ABSENCE OF SUPERCONDUCTIVITY IN METALS OF THE BISMUTH TYPE

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It is shown that the reason for the absence of superconductivity in metals of the bismuth type is the weakness of the interaction between electron excitations.

PREVIOUSLY [1] we have obtained the dielectric tensor for metals with crystal lattices of the bismuth type. The calculation was based on a theory of the electron spectrum developed in the article by the author and Fal'kovskiĭ. [2] It was found that the principal values of the dielectric tensor, regarded as functions of frequency and wave vector $\epsilon_{ik}(\omega, \mathbf{k})$ in the frequency range $\omega \sim \gamma \sim 0.01$ to 0.1 eV (for momenta $k \lesssim \gamma/v$, where $v \sim 10^8$ cm/sec), and also in the region of momenta $k \sim \gamma/v$ (at frequencies $\omega \lesssim \gamma$), are the order of $(E_0/\gamma)^2 \sim 100$ ($E_0 \sim 0.1$ to 1 eV). These regions of frequencies and momenta are of the greatest interest, since it is precisely in these regions that the Fermi energy and limiting Fermi momentum of electron and hole excitations occur.

The large dielectric constant reduces the mutual interaction between electrons at momentum and energy transfers in the ranges mentioned, and also weakens the interaction of electrons with phonons of corresponding momenta and frequencies. It is shown in the present article that this circumstance is the reason for the absence of superconductivity in the metals under consideration.

As is well-known (see, for example, [3]), in an investigation of the superconductivity, it is first of all necessary to consider two types of elementary electron interactions. The first of these interactions—the direct Coulomb interaction of electrons in the momentum representation—has the form

$$U_1(\omega, \mathbf{k}) = 4\pi e^2/k_i k_k \varepsilon_{ik}(\mathbf{k}, \omega). \quad (1)$$

The second interaction results from phonon exchange. As indicated in [1], the vertex which describes the interaction of an electron with a phonon in the case under consideration is

$$\zeta [a^{3/2} m^{1/2} k_i k_k \varepsilon_{ik}(\mathbf{k}, \omega)]^{-1}, \quad (2)$$

where $\zeta \sim 1$, a is the lattice period and m is the mass of the free electron.

Intending only to estimate the effect, we examine the Debye model for the phonons. In this case, the D-function of the longitudinal phonons has the form $\omega_0^2(\mathbf{k})/[\omega^2 - \omega_0^2(\mathbf{k})]$ (see [4]). Therefore, the vertex which describes the elementary interaction of electrons via phonon exchanges is

$$U_2(\omega, \mathbf{k}) = \frac{\zeta^2}{a^3 m [k_i k_k \varepsilon_{ik}(\mathbf{k}, \omega)]^2} \frac{\omega_0^2(\mathbf{k})}{\omega^2 - \omega_0^2(\mathbf{k})}. \quad (3)$$

According to Eq. (3), $U_2(\omega, \mathbf{k})$ decreases rapidly when ω significantly exceeds $\omega_0(\mathbf{k})$. If $\omega \lesssim \omega_0(\mathbf{k})$, then one can replace $D(\mathbf{k})$ in $U_2(\omega, \mathbf{k})$ by -1. Thus U_2 is negative in the important frequency region.

Inasmuch as the Fermi surface in the metals under consideration consists of very small regions in the neighborhood of specific points in momentum space, two cases are possible. Upon emission of a phonon, the electron either remains in its "own" region, or else it goes over to one of the "foreign" regions.

First we consider $U_2^{(1)}$ for the case when the electron remains in its "own" region. In this case the phonon momentum must be less than γ/v, and the energy less than $\omega_0(\gamma/v) \sim u\gamma/v \ll \gamma$. According to the results of the previous article, [1] $\epsilon_{ik} \sim \epsilon_0 \gamma^2/v^2 k^2$ for $\omega \lesssim vk$, where $\epsilon_0 \sim (E_0/\gamma)^2 \sim 100$. This means that the Coulomb interaction is screened over distances of the order of γ/v, and therefore small $\mathbf{k}$ do not play a preferred role. Substituting ϵ_{ik} into (3) and taking the rapid decrease of U_2 for $\omega > \omega_0(k)$ into account, we obtain instead of (3) the approximate expression

$$U_2^{(1)} \approx \begin{cases} -v^4 C_1'/a^3 m \gamma^4 \epsilon_0^2 = -C_1 a/m, & \omega \leqslant \omega_1 \sim u\gamma/v \\ 0, & \omega > \omega_1 \end{cases} \quad (4a)$$

for $k \sim \gamma/v$. We use here the fact that $v/a \sim E_0 \sim 1$ eV and $\epsilon_0 \sim (E_0/\gamma)^2$. The constants C_1' and C_1 are positive and of order unity.

