INVESTIGATION OF THE $\pi^+\pi^-$ INTERACTION AT LOW ENERGY BY THE CHEW-LOW METHOD

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The $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ reaction in the 210—230 MeV range was studied. For events involving small momentum transfer to the nucleon, the experimental data are linearly extrapolated to the unphysical region by the Chew-Low method. The $\pi^+ + \pi^- \rightarrow \pi^+ + \pi^-$ elastic scattering cross section for ω^2 from 4 to 7 obtained by linear extrapolation is 34 ± 9 mb.

1. In [1] we studied the mass spectrum of the dipion system in the 280—350 MeV interval for the reaction

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n \qquad (1)$$

with 240-MeV incident π^- mesons. It was shown that the matrix element for reaction (1) increases with the increasing mass of the $\pi^+\pi^-$ system. However, no peaks indicating that the interaction has a resonant character in the 280—350 MeV interval were found. The nonresonant character of the mass spectrum in the low-energy region leads to the conclusion that a more complex analysis of reaction (1) is needed in order to obtain quantitative information on the $\pi\pi$ interaction.

It was also shown[2] that the cross section for the charge-exchange process $\pi^+ + \pi^- \rightarrow \pi^0 + \pi^0$ at zero energy can be determined from the study of the relative-momentum distribution of the secondary particles in reaction (1) close to the threshold by the method of Ansel'm and Gribov.[3] In principle, the pion-pion elastic scattering cross sections can, at the present time, be determined at any energy only by means of the Chew-Low method.[4] Data on the cross sections for the processes $\pi^\pm + \pi^0 \rightarrow \pi^\pm + \pi^0$ [5,6] and $\pi^+ + \pi^+ \rightarrow \pi^+ + \pi^+$ [7] obtained by this method from analysis of the $\pi N \rightarrow \pi\pi N$ reaction have already been discussed in the literature. The aim of the present work is to obtain data on the cross sections for the process $\pi^+ + \pi^- \rightarrow \pi^+ + \pi^-$ at low energy by the Chew-Low extrapolation technique.

As is known, the Chew-Low technique involves the separation of peripheral collisions, which are described by a one-pion-exchange diagram (Fig. 1a). Thus far, the Chew-Low extrapolation method has been applied primarily to a region of rather high primary pion energy (~ 1 GeV). Use of this

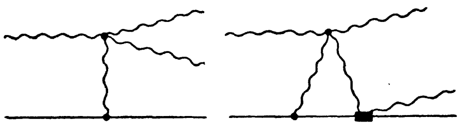

FIG. 1. Diagrams for the process $\pi N \rightarrow \pi\pi N$: a — pole diagram, b — diagram with rescattering.

method in the low-energy region entails some difficulties. At low energies the wavelength of the incident pion is comparable with the size of the nucleon, and consequently the probability of a peripheral collision is small. However, the range of energies and momentum transfers to the nucleon for which the Chew-Low method is applicable has not yet been established; this question can be investigated experimentally.

2. In this experiment we analyzed 545 events corresponding to reaction (1) at energies between 210 and 310 MeV. The events were recorded in emulsion. The scanning method and selection criteria have been described earlier.[8] As was shown by Chew and Low,[4] the differential cross section $d^2\sigma/dp^2d\omega^2$ for reaction (1) in the case of events described by the one-pion-exchange diagram, tends, as $p^2 \rightarrow -1$, to a limit completely determined by the pion-nucleon coupling constant (f^2) and the $\pi^+\pi^-$ elastic scattering cross section $\sigma_{\pi^+\pi^-}$:

$$\lim_{p^2 \rightarrow -1} \frac{d^2\sigma}{dp^2 d\omega^2} = \frac{f^2}{\pi} \frac{p^2}{(p^2+1)^2} \frac{\omega \sqrt{\omega^2/4 - 1}}{q_0^2} \sigma_{\pi^+\pi^-}(\omega). \qquad (2)$$

Here p is the change in the 4-momentum of the nucleon, ω is the total energy of the two pions in their rest frame, q_0 is the momentum of the primary π^- meson in the laboratory system. All these quantities are expressed in units of the pion mass.[1)] But the point $p^2 = -1$ is situated outside the physical region, and in order to determine $\sigma_{\pi^+\pi^-}$ for fixed ω, the function

$$F(p^2, \omega^2) = \frac{\pi q_0^2}{f^2 \omega \sqrt{\omega^2/4 - 1}} \frac{d^2\sigma}{dp^2 d\omega^2} (p^2 + 1)^2 \tag{3}$$

has to be extrapolated from the region $p^2 > 0$ to the point $p^2 = -1$. At this point we have $F(-1, \omega^2) = -\sigma_{\pi^+\pi^-}$.

