FACTORIZATION AT THE REGGE POLE OF SCATTERING AMPLITUDES OF PARTICLES WITH SPIN

D. V. VOLKOV

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It is shown that by making use of derivatives with respect to particle momenta the contribution of a boson Regge pole to the amplitude for scattering of particles with spin may be represented in an explicitly factorized form.

RECENTLY Gell-Mann[1] and Gribov and Pomeranchuk[2] have shown that the residues of different amplitudes at the Regge pole satisfy simple factorization relations. This gives rise to important relations between scattering cross sections for different processes at high energies.

The factorization of the residues at the Regge pole when the spin structure of the amplitude is taken into account has been considered in a number of papers (see, e.g., [3-5]). In particular, Gribov and Pomeranchuk[3] have remarked that the factorization of the residue makes it possible to write the contribution of each Regge pole to the scattering amplitude as if it came from the Feynman diagram corresponding to the exchange of a "reggeon," and they obtained asymptotic expressions for the Green's function of the "reggeon" and the vertex parts. The expression for the vertex part proposed in [3] contained, however, a dependence on the momenta of particles belonging to an "alien" vertex. This circumstance upset the simple interpretation of the contribution from the Regge pole as corresponding to the transition initial state → reggeon → final state.

In the work of Gribov and the author[4] the contribution from the Regge pole was expressed in a form in which none of the vertices contained "alien" momenta. However, just as in [3], that representation was valid only asymptotically for large values of cos θ.

Here we consider a representation of the contribution from the Regge pole to the amplitude for the scattering of particles with spin, which is valid for arbitrary cos θ and which takes explicitly into account the factorization property of the residues. Use of this representation makes it possible to avoid the usual awkward procedure of "reggeization" (see, e.g., [5]) and may be useful in the study of the analytic properties of scattering amplitudes.

Let us consider the process $a_1 + a_2 \rightarrow a_1' + a_2'$ where a_1 and a_1' are some particles and a_2 and a_2' are some antiparticles. We assume that the spin of every particle equals a half and that the masses are arbitrary. We denote the particle and antiparticle momenta by respectively p_1, p_1', p_2 and p_2'. As independent vectors we choose the following combinations of 4-momenta:

$$P_\mu = (p_1 + p_2)_\mu = (p_1' + p_2')_\mu,$$
$$Q_\mu = (p_1 - p_2)_\mu - (p_1 + p_2)_\mu (m_1^2 - m_2^2)/t, \qquad (1)$$
$$Q_\mu' = (p_1' - p_2')_\mu - (p_1' + p_2')_\mu (m_1'^2 - m_2'^2)/t,$$

and as independent invariants

$$t = (p_1 + p_2)^2, \qquad \cos\theta = -QQ'/\sqrt{Q^2 Q'^2}.$$

For the process under consideration the contribution of the Regge pole to the scattering amplitude has the form

$$-(2j(t) + 1)(A' + B'\hat{D}')$$
$$\times (A + B\hat{D})[P_{j(t)}(-\cos\theta) \pm P_{j(t)}(\cos\theta)]\pi/2\sin\pi j(t) \quad (2a)$$

for the Regge pole with parity $(-1)^j$ and

$$-(2j(t) + 1)(A'\gamma_5' + B'\gamma_5'\hat{D}') \times (A\gamma_5 + B\gamma_5\hat{D})[P_{j(t)}(-\cos\theta)$$
$$\pm P_{j(t)}(\cos\theta)]\,\pi/2\sin\pi j(t) \quad (2b)$$

for the Regge pole with parity $(-1)^{j+1}$, where A, B, A′ and B′ are functions of t; $\hat{D} = \gamma_\mu D_\mu$, $\hat{D}' = \gamma_\mu' D_\mu'$;

$$D_\mu = \partial/\partial Q_\mu - Q^{-2} Q_\mu (Q\partial/\partial Q) - t^{-1} P_\mu (P\partial/\partial Q),$$
$$D_\mu' = \partial/\partial Q_\mu' - Q'^{-2} Q_\mu' (Q'\partial/\partial Q') - t^{-1} P_\mu (P\partial/\partial Q'); \quad (3)$$

