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TEMPERATURE DEPENDENCE OF THE SHAPE OF THE DOMAIN WALL IN FERROMAGNETICS AND FERROELECTRICS

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The 180° domain wall in ferromagnetic and ferroelectric substances is considered over a broad temperature range. Analytical solutions, however, are derived only near the Curie temperature; at lower temperatures only a qualitative picture is presented. The shape of the wall and its character as a whole should in general change significantly on going from low to high temperatures (region of Curie point). For example, in uniaxial ferromagnetic crystals, beginning at a certain temperature and up to the Curie point, the magnetization in the domain wall as well as in the domain volume is directed only along the axis of easiest magnetization. In Rochelle salt the solution should be exactly of this type if use is made of all known constants in the expression for the free energy.

THE free energy density F of ferromagnetics in the low-temperature region (far from the Curie point) has a rather deep minimum at some value of the magnetization M_0. Under such conditions it is natural to assume that the magnitude of M_0 is given, and only the direction of the vector $\mathbf{M}_0$ changes. It is just in this approximation that calculations of the shape of the domain wall in ferromagnetics have been carried out in the literature known to us. In the case of rotation of the vector M by 180° (180° wall), if the x axis is directed perpendicular to the wall, then $M_x = 0$, and the functions $M_y(x)$ and $M_z(x) = \sqrt{M_0^2 - M_y^2(x)}$ are known (see [1], sec 39,[2], and below).

It is easy to see, however, that a result obtained in this fashion can be invalid at elevated temperatures. Suffice it to say that near the Curie point, if anisotropy is neglected, the free energy density of a ferromagnet has the form

$$F(T, M) = F_0(T) + \tfrac{1}{2}\alpha(T) M^2 + \tfrac{1}{4}\beta M^4 \qquad \text{(A)}$$

(see [3] and [1], Sec. 36), and the absolute magnitude of the vector M in the domain wall cannot be assumed fixed. Therefore, in principle, the vector M may altogether not rotate in the transition layer (in this case $M_x = M_y = 0$, $M_z = M_z(x)$, $M_z(+\infty) = -M_z(-\infty) = M_0$). Moreover, similar solutions obviously exist, as is clear by analogy with the case of ferroelectrics.[4]

Thus, as has already been noted,[5] the question arises of the dependence of the shape of the domain wall in ferromagnetics on temperature, and we are concerned not only with changes in the parameters of the wall but also with a qualitative picture of the distribution of magnetization. Of course, significant changes in the shape of the wall can be expected only in the high-temperature region, i.e., at temperatures comparable with the Curie temperature Θ.

The above applies also to ferroelectrics, in which the spontaneous electrical polarization P is in many respects analogous to the magnetization in ferromagnetics. We note that for ferroelectrics various solutions for the domain wall are known,[4,6] but it is still not clear which of them is realized.

FERROMAGNETICS

We shall consider the behavior of the domain wall in a ferromagnetic in the vicinity of the Curie point. Then in the expansion of the free energy density (or thermodynamic potential) F in powers of the magnetization M we can limit ourselves to fourth-order terms:

$$\begin{aligned} F = F_0 &+ \tfrac{1}{2}\varkappa\,[(dM_y/dx)^2 + (dM_z/dx)^2] \\ &- \tfrac{1}{2}A(M_y^2 + M_z^2) + \tfrac{1}{2}KM_y^2 + \tfrac{1}{4}GM_y^4 \\ &+ \tfrac{1}{4}DM_z^4 + \tfrac{1}{2}RM_y^2M_z^2, \end{aligned} \qquad (1)$$

where the magnetic field and stresses are assumed to be equal to zero. In Eq. (1) there are no terms with x-component magnetization, since we shall assume later that the wall is parallel to the yz plane, and the entire change in M occurs along the x axis. Then, from the condition div $\mathbf{M} = 0$ it follows that $M_x = 0$ (solutions with $M_x \neq 0$, interesting in the case of films, will not be considered here).