A basic difficulty of the Chew-Low method is that the theory does not provide information on the dependence of F on p^2 in the physical region, and hence it is not known how to extrapolate the function F. It is usually assumed that the function $F(p^2)$ is analytic and, consequently, can be expanded into a series in powers of p^2. The series is written in the form $F(p^2) + A_0 + A_1(p^2 + 1) + A_2(p^2 + 1)^2 + \ldots$ In this case, for $p^2 = -1$, we have $F(1) = A_0 = -\sigma_{\pi^+\pi^-}$. In order to determine how many terms of the polynomial should be retained, we should, generally speaking, take polynomials of degree greater than those considered thus far and keep increasing the degree until the effect of the remaining terms is negligible. In practice, however, owing to insufficient statistics, we limit ourselves to quadratic or linear polynomials. Of course, some uncertainty then arises in the determination of the value of $\sigma_{\pi^+\pi^-}$.

3. The role of the one-pion exchange diagram at low energies is not yet known. Other more complicated diagrams can contribute to the $\pi N \rightarrow \pi\pi N$ process. Thus, for example, Goebel and Schnitzer[10,11] considered, along with the one-pion-exchange diagram (Fig. 1a), a diagram with rescattering (Fig. 1b) to describe the $\pi N \rightarrow \pi\pi N$ reaction in the energy region considered by us. Contributions from several diagrams complicate the description of the process, in particular, as a result of the occurrence of terms representing interference between the matrix elements of different diagrams. Moreover, even the one-pion exchange diagram (Fig. 1a) cannot be calculated exactly; formula (2) is valid only in the limiting case. For this reason, comparison of experimental data with calculations based on the one-pion-exchange diagram will not allow us to determine whether the difference is due to contributions from other diagrams or to the approximate character of the calculation for the one-pion-exchange diagram (pole approximation).

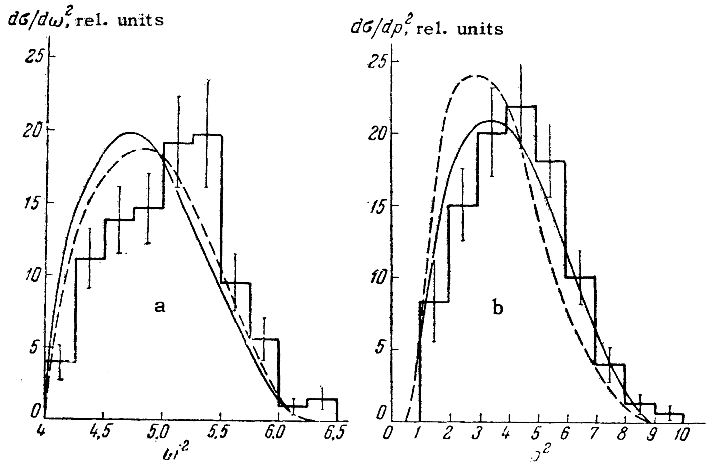

FIG. 2. Distributions of events from the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at a mean energy of 240±15 MeV; a – ω^2 distribution; b – p^2 distribution. The histogram represents the experimental data; the solid curve represents phase space; the dashed curve was calculated from the one pion exchange diagram with a constant value of $\sigma_{\pi^+\pi^-}$. The histogram and the curves were normalized to the same area. (Here and in what follows ω^2 and p^2 are given in units of μ^2.)