In the second case, when the electron goes from one isolated part of the Fermi surface to another,

the momentum of the emitted phonon is of the order of the period of the reciprocal lattice, i.e., its energy $\omega_0 \sim \omega_D \lesssim \gamma$. Hence once more $\omega \ll kv$ and $\epsilon_{ik}k_ik_k \sim E_0^2/v^2$. An approximate expression for the interaction is in this case

$$U_2^{(2)} \approx \begin{cases} -C_2 a/m, & \omega \leqslant \omega_2 \sim \omega_D \sim \gamma \\ 0, & \omega > \omega_2 \end{cases} \quad (4b)$$

for $k \sim E_0/v$. The constant C_2 is positive and of order unity. Here it is essential to note that, in contrast to the interaction $U_2^{(1)}$ which can be regarded as approximately isotropic with respect to the direction of $\mathbf{k}$, the interaction $U_2^{(2)}$ is effective only in very narrow regions of solid angles [of order $(\gamma/E_0)^2$]. This is connected with the fact that upon absorption or emission of a phonon with momentum $k \sim E_0/v$ the electron must go from one part of the Fermi surface to another. But the linear dimensions of these segments in momentum space are of order γ/v. As is well-known, U multiplied by the density of final states of the electron plays the role of a dimensionless coupling constant. As a result it turns out that $U_2^{(2)}$ is equivalent to an isotropic interaction of the type (4b), for which, however, $k \sim \gamma/v$. As we shall presently see, the Coulomb interaction has approximately the same form (apart from the sign).

It is also possible to represent the direct Coulomb interaction of the electrons (1) in an approximate form. Here, too, two cases are possible: either the electrons (from the same or from different isolated segments) remain in their "own" segments, or electrons are exchanged by two different segments. In the first case $k \sim \gamma/v$ and, as we see below, $\omega \lesssim kv$ are important. In the second case $k \sim E_0/v$, $\omega \lesssim \gamma$, i.e., $\omega \ll kv$; hence, in both cases $\omega \lesssim kv$ and $\epsilon_{ik}k_ik_k \sim E_0^2/v^2$.

With regard to transitions of the second type, the same considerations pertaining to the restriction of solid angles are also valid for the interaction by means of exchanges of short-wave phonons. In view of this, both types of electron transitions give one and the same effect, approximately equivalent to the contribution from the isotropic interaction:

$$U_1 \approx \begin{cases} e^2v^2C_3'/\varepsilon_0\gamma^2 = C_3 a/m, & \omega \leqslant \omega_2 \sim \gamma \\ 0, & \omega > \omega_2 \end{cases} \quad (5)$$

for $k \sim \gamma/v$ (C_3 is a positive constant of order unity). Adding the two contributions, we obtain

$$U = U_1 + U_2 = \begin{cases} B_1 a/m, & \omega < \omega_1 \sim \gamma u/v \\ B_2 a/m, & \omega_1 < \omega < \omega_2 \sim \gamma, \\ 0, & \omega > \omega_2 \end{cases} \quad (6)$$

where $B_1 = C_3 - C_1$, $B_2 = C_3 - C_2$, where B_1 and B_2 are of order unity and may have arbitrary sign.