Equation (1) is written for a ferromagnetic of arbitrary symmetry. For cubic crystals $K = 0$, $G = D$, and $R > D$; in uniaxial crystals (with the axis of easiest magnetization along z) $K > 0$, $G \geq D = R$. An inhomogeneous magnetostrictive deformation of the body also gives a contribution to the free energy. The condition of vanishing of the stress tensor in the center of the domain allows the strain tensor to be expressed in the free energy formula via the components of the magnetization vector (see e.g.,[2]). In this the coefficients A, K, G, D, and R are changed somewhat, and to F_0 are added terms proportional to the fourth power of the magnetization in the domain volume, which is practically equal to M_0. Since the ratio of the magnetoelastic energy to anisotropy energy is not greater than 10^{-3}, the corresponding corrections to the coefficients are small and can usually be neglected [consideration of magnetostriction is essential only for cubic crystals, where it leads to a value $K \neq 0$ in Eq. (1)]. All magnetostriction corrections to the coefficients proportional to M_0^2 also disappear when the temperature T approaches the Curie point Θ.

We now consider the general case and turn to estimates for specific ferromagnetics later. From the requirement that the free energy

$$\mathcal{F} = \int F\,dV \tag{2}$$

be a minimum, we obtain equations for the magnetization in the domain wall

$$\begin{aligned} \varkappa d^2M_y/dx^2 &= -(A-K)\,M_y + GM_y^3 + RM_yM_z^2, \\ \varkappa d^2M_z/dx^2 &= -AM_z + DM_z^3 + RM_zM_y^2. \end{aligned} \tag{3}$$

Of course, Eq. (2) does not take into account magnetic energy, something impossible to do for finite samples (see below).

In the Eqs. (3) it is convenient to transform to the dimensionless quantities:

$$t = x\sqrt{A/\varkappa}, \quad f = R/D, \quad g = G/D,$$
$$\beta = K/A, \quad \mathbf{m} = \mathbf{M}/M_0, \quad M_0 = \sqrt{A/D}. \tag{4}$$

Then the equations take the form (the primes indicate differentiation with respect to t)

$$\begin{aligned} m_y'' &= -(1-\beta)\,m_y + m_y\,(gm_y^2 + fm_z^2), \\ m_z'' &= -m_z + m_z\,(m_z^2 + fm_y^2). \end{aligned} \tag{5}$$

We must specify for these equations boundary conditions such that the solutions will be periodic functions of x with period 2L, where L is the domain thickness. Similar solutions for ferromagnetics for $T \ll \Theta$ were considered by Shirobokov.[7]

Equations (5) are analogous to the equations for the two-dimensional motion of a material point of unit mass, with coordinates $z \equiv m_z$ and $y \equiv m_y$ and potential

$$U(m_z, m_y) = \tfrac{1}{2}(m_z^2 + m_y^2)$$
$$-\tfrac{1}{2}\beta m_y^2 - \tfrac{1}{4}m_z^4 - \tfrac{1}{4}gm_y^4 - \tfrac{1}{2}fm_y^2m_z^2. \tag{6}$$

From the form of this potential it follows that for $\beta < 1$ there exists a two-dimensional potential well between the points $m_z = \pm 1$, $m_y = 0$, and two-dimensional periodic motions about the point $m_y = 0$, $m_z = 0$ are possible. For $\beta > 1$ there are possible only one-dimensional periodic solutions with $m_y \equiv 0$—only for these does a potential well exist.

The characteristic width of the domain wall is usually small in comparison with the depth of the domains L. Hence, it is practically possible to consider that the magnetization in the domain volume is the equilibrium magnetization $M_0(T)$. We assume that M_0 is parallel or antiparallel to the z axis (or one of the axes of easiest magnetization). Then in the domain volume we have $m_z = \pm 1$ and $m_y = 0$. This means, at the same time, that we are interested in those periodic solutions to Eqs. (5) for which the integral curves approach very closely to the singular points of these solutions $m_z = \pm 1$, $m_y = 0$, and $m_z' = m_y' = 0$. The period of the motion (the domain thickness) is thus determined by the proximity of the trajectory to the singular points and will be many times greater than the characteristic dimensions corresponding to the Eqs. (5). Thus, the problem actually comes down to finding all integral curves of Eqs. (5) that pass through the points $m_z = \pm 1$, $m_y = 0$, $m_z' = 0$, and $m_y' = 0$. The integrals so found will be close to the desired solutions for domains of finite thickness in view of the continuous dependence of the solutions on the initial conditions (the singular points are formally attained only for infinite domains).