In Fig. 2 the experimental ω^2 and p^2 distributions for part of the events corresponding to reaction (1) with a mean pion energy of 240 ± 15 MeV are compared with the theoretical calculations based on the one-pion exchange diagram under the assumption that $\sigma_{\pi^+\pi^-}$ does not depend on ω^2 and p^2. As is seen from Fig. 2, the experimental distributions are not described by the theoretical curves and are shifted relative to these curves toward higher values of ω^2 and p^2. (According to the V^2 test, the probability of agreement is less than 5%.) We also carried out calculations under the assumption that $\sigma_{\pi^+\pi^-}$ is a decreasing ($\sigma_{\pi^+\pi^-} \sim \omega^{-2}$) or increasing ($\sigma_{\pi^+\pi^-} \sim \omega^2$) function of ω^2. But since the quantity ω^2 changes slowly (from 2 to 2.6), the theoretical distributions under these assumptions shift only slightly to one side or the other and agreement with experiment cannot be obtained. Hence the ω^2 and p^2 distributions show that in the energy region under consideration the $\pi N \rightarrow \pi\pi N$ process cannot be described only by a one-pion-exchange diagram in the pole approximation.

4. In order to enhance the relative role of the one-pion-exchange diagram, events with a small momentum transfer to the nucleon $p^2 \leq 7-10$ are usually considered separately in the hope that the pole has a significant effect in events of this type. We separated out two groups of events with rela-

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1)It should be noted that formula (2) is valid only if it is assumed that no other diagrams have a singularity in the region close to the pole $p^2 = -1$. In fact, the validity of this assumption is not obvious and a contribution from other diagrams would complicate the problem. Thus, for example, Landshoff and Treiman[9] showed that a branching point quite close to the pole can occur in the amplitude of the diagram (Fig. 1b).

tively small momentum transfers, one group with $p^2 \le 7$ (346 events) and the other with $p^2 \le 4$ or 5 (if the energy of the primary π^- meson was less than 260 MeV, we selected events with $p^2 \le 4$ and if the energy was greater than 260 MeV, it was required that p^2 be ≤ 5; this group contained 146 events). The maximum value of p^2 was 13. In the first group of events, the c.m.s. angular distribution of the neutrons was isotropic. In the second group, the neutrons were emitted in the backward hemisphere, which is more characteristic for peripheral collisions. But, as is seen from Fig. 2, the p^2 distribution calculated from the one-pion-exchange diagram differed little from the statistical distribution, and hence the selection of events with small momentum transfer to the nucleon in the studied energy region may prove to be not very effective for enhancing the role of the one-pion-exchange diagram. For this reason, it should first be shown that the role of the one-pion-exchange interactions in the selected events is actually large.

5. This question was discussed, for example, by Gramenitskiĭ, Podgoretskiĭ, and Khrustalev,[12] who suggested that the fulfillment of the so-called "isobaric kinematics" be checked for events with a small momentum transfer to the nucleon. The existence of a large number of events satisfying the kinematical relations connected with the decay of the "isobar" would indicate a large contribution of a diagram with rescattering (Fig. 1b).

Recently, Treiman and Yang[13] suggested a more general and rather simple criterion for checking the role of the one-pion-exchange diagram. These authors noted that since the exchange of particles emitted from the meson and nucleon vertices in a one-pion-exchange diagram takes place through the intermediary of a spinless particle (pion), there should be no coupling other than kinematical between these two groups of particles. Consequently, there should be no correlation between the planes formed by the trajectories of these particles. In particular, in the "antilaboratory" system (rest frame of the primary π^- meson), the distribution of the angle between the plane formed by the trajectories of the nucleon before and after the collision and the plane formed by the trajectories of the two pions should be isotropic if the process is due to a one-pion-exchange diagram.

We constructed the angular distribution between the aforementioned planes in the "antilaboratory" system for events with small momentum transfer ($p^2 \le 5$ and $p^2 \le 7$) and for events with relatively large momentum transfer to the nucleon ($p^2 > 7$). All distributions, constructed for 20° intervals, proved to be isotropic within the limits of statistical error.

