha$ particles in the case of grazing (or nearly grazing) collisions between nuclei, as is confirmed by the presence of maxima at angles $\theta = 0$ and $\theta = \pi$ in the angular correlations (Fig. 5).

## 3. CORRECTIONS TO THE STATISTICAL MODEL OF THE DIRECT NUCLEAR DISINTEGRATION

In the earlier analysis [1] of a system consisting of $\alpha$ particles, the system was assumed to be classical and was accordingly described by a Boltzmann distribution, while the intermediate volume was assumed spherical, so that good qualitative agreement was obtained with the experimental data. We are considering here corrections to the Boltzmann distribution (1), connected with the Bose-Einstein statistics and with the non-sphericity of the intermediate volume.

An exact description of the $\alpha$-particle system can be made with the aid of a Bose-Einstein distribution, which, however, leads to definite mathematical difficulties. Since the deviations from the Boltzmann distribution cannot be large, it is convenient to seek corrections to all the quantities obtained with the aid of the distribution (1). Putting $e^{\alpha\mu} = C$ ($\mu$ — chemical potential of the system), we can represent the Bose-Einstein distribution in a power series in $C$ (the chemical potential $\mu$, as can be seen from our earlier data [1] is negative and is not very close to zero, $\mu = -1.1$ MeV):

$$dn = \sum_{k=1}^{\infty} C^k e^{-k\alpha w + k\beta m_x} d\mathbf{p}\, d\mathbf{r}. \tag{5}$$

The first term in the sum of (5) is the Boltzmann distribution, and all the other terms are the corresponding corrections. The constants C, $\alpha$, and $\beta$ should now be determined from

$$N = \frac{2}{\sqrt{\pi}\beta^3\hbar^3}\sum_{k=1}^{\infty}\frac{C^k e^{ku}\Gamma(^3/_2,\ ku)}{k^3},$$

$$E = \frac{u}{\sqrt{\pi}\alpha\beta^3\hbar^3}\sum_{k=1}^{\infty}\frac{C^k e^{ku}\Gamma(^1/_2,\ ku)}{k^3},$$

$$M = \frac{4u}{\sqrt{\pi}\beta^4\hbar^3}\sum_{k=1}^{\infty}\frac{C^k e^{ku}}{k^3}\left[\Gamma(^3/_2,\ ku) - \frac{\Gamma(^5/_2,\ ku)}{ku}\right]. \quad (6)$$

The angular and energy distributions of the emitted $\alpha$ particles will now be given by

$$\frac{dn}{d\Omega} = \frac{1}{2\pi^{3/2}\beta^3\hbar^3}\sum_{k=1}^{\infty}\frac{C^k}{k^3}\int_0^{ku}\exp[x(1-\sin^2\vartheta\cos^2\chi)]\,x^{1/2}dx, \quad (7)$$

$$\frac{dn}{dw} = \frac{4\alpha u}{\pi\beta^3\hbar^3}\sum_{k=1}^{\infty}\frac{C^k e^{-k\alpha w}}{k}\int_0^{\pi/2}\operatorname{sh}\left(k\sqrt{\frac{w}{\varepsilon}}\sin x\right)\cos^2 x\,dx, \quad (8)$$

where

$$\varepsilon = 1/2m\beta^2R^2.$$

It is seen from (7) and (8) that the angular and energy distributions experience no essential changes when the corresponding corrections are taken into account. Inclusion of the first few terms in (7) and (8) is important if the experimental accuracy and the number of considered stars is increased, as will occur at higher energies. The numerical estimates show that an account of the second term in (6) and (7) reduces the angular distribution (7) in the direction of $\vartheta = 0$ by approximately 16%.

In the calculation of the angular and energy distributions by the statistical model of direct nuclear disintegration it was assumed that the intermediate volume is in the form of a sphere of radius R[1]. All the calculations can be made by assuming that the intermediate volume has the form of an ellipsoid, the major semiaxis of which lies in the plane perpendicular to the direction of the total angular momentum, if the ellipsoid does not differ strongly from a sphere. If the eccentricity $\epsilon$ of the ellipsoidal volume is small compared with unity ($\epsilon = \sqrt{1 - R^2/b^2}$, b —half the symmetry axis, R — radius of maximum circular cross section), then we obtain for the angular distribution averaged over all orientations of the ellipsoid, in the plane perpendicular to the direction of the total angular momentum, the expression

$$\frac{dn}{2\pi\sin\vartheta d\vartheta} = \frac{Ne^{-u}}{\sqrt{2}\pi\Gamma(^3/_2, u)}$$

$$\times\left\{F^0_{1/2}(\vartheta) + \frac{\varepsilon^2}{4}\left[\left(3 - \frac{u\Gamma(^1/_2,\ u)}{\Gamma(^3/_2,\ u)}\right)F^0_{1/2}(\vartheta) + 2\cos^2\vartheta F^0_{3/2}(\vartheta)\right.\right.$$

$$\left.\left. + \sin^2\vartheta(F^0_{3/2}(\vartheta) - F^1_{3/2}(\vartheta))\right]\right\}, \quad (9)$$

where the functions $F^m_n(\vartheta)$ are defined as

$$F^m_n(\vartheta) = \int_0^{u/2} e^{x(1+\cos^2\vartheta)} I_m(x\sin^2\vartheta)x^n dx. \quad (10)$$

Numerical calculations show that for sensible values of $\epsilon$ ($\epsilon \sim 0.4$) the angular distribution increases in the direction of $\vartheta = 0$ by approximately 20%. The corrections connected with the Bose-Einstein distribution and the non-sphericity of the intermediate volume are of opposite signs and therefore cancel each other.

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