Equations (5) have an integral (analogous to the energy integral)

$$(m_z')^2 + (m_y')^2 + m_z^2 + m_y^2$$
$$-\beta m_y^2 - \tfrac{1}{2}m_z^4 - \tfrac{1}{2}gm_y^4 - fm_z^2m_y^2 = \tfrac{1}{2}, \tag{7}$$

where the constant is chosen already for the integral curves that pass through the singular points. Using (7), it is possible to go over from (5) to a single equation of the second order for the functions

$$u(\xi) = -\int_0^\xi m_y^2(\xi)\, d\xi, \qquad \xi = 1 - m_z^2. \tag{8}$$

The equation for $u(\xi)$ has the form

$$(1-\xi)(u'')^2\left(\tfrac{1}{2}\xi^2 + fu\right)$$
$$= u'\left[fu + (f - 1 + \beta)u' - f\xi u' - \tfrac{1}{2}g(u')^2\right]. \tag{9}$$

This is to be solved with boundary conditions $u(0) = u'(0) = 0$.

It is obvious that ξ, u, and u' are much smaller than unity near the singular points, and Eq. (9) takes the form

$$(u'')^2\left(\tfrac{1}{2}\xi^2 + fu\right) = \delta(u')^2, \quad \delta = f - 1 + \beta. \tag{10}$$

From this, for $\delta < 1/2$, we obtain $u(\xi) = -(1 - 2\delta)\xi^2/2f$.

By successive approximation we further obtain a solution to Eq. (9) in the form of a series

$$u(\xi) = -\frac{1}{2f}(1-2\delta)\left[\xi^2 + \sum_{k>2} a_k\xi^k\right]$$
$$+ \left[C + \xi^{1+1\sqrt{2\delta}} + \sum_{\substack{k>1\\ l\geqslant 1}} b_{kl}C^l\xi^{k+l/\sqrt{2\delta}}\right], \tag{11}$$

where C is a constant. This constant is limited by the requirement that $m_y^2 \geq 0$ and the series (11) converge.

For $2\delta n^2 \to 1$ ($n = 2, 3, \ldots.$) the solution (11) transforms to

$$u(\xi) = -\frac{1}{2f}(1-2\delta)\left[\xi^2 + \sum_{k>2}^{n-1} a_k\xi^k + \sum_{\substack{k>n\\ l\geqslant 1}} d_{kl}\xi^k(\ln\xi)^l\right]$$
$$+ C\sum_{k\geqslant n} d_k(C)\xi^k. \tag{12}$$

For $\delta > 1/2$, we obtain similarly

$$u(\xi) = C\left[\xi^{1+\sqrt{2\delta}} + \sum_{\substack{k>1\\ l\geqslant 1}} a_{kl}C^l\xi^{k+l\sqrt{2\delta}}\right]. \tag{13}$$

For $\delta = 1/2$ the solutions (11) and (13) coincide. By differentiating $u(\xi)$ with respect to ξ we obtain m_y^2 as a function of $\xi = 1 - m_z^2$. After this, Eqs. (3) are easily integrated.

The solutions (11) to (13) are formal, since the series in powers of ξ can diverge. Equation (9) is nonlinear; hence the coefficients of the series (11)—(13) cannot be found in general form, and consequently their convergence cannot be investigated. In accordance with remarks made earlier, it is only possible to confirm that for $\beta > 1$ and the remaining arbitrary constants g and $f \geq 0$, the series diverge (there is only the solution (11) with C = 0, which is called linear below).

Besides solutions (11) and (12), for arbitrary values of the parameters there is a solution $U(\xi) \equiv 0$, in which case

$$m_y \equiv M_y/M_0 \equiv 0,$$
$$m_z = M_z/M_0 = \text{th}\,(t/\sqrt{2}) = \text{th}\,(x\sqrt{A/2\varkappa}). \tag{14*}$$

This solution we shall also call linear. For $\delta < 1/2$ we have both the solution (14) and the one-parameter family of rotational solutions (11) with

$$m_y^2 = \frac{1}{f}(1-2\delta)\left[(1-m_z^2) + \sum_{k>1} a'_k(1-m_z^2)^k\right]$$
$$+ C\left[(1-m_z^2)^{1/\sqrt{2\delta}} + \sum_{\substack{k\geqslant 1\\ l\geqslant 1}} b'_{kl}C^l(1-m_z^2)^{(k+l/\sqrt{2\delta})}\right]. \tag{15}$$

All the solutions (11) and (15) have the same free energy, since they are derived from the variational principle.[1)] It can be shown that the linear solution (14) has, for $\delta < 1/2$, a greater free energy than solution (15). In fact, for f = g = 1 (incidentally, these values are close to those realized in uniaxial crystals), there is among the solutions (15) the exact solution:

$$m_y^2 = (1-2\beta)(1-m_z^2),\ m_z(t) = \text{th}\,(t\sqrt{\beta}) = \text{th}\,(x\sqrt{K/\varkappa}) \tag{16}$$