6. Hence with the aid of the Treiman-Yang criteria, we were unable to determine how the role of the one-pion-exchange diagram varies as a function of p^2. However, to obtain information on the $\pi^+\pi^-$ interaction, we analyzed events with relatively small momentum transfer to the nucleon ($p^2 \le 7$ and $p^2 \le 5$). In order to analyze events with different primary energies, the events were divided into six intervals of the primary π^--meson energy: 210—230, 230—245, 245—260, 260—280, 280—295, 295—310 MeV. For each interval we constructed the kinematical region in the (p^2, ω^2)

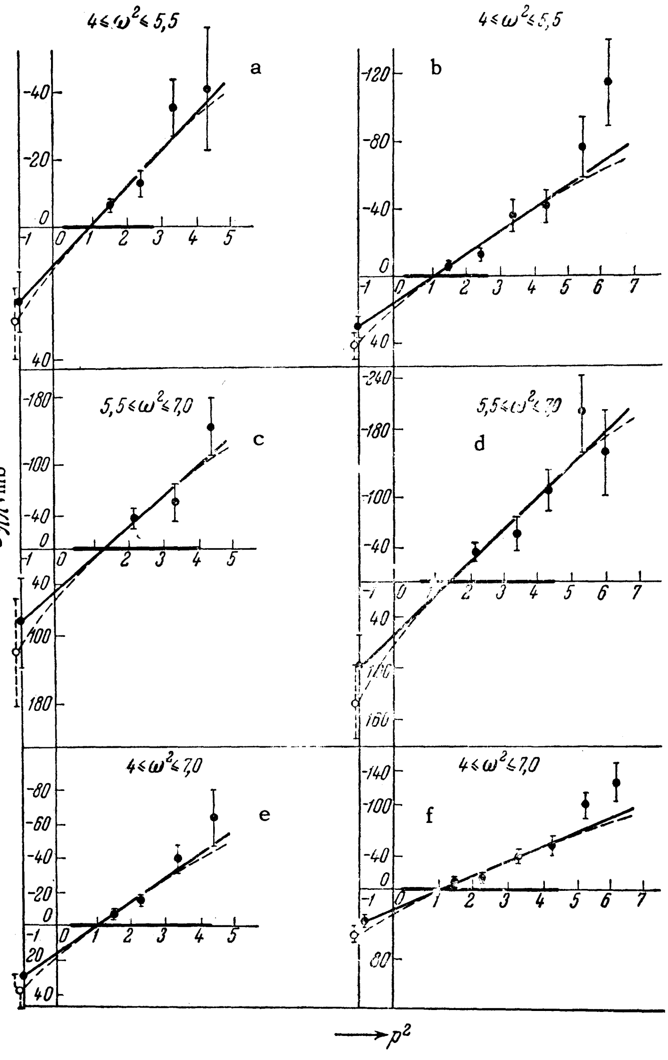

FIG. 3. Extrapolation of the function $F(p^2, \omega^2)$. The solid lines represent the polynomial $F(p^2, \omega^2) = A_0 + A_1(p^2 + 1)$; the dashed line represents the polynomial $F(W, \omega^2) = A_0' + A_1' W$; a, c, and e represent results for groups of events with p^2 $p^2 \le 5$; b, d, and f represent results for the group of events with $p^2 \le 7$. The thick line on the abscissa axis corresponds to the intervals of minimum values of p^2 for the given interval of ω^2.

Extrapolation polynomial	$F=A_0+A_1(p^2+1)$			$F=A_0'+A_1'W$		$\overline{\sigma}_{\pi\pi}$, mb	No. of events
	$\Delta\omega^2, \mu^2$	$\sigma_{\pi\pi}$, mb	$\chi^2/\overline{\chi}^2$	$\sigma_{\pi\pi}$, mb	$\chi^2/\overline{\chi}^2$		
$p^2 \leqslant 7$	4–7	39 ± 6	1.6	52 ± 8	1.6	44 ± 7	346
	4–5.5	32 ± 6	1.2	41 ± 8	1.3	35 ± 7	238
	5.5–7	99 ± 36	1.4	141 ± 45	1.4	115 ± 41	108
$p^2 \leqslant 5$	4–7	30 ± 8	1.2	39 ± 10	1.2	34 ± 9	156
	4–5.5	22 ± 8	1.0	29 ± 10	1.0	25 ± 9	107
	5.5–7	88 ± 52	0.7	120 ± 64	0.7	101 ± 58	49