γ_μ and γ_μ' are two sets of Dirac matrices referring respectively to the initial and final states; the signs ± refer to the pole signature. For brevity we have omitted in Eqs. (2a) and (2b) the spinors $u(p_1, \mu_1)$, $\bar{v}(-p_2, \mu_2)$, $\bar{u}(p_1', \mu_1')$ and $v(-p_2', \mu_2')$, it being understood that each of the brackets $(A + B\hat{D})$ is multiplied by the spinors of the initial or final state depending on which set of Dirac matrices is involved in the bracket in question.

To justify the Eqs. (2a) and (2b) we consider the functions

$$\langle j, m, 1 | \mathbf{n}, \mu_1, \mu_2\rangle = \left[\frac{2m_1m_2}{t-(m_1+m_2)^2}\right]^{1/2} \bar{v}(-p_2, \mu_2)\, u(p_1, \mu_1) Y_{jm}(\mathbf{n}), \tag{4a}$$

$$\langle j, m, 2 | \mathbf{n}, \mu_1, \mu_2\rangle = \left[\frac{2m_1m_2(t-(m_1+m_2)^2)}{j(j+1)t}\right]^{1/2} \times \bar{v}(-p_2, \mu_2)\gamma_\mu u(p_1, \mu_1) D_\mu Y_{jm}(\mathbf{n}), \tag{4b}$$

$$\langle j, m, 3 | \mathbf{n}, \mu_1, \mu_2\rangle = \left[\frac{2m_1m_2}{t-(m_1-m_2)^2}\right]^{1/2} \bar{v}(-p_2, \mu_2)\gamma_5 u(p_1, \mu_1) Y_{jm}(\mathbf{n}), \tag{4c}$$

$$\langle j, m, 4 | \mathbf{n}, \mu_1, \mu_2\rangle = \left[\frac{2m_1m_2(t-(m_1-m_2)^2)}{j(j+1)t}\right]^{1/2} \bar{v}(-p_2, \mu_2) \times \gamma_5\gamma_\mu u(p_1, \mu_1) D_\mu Y_{jm}(\mathbf{n}) \tag{4d}$$

for integer values of j.

Since the coefficients in front of the spherical harmonics $Y_{jm}(\mathbf{n})$ are invariant with respect to spatial rotations and depend on the variables of the initial state only it follows that the functions (4a) – (4d) describe the initial state of a system with angular momentum j, z-component m, and definite parity. The parity of the states (4a) and (4b) is equal to $(-1)^j$, of the states (4c) and (4d) $-(-1)^{j+1}$. The functions (4a) – (4d) are orthogonal and normalized in such a way that

$$\sum_{\mu_1\mu_2}\int\frac{d\Omega}{4\pi}\langle j, m, \alpha | \mathbf{n}, \mu_1, \mu_2\rangle\langle \mathbf{n}, \mu_1, \mu_2 | j, m, \alpha'\rangle = \delta_{\alpha\alpha'}.$$

Let us note that the appearance in Eqs. (4a) – (4d) of derivatives with respect to particle momenta is due to the requirement that the functions (4a) – (4d), and consequently the vertices of the type $A + B\hat{D}$ and $A\gamma_5 + B\gamma_5\hat{D}$ in Eqs. (2a) and (2b), be independent of the particle momenta in the final states. Indeed, in order to construct independent spin combinations one must make use of the matrix γ_μ, but the product of γ_μ by the momenta of their "own" particles (p_1 and p_2) reduces to a mass as a consequence of Dirac's equation and does not give rise to new invariants.