(this solution is obtained from (15) if one sets C = 0). For this solution the surface energy of the domain wall is

$$\sigma = \int_{-\infty}^{+\infty}\left[F(x) - F_0 + \tfrac{1}{4}AM_0^2\right]dx = 2AM_0^2\left(1 - \tfrac{2}{3}\delta\right)\sqrt{\varkappa\delta},$$

since at equilibrium we have in the domain volume $F = F_0 = \frac{1}{4}AM_0^2$. For the linear solution (14), on the other hand,

$$\sigma = \sqrt{8}AM_0^2\sqrt{\varkappa}/3, \tag{18}$$

which for $\delta < 1/2$ is greater than the surface energy (17) for the elliptical rotational solution (16) and, as mentioned, for all solutions (15). For $f \approx g \approx 1$, the situation is unchanged because of the continuity of σ as a function of f and g. Hence it can be assumed that for $\delta < 1/2$ a solution from the family (15), and not the linear solution (14), is realized.

For $\delta > 1/2$, we have from (13)

$$m_y^2 = C\left[(1-m_z^2)^{\sqrt{2\delta}} + \sum_{\substack{k\geqslant 1\\ l\geqslant 1}} a'_{kl}C^l(1-m_z^2)^{(k+l\sqrt{2\delta})}\right]. \tag{19}$$

The linear solution (14) is obtained from this for C = 0. The free energy for all solutions of the

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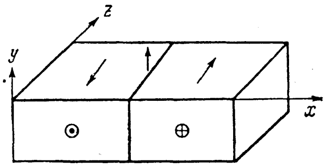
*th = tanh.

[1)] All solutions of the variational problem lead to extremal values of $\mathscr{F}$; hence, to solutions obtained from each other by a continuous variation of a parameter there corresponds one value of $\mathscr{F}$.



type (19) is, of course, the same. The choice of solutions from the family (15) for $\delta < 1/2$ and from the family (19) for $\delta > 1/2$ obviously cannot be made solely from the requirement of minimum energy (2). However, a well-defined choice can be made by taking into account the energy of the magnetic field outside of the ferromagnet.

We shall consider, for example, a ferromagnet in the shape of a parallelepiped (see the figure), where the arrows indicate the direction of magnetization on the upper surface. The field on the end of the sample—the xy plane—determines the dimensions of the domain. However, a magnetic field exists also on the other face—the xz plane, but only near the domain wall. The energy of this field, for the linear solution with $M_y \equiv 0$, will be the smallest of all the solutions of the family (19) (in this case there is in general no field in the vicinity of the domain wall in the xz plane); therefore, for $\delta > 1/2$ in the present case we obtain only a linear solution. In the family (15) minimum field energy is evidently obtained for a solution in which $|M_y|$ is a minimum in the center of the domain wall.

We turn now to a consideration of the temperature dependence of the shape of the domain wall. The structure of the domain wall is determined by the parameter $\delta = f - 1 + \beta = R/D - 1 + K/A$. For uniaxial ferromagnetics $R = D$, and near the Curie point Θ the coefficient K, generally speaking, does not vanish. According to the theory of second-order phase transitions, the coefficient $A = a(\Theta - T)$ for $T \to \Theta$, and therefore, above the temperature

$$T_{1/2} \equiv T\left(\delta = \tfrac{1}{2}\right) = \Theta - 2K/a \tag{20}$$

the parameter $\delta > 1/2$, and under the conditions represented in the figure, we should have the linear solution (14) with surface energy (18) and a domain wall thickness of the order $l \approx \sqrt{\kappa/A}$, which in the vicinity of the Curie point increases as $(\Theta - T)^{-1/2}$.

For $T < T_{1/2}$ the parameter $\delta < 1/2$, and we obtain a solution of a more complex type (15). For this $M_z^2 \ll M_y^2$ and $M_y^2 \approx (1 - 2\delta) M_0^2/f$ in the center of the domain wall. For $T \to T_{1/2}$ the modulus of the magnetization vector in the domain wall decreases to zero, and the solution goes over continuously into the linear one.