plane and the distribution of events in each kinematical region. We then constructed the function $F(p^2, \omega^2)$ for all kinematical regions. The total cross sections for reaction (1) in the respective kinematic regions were 0.03 ± 0.01, 0.07 ± 0.02, 0.12 ± 0.02, 0.25 ± 0.03, 0.40 ± 0.04, 0.54 ± 0.05 mb. The value of ω^2 changed at most from 4 to 7. The mean value of ω^2 for all kinematic regions was 5.8. In view of the poor statistics, we limited ourselves to the linear expansion of the function F: $F(p^2, \omega^2) = A_0 + A_1(p^2 + 1)$. The linear extrapolation for the ω^2 interval from 4 to 7 is shown in Fig. 3 for two groups of events: $p^2 \leq 7$ and $p^2 \leq 5$. The points for $F(p^2)$ shown in Fig. 3 are the mean values for all the kinematic regions. The results of the extrapolation are shown in the table. The experimental points are described rather well by the function $F = A_0 + A_1(p^2 + 1)$. The value of the cross section for the process $\pi^+ + \pi^- \rightarrow \pi^+ + \pi^-$ for values of ω^2 between 4 and 7 is 39 ± 6 mb for events with $p^2 \leq 7$ and 30 ± 8 mb for events with $p^2 \leq 5$. These two cross sections do not differ within the limits of experimental error, but preference should be given to the cross section obtained from analysis of the events with smaller momentum transfers, i.e., with $p^2 \leq 5$.

7. Apart from the pole, the closest thing to a singularity in the amplitude of the process $\pi N \rightarrow \pi\pi N$ is a cut from $p^2 = -9$ to ∞. In order to improve the convergence of the series $F(p^2)$, we used an expansion of the function F in terms of a new variable W which leads to a conformal mapping of the p^2 plane with the cut from -9 to ∞ into a circle of unit radius. The point $p^2 = -1$ is then mapped into the center of the circle, while the cut itself is mapped into the circumference of the circle.[2)] The function W was obtained by V. A. Meshcheryakov:

$$W = \frac{18p^{-2} - 6p^{-2}\sqrt{p^2+9} + 1.03}{1 + 0.03\,(18p^{-2} - 6p^{-2}\sqrt{p^2+9} + 1)}. \tag{4}$$

[2)]The method of conformal mapping for an analogous problem—determination of the coupling constant for πN scattering was suggested by Frazer[14] and was used by Kazarinov, Legar, and Silin.[15]

The extrapolation corresponding to the polynomial $F(W) = A_0 + A_1W$ is shown by the dashed line in Fig. 3. The results of the extrapolation are given in the table. The experimental data are also described very well by the polynomial $F(W)$. The $\sigma_{\pi^+\pi^-}$ cross section obtained from the extrapolation of the polynomial $F(W)$ does not differ, within the limits of experimental error, from the data obtained with the aid of the polynomial $F(p^2)$. The values of $\chi^2/\overline{\chi}^2$ are practically the same. For this reason, in what follows we shall consider the mean values of $\sigma_{\pi^+\pi^-}$ from the results of both extrapolations. The mean value of the cross section for the process $\pi^+ + \pi^- \rightarrow \pi^+ + \pi^-$ obtained from the linear extrapolation for ω^2 between 4 and 7 and $p^2 \leq 5$ is 34 ± 9 mb.

8. We attempted to compare our results with the estimates for the $\sigma_{\pi^+\pi^-}$ from an analysis of reaction (1) for primary pions at higher energies, although no extrapolated data on $\sigma_{\pi^+\pi^-}$ are available. The only data concern the resultant cross section $\sigma = \frac{1}{3}(2\sigma_{\pi^+\pi^-} + 2\sigma_{\pi^-\pi^0})$ obtained from the application of formula (2) to the physical region for primary π^--meson energies of 1.75 GeV[16] and 1.25 GeV[17]. These cross sections for ω^2 in the interval between 4 and 7 are 80 ± 30 mb according to Erwin et al.[16] and 35 ± 8 mb according to Pickup et al.[17] If we use the estimate for $\sigma_{\pi^+\pi^-}$ from the analysis of the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ at 1.45 GeV[18] and use formula (2) in the physical region, then we can obtain the following estimates for $\sigma_{\pi^+\pi^-}$ from these data: 115 ± 40 mb from [16,18] and 45 ± 12 mb from [17,18]. As we see, our result is in better agreement with the second estimate. However, the errors are rather large and a quantitative comparison of the results requires more accurate data.