The form of the derivative D_μ [Eq. (3)] chosen by us corresponds to the exclusion from the conventional derivative $\partial/\partial Q_\mu$ of the longitudinal and time components. With this choice of differentiation particles remain on their mass shells so that no ambiguities are produced in differentiation of the invariants.

In going over to nonrelativistic notation the derivative D_μ goes into $\nabla_\mathbf{n}$ (where $\mathbf{n}$ is a unit vector in the direction of the relative momentum of the particles in the initial state), and the functions (4a) – (4d) go over respectively into $\chi_2^*\boldsymbol{\sigma}\cdot\mathbf{n}\chi_1 Y_{jm}(\mathbf{n})$, $\chi_2^*\boldsymbol{\sigma}\cdot\nabla_\mathbf{n}\chi_1 Y_{jm}(\mathbf{n})$, $\chi_2^*\chi_1 Y_{jm}(\mathbf{n})$ and $\chi_2^*\boldsymbol{\sigma}\cdot(\mathbf{n}\cdot\nabla)\chi_1 Y_{jm}(\mathbf{n})$ [where χ_1 and χ_2 are two-component spinors corresponding to $u(p_1, \mu_1)$ and $\bar{v}(-p_2, \mu_2)$]. The first two are triplet states and correspond to combinations of states with $j = l \pm 1$, that appear in the spherical vectors.[6] The third and fourth are respectively singlet and triplet states with $j = l$.

In order to obtain an expression for the contribution of individual Regge poles to the scattering amplitude we write the latter in the form

$$\langle \mathbf{n}', \mu_1', \mu_2' | F | \mathbf{n}, \mu_1, \mu_2\rangle = \sum_{jm}\sum_{\alpha,\alpha'=1}^{4}\langle \mathbf{n}', \mu_1', \mu_2' | j, m, \alpha'\rangle f^j_{\alpha'\alpha}(t)\langle j, m, \alpha | \mathbf{n}, \mu_1, \mu_2\rangle. \tag{5}$$

Applying to Eq. (5) the usual Sommerfeld-Watson transformation, making use of Eqs. (4a) – (4d), and taking into account that the residues of the partial amplitudes $f^j_{\alpha'\alpha}(t)$ factorize at the pole we obtain for the contribution of an individual pole the expressions (2a) and (2b). It follows from the normalization condition of the functions (4a), (4b) that the coefficients A, A, A′ and B′ in Eq. (2a) are related to the residues $r_{\alpha'\alpha}(t)$ of the amplitudes $f_{\alpha'\alpha}(t)$ as follows:

$$A = \left[\frac{2m_1m_2}{t-(m_1+m_2)^2}\right]^{1/2} a, \qquad B = \left[\frac{2m_1m_2(t-(m_1+m_2)^2)}{j(j+1)t}\right]^{1/2} b,$$

$$A' = \left[\frac{2m_1'm_2'}{t-(m_1'+m_2')^2}\right]^{1/2} a', \qquad B' = \left[\frac{2m_1'm_2'(t-(m_1'+m_2'))}{j(j+1)t}\right]^{1/2} b';$$

$$aa' = r_{11}, \quad bb' = r_{22}, \quad ab' = r_{12}, \quad a'b = r_{21}. \tag{6}$$

The analogous relations for the coefficients in Eq. (2b) may be obtained from Eq. (6) by the replacement of $m_1 + m_2$ by $m_1 - m_2$ and of the states 1 and 2 by the states 3 and 4.

We have considered here the factorization of the scattering amplitude in the case when the spin of every particle is equal to a half. The distinction between particle and antiparticle made by us is not essential since the replacement $v(-p, \mu) \leftarrow C\bar{u}(p, \mu)$ allows one to replace any of the antiparticles by a particle and vice versa.

The factorization at the boson Regge pole of amplitudes for other processes may be carried out in an analogous manner. The vertices for each specific pair of particles coupled to the "reggeon" are easily found from considerations of invariance. The factorization of the amplitude at a fermion Regge pole is somewhat more complicated and will be considered separately.

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