In order to analyze the question of superconductivity, we apply the method of Cooper in a manner similar to [5]. Here U plays the role of Fourier component of the potential, and it is necessary to set $\mathbf{k} = \mathbf{p} - \mathbf{p}'$, $\omega = \xi(\mathbf{p}) - \xi(\mathbf{p}')$, $\xi = \epsilon(\mathbf{p}) - \mu$. Near the Fermi level $\xi(\mathbf{p}) \approx v(p - p_0)$, i.e., $\omega = \xi(\mathbf{p}) - \xi(\mathbf{p}') \approx v(p - p') \lesssim v(|\mathbf{p} - \mathbf{p}'|) = vk$. This justifies approximations (4) and (5) for the potentials U_1 and U_2. The Schrödinger equation for the Cooper-pair wave function has the form

$$(2\xi(\mathbf{p}) - E)\, a_{\mathbf{p}} + B_1 \frac{a}{m} \frac{1}{V} \sum_{\mathbf{p}'}^{(1)} a_{\mathbf{p}'} + B_2 \frac{a}{m} \frac{1}{V} \sum_{\mathbf{p}'}^{(2)} a_{\mathbf{p}'} = 0. \quad (7)$$

Here $a_{\mathbf{p}}$ is the coefficient of the expansion of the pair function $\psi(\mathbf{r}, \mathbf{r}')$, with total momentum equal to zero, in terms of the functions $\exp[i\mathbf{p}\cdot(\mathbf{r} - \mathbf{r}')]$; the first summation goes over the region $|\xi - \xi'| < \omega_1$, $\xi' > 0$, the second is over the range $\omega_2 > |\xi - \xi'| > \omega_1$, $\xi' > 0$. Expression (6) is substituted for the potential energy.

We introduce the symbols

$$\alpha = \frac{1}{V} \sum_{\mathbf{p}'}^{(1)} a_{\mathbf{p}'}, \qquad \beta = \frac{1}{V} \sum_{\mathbf{p}'}^{(2)} a_{\mathbf{p}'}. \quad (8)$$

From (7) we obtain

$$(2\xi(\mathbf{p}) - E)\, a_{\mathbf{p}} = -\frac{a}{m} B_1 \alpha - \frac{a}{m} B_2 \beta, \qquad \xi < \omega_1,$$

$$(2\xi(\mathbf{p}) - E)\, a_{\mathbf{p}} = -\frac{a}{m} B_2 (\alpha + \beta), \qquad \omega_2 > \xi > \omega_1.$$

From this and from (8) we obtain equations for α and β. Since the integrals with respect to $\xi(\mathbf{p})$ appearing in these (equations) are logarithmic, it turns out that (values of) $\xi \ll \epsilon_F \sim \gamma \sim \omega_2$ are important, i.e., the integration is near the Fermi surface. In view of this, the system of equations takes the form

$$\alpha\left(1 + K_1 \ln \frac{\omega_1}{\Delta}\right) + \beta K_2 \ln \frac{\omega_1}{\Delta} = 0,$$
$$\alpha K_2 \ln \frac{\omega_2}{\omega_1} + \beta\left(1 + K_2 \ln \frac{\omega_2}{\omega_1}\right) = 0, \quad (9)$$

where $\Delta = -E/2$, and the constants K_1 and K_2 are of order $|K_{1,2}| \sim \gamma^2 a/v^3 m \sim \gamma^2/E_0^2 \sim 1/\epsilon_0 \sim 10^{-2}$, where the signs of the constants K_1 and K_2 can be arbitrary.

Since $v/u \sim 10^2$, $K_2 \ln(\omega_2/\omega_1) \sim K_2 \ln(v/u) \ll 1$. Therefore the equation for the gap in the spectrum has the form

$$1 + K_1 \ln(\omega_1/\Delta) = 0. \quad (10)$$

If K_1 is negative, then this equation has the solution

$$\Delta = \omega_1 \exp(1/K_1).$$

In other words,

$$\Delta \sim uv^{-1}\gamma \exp(1/K_1) \sim \exp(-1/|K_1|) \text{ deg} \quad (11)$$

where $1/|K_1| \sim 100$. This means that superconductivity can exist in principle, but at temperatures which cannot be reached experimentally.

This also explains the well-known fact that superconductivity is not observed in metals of the bismuth type. Such a conclusion is a consequence of the special structure of the electron spectrum, i.e., in the final analysis, a consequence of the crystal structure of the metals in question. The other modifications of bismuth which exist under large pressures are, as is well-known, superconducting.[6]

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