9. In order to account for the energy dependence of $\sigma_{\pi^+\pi^-}$, the experimental data were divided into intervals of ω^2 from 4 to 5.5 and from 5.5 to 7. The extrapolation curves for the two groups of events with $p^2 \leq 7$ and $p^2 \leq 5$ are shown in Fig. 3. The results of the extrapolation are given in the table. As is seen from the table, the cross section $\sigma_{\pi^+\pi^-}$ for events with $p^2 \leq 7$ increases with increasing ω^2. Although results were also obtained for the group of events with $p^2 \leq 5$, the difference in the cross sections for the two intervals of ω^2 is not greater than the error.

10. From the data for high-energy primary π^- mesons it is difficult to obtain information on the energy dependence of $\sigma_{\pi^+\pi^-}$ for values of ω^2 between 4 and 7. However, in a broader interval of ω^2 it can be established that the mean cross section $\bar{\sigma}$ decreases with increasing ω^2. Since $\sigma_{\pi^-\pi^0}$ is small and approximately constant [18] in the region of small ω^2, it can be assumed that $\sigma_{\pi^+\pi^-}$ also decreases with increasing ω^2 in this interval. In order to remove these disagreements, we should first of all improve the accuracy of the data for primary π^--mesons at both high and low energies. It is possible that the reason for the contradiction involves the fact that at low energies the contribution from one pion exchange interactions, even at very small momentum transfers to the nucleon, is small and the linear extrapolation is not sufficient to obtain quantitative information on $\pi^+\pi^-$ scattering.

11. It is interesting to compare our results with the results of Aref'ev et al, [19] who studied reaction (1) with the aid of spark chambers for a primary π^--meson energy of 310 MeV. The cross section $\sigma_{\pi^+\pi^-}$ was investigated for values of ω^2 in the interval 4—5 with the aid of the Chew-Low formula in the physical region for very small momentum transfers to the nucleon: $p^2 = 0.3$—2.0. The main results [19] can be formulated in the following way:

1) the cross section $\sigma_{\pi^+\pi^-}$ drops rapidly with increasing ω^2;

2) there is a possible resonance at $\omega^2 = 4.2$.

For a comparison with these results, we also used the Chew-Low formula in the physical region for the group of events with $p^2 \leq 4$ ($\bar{p}^2 = 2.6$) for a mean primary π^--meson energy of 240 MeV. The value of $\sigma_{\pi^+\pi^-}$ in the interval of ω^2 from 4 to 5 obtained in this way is shown in Fig. 4. The absolute values of the $\pi\pi$ cross section were found

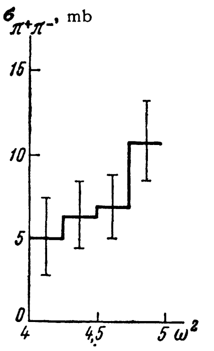

FIG. 4. Dependence of the cross section for the process $\pi^+ + \pi^- \to \pi^+ + \pi^-$ on the value of ω^2 obtained from the application of the Chew-Low formula in the physical region of momenta ($p^2 \leq 4$) for primary π^--meson energy of 240 ± 15 MeV.

to be smaller than in the extrapolation, but the energy behavior of $\sigma_{\pi^+\pi^-}$ was preserved. As in the case of the extrapolation, $\sigma_{\pi^+\pi^-}$ increases with increasing ω and there are no indications of a resonance in the region of small ω^2.

A detailed discussion of the reason for the differences is, however, premature, since the data of [19] were obtained on a polyethylene target, while the final analysis of the experiment with a hydrogen target has not yet been completed. (This question was discussed with the authors of [19].)

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