As the temperature is lowered to the region $T \ll \Theta$ the solutions (15) should transform to the Bloch solution with $m_z^2 + m_y^2 = 1$. We have not investigated how this transition takes place because Eq. (1) for the free energy density cannot describe a ferromagnetic in this region. Obviously, at temperatures $T \ll \Theta$ the free energy has a sharp minimum for $M_z^2 + M_y^2 = M_0^2$, which requires the calculation of a larger number of terms in the expansion of the free energy density in powers of the magnetization.

The general picture of the variation of the domain wall with temperature can be visualized in this way. At sufficiently low temperatures, $M_z^2 + M_y^2$ is practically a constant (independent of direction), and in the domain wall of a uniaxial ferromagnetic we have the well-known circular rotational solution (see [1], Sec. 39)

$$M_z^2 + M_y^2 = M_0^2, \quad M_z = M_0 \operatorname{th}\left(x\sqrt{K/\varkappa}\right). \tag{21}$$

With rise in temperature, the free energy density begins to depend on M less strongly, and in a state of thermodynamic equilibrium, the magnitude of the vector M already depends on direction. In the domain wall therefore one has a rotational solution which is not quite, but almost, circular (at low and middle temperatures).[13] If we neglect magnetic energy, we then have many (a "bundle" of) solutions with the same value of σ, but not one of them corresponds to M becoming zero in the wall. Rather, for rotational solutions $M \neq 0$ up to a point $T_{1/2}$, although the following possibility is also logically conceivable: with an increase in temperature (at $T = T_0$) there appear solutions with $M = 0$ in the center of the wall. The behavior of the curves in the region $T \approx T_0$ is still unclear. In any case, we have obtained solutions for $T > T_0$ outside of the immediate vicinity of the temperature T_0 (obviously, $T_0 < T_{1/2} < \Theta$). It seems that if the point T_0 exists, then $M_0(T_0)$ is of the order of the difference in the moments M along the axes of easiest and hardest magnetization.

It has been assumed above that for uniaxial crystals K does not become zero at the Curie temperature. This assumption leads to a dependence of the Curie point on the direction of magnetization,[2] which is in agreement with the results

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[2] The question, of course, is not one of an equilibrium phase transition in the absence of an external field (in this case the moment is automatically directed along the axis of easiest magnetization). The Curie temperature in the direction of hard magnetization is conventionally understood to mean the temperature at which the relative minimum in the thermodynamic potential along this axis disappears (in a vanishing field).

of Callen.[8] The anisotropy constant K near the Curie temperature, as all other parameters in the equation for the free energy, can be found from measurements of the magnetization $M_0(T)$ and the magnetic permeability in the corresponding temperature region for different directions of the magnetic field relative to the crystal axes. The order of magnitude of the constant K can be obtained from the theory of the temperature dependence of anisotropy energy (see, e.g.,[9]). We point out that Eq. (1) also agrees with this theory. For classical ferromagnetics with anisotropy field $H_a = KM_0$ ($T = 0$) of the order of 10^4 Oe and with $a \sim 1$ to 10, the separation $\Theta - T_{1/2}$ is approximately 1 to 10°.

The authors do not know of any experiments that confirm the existence of a linear solution in the domain wall. In this connection, the work of Gossard et al.[10] would seem to be of great interest. Using nuclear magnetic resonance, they measured the magnetization in the center of a domain inside of a domain wall in the uniaxial ferrodielectric $CrBr_3$, which has a Curie temperature of 37°K and an anisotropy field of 6850 Oe. As the temperature was increased from 1.5 to 7°K, the magnetization inside the domain wall fell off faster than in the domain volume, which agrees qualitatively with our calculations. Unfortunately, no measurements were made of the behavior of the magnetization in the wall for higher temperatures. It should be noted that the ferrodielectric $CrBr_3$ is very convenient for the observation of the temperature dependence of the domain wall because of its large anisotropy and relatively weak exchange interaction. No less interesting is ludlamite, $Fe_3(PO_4)\cdot 4H_2O$, with $\Theta = 20$°K and a very strong anisotropy.[11]

The transition of the distribution of the magnetization in the wall to a linear solution with $M_y \equiv 0$, i.e., temperature $T_{1/2}$, can apparently be determined by means of magneto-optical changes. This transition in multi-domain samples is accompanied by a discontinuity in the heat capacity, which in order of magnitude amounts to $\Delta c l/L$, where Δc is the heat capacity discontinuity at the Curie point, and $l \approx \sqrt{\kappa/K}$ is the thickness of the wall at the temperature $T_{1/2}$.

In cubic crystals the quantity δ is everywhere very small. In fact, for cubic crystals, as has already been mentioned, the quantity K is different from zero only because of magnetostriction and hence is small everywhere (in order of magnitude it is not greater than 10^{-7}). The quantity $R/D - 1$ is also small; it is of order 10^{-4} at room temperature and does not increase with increasing temperature. Therefore, the linear solution does not exist in cubic crystals, and the shape of the domain wall on approach to the Curie point cannot be of the Bloch type but is described by a more complicated solution like (15). Therefore, an investigation of the properties of the domain wall near the Curie temperature is of interest also in cubic crystals.

FERROELECTRICS

All the results obtained previously are in essence valid also for ferroelectrics in which the phase transition can be described by the polarization vector (this is not always so, see [12]). For the change to ferroelectrics, it is only necessary to replace the magnetization **M** with the polarization **P** in all the formulas and, of course, choose different values for the coefficients. In contrast to ferromagnetics, striction effects are more significant in the case of ferroelectrics. It is also necessary to consider the piezoelectric effect for ferroelectrics.

For example, the free energy of a rhombohedral crystal of class D_2 (Rochelle salt), including piezo-effects and electrostriction, has the form

$$\begin{aligned} F = {} & \tfrac{1}{2}(\alpha_{22}P_y^2 + \alpha_{33}P_z^2) + \tfrac{1}{4}(\beta_{22}P_y^4 + \beta_{33}P_z^4) + \tfrac{3}{2}\beta_{23}P_y^2P_z^2 \\ & + \tfrac{1}{2}(c_{11}u_{xx}^2 + c_{22}u_{yy}^2 + c_{33}^2u_{zz}^2 + 4c_{12}'u_{xy}^2 + 4c_{13}'u_{xz}^2 \\ & + 4c_{23}'u_{yz}^2 + 2c_{12}u_{xx}u_{yy} + 2c_{13}u_{xx}u_{zz} + 2c_{23}u_{yy}u_{zz}) \\ & + 2\lambda_{231}P_yu_{zx} + 2\lambda_{312}P_zu_{xy} + \tfrac{1}{2}(q_{22}P_y^2u_{yy} + q_{33}P_z^2u_{zz} \\ & + q_{21}P_y^2u_{xx} + q_{23}P_y^2u_{zz} + q_{31}P_z^2u_{xx} \\ & + q_{32}P_z^2u_{yy} + 4q_{2323}P_yP_zu_{yz}). \end{aligned} \tag{22}$$

We exclude the strain tensor u_{ik} from the condition of vanishing of the stress tensor $\sigma_{ik} = \partial F/\partial u_{ik}$ in the domain volume. Having done this, we obtain Eq. (1) with **M** replaced by **P** and with coefficients

$$\begin{aligned} A &= \alpha_{33} - \lambda_{312}^2/c_{12} + P_0^2[\mu q_{32} + \lambda q_{33} - q_{31}c_{11}^{-1}(\mu c_{12} + \lambda c_{13})] \\ A - K &= \alpha_{22} - \lambda_{231}^2/c_{13} \\ &\quad + P_0^2[\mu q_{22} + \lambda q_{23} - q_{21}c_{11}^{-1}(\mu c_{12} + \lambda c_{13})], \\ D &= \beta_{33} - q_{31}^2/2c_{11}, \qquad G = \beta_{22} - q_{21}^2/2c_{11}, \\ R &= 3\beta_{23} - q_{31}q_{21}/2c_{11}, \end{aligned} \tag{23}$$

where P_0 is the polarization in the domain volume and λ, ρ, μ is the solution of the system of equations

$$c_{i1}\rho + c_{i2}\mu + c_{i3}\lambda = -\tfrac{1}{2}q_{i3}, \quad i = 1, 2, 3. \tag{24}$$

As D. G. Sannikov has so kindly informed us, according to the literature data $A(T) = (\Theta - T)/C_\Theta$, $C_\Theta = 178°$, and $A - K = 1.35$ or 1.55 for room temperatures and above depending on direction of the x axis relative to the crystallographic axes. For these coefficients it is always true that $K > A$; hence in Rochelle salt in the indicated temperature interval the linear solution found by Zhirnov[4] should obtain. Similarly, we can find the structure of the domain wall for other ferroelectrics for which we can limit the free energy expansion to fourth-order terms. Ferroelectrics of the barium titanate type are described by free energies involving the polarization to the sixth power. The calculation for this case will be presented later.[14]

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