PHASE SHIFT ANALYSIS OF NUCLEON-NUCLEON SCATTERING AT 147 *MeV*

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The phase shifts previously determined by the authors are evaluated with higher accuracy on the basis of recent experimental data. It is shown that this solution is unique in the vicinity of ±5°. In quite a few of the cases the phase shifts differ by three or more standard deviations from those derived by Breit et al [8] as well as from those calculated from the Hamada–Johnston potentials [10].

SEVERAL papers published following our phase shift analysis of nucleon-nucleon scattering at 147 MeV have added greatly to our knowledge of the characteristics of np and pp scattering at this energy [1–5]. It must be noted that the new data have fitted satisfactorily on the curves calculated from the phase shifts of the previously obtained single solution [6,7]. However, the desire to reduce the errors of the phase shifts has induced the authors to use the additional information and to make more precise the previously obtained solution. This is also of interest, in part, when attempting to ascertain more precisely the extent to which the results of the phase-shift analysis agree with the phase shifts obtained in other ways [8–10]. Unfortunately, the phase shift analysis of Perring [3], which was made relatively recently, was carried out without an error calculation and did not answer this question unambiguously.

The data used for the phase shift analysis are listed in Table I. The analysis yielded more accurate phase shifts for $l_{\max} = 3$, when the amplitude of the nucleon-nucleon scattering is taken in the 1-meson approximation, starting with angular momentum values $l = 4$ and higher. A solution was found for $l_{\max} = 4$. Plots of the experimentally measured quantities against the scattering angle and elements of the transition matrix were calculated (see Figs. 1–4, where the points indicate the experimental data used for the phase shift analysis, and the vertical segments show the error bars in the case when the errors exceed 5%). The stability of the solutions under small changes in the effective energy (147–143 MeV) was considered. At $l_{\max} = 3$, a search was also made for solutions under random displacements of ± 5% in the phases of the S, P, and D waves and ± 2.5% in the phases of the F waves. This yielded no new solutions.

Table I

Measured quantity	Energy (MeV) at which the measurements were made and reference	Number of points
	Data used in [6]	
σ_{pp}	147 [11]	21
P_{pp}	147 [11]	14
D_{pp}	147 [12], 143 [13]	9
R_{pp}	140 [14], 142 [15]	14
σ_{np}	156 [16]	23
P_{np}	143 [17]	8
	New data	
A_{pp}	139 [1]	6
A_{pp}	143 [2]	6
R_{pp}	140 [3]	1
A_{pn}	140 [4]	5
R_{pn}	140 [4]	5
P_{pn}	140 [5]	12

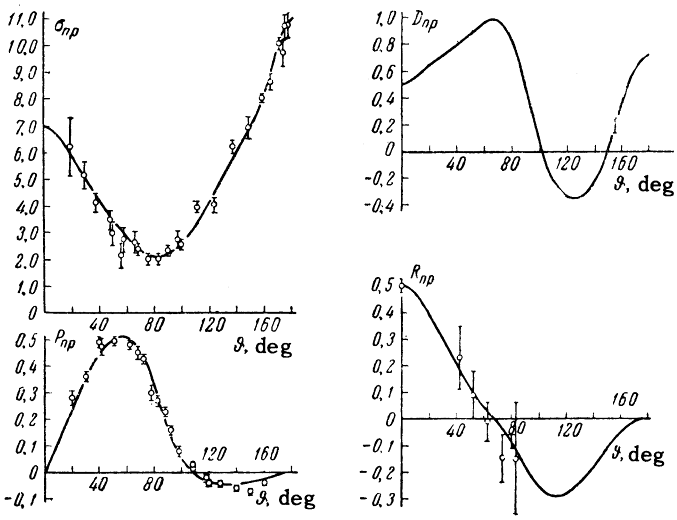

FIG. 1

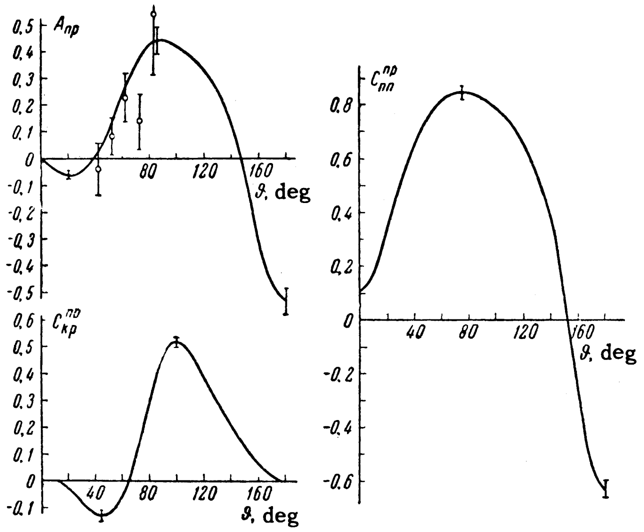

FIG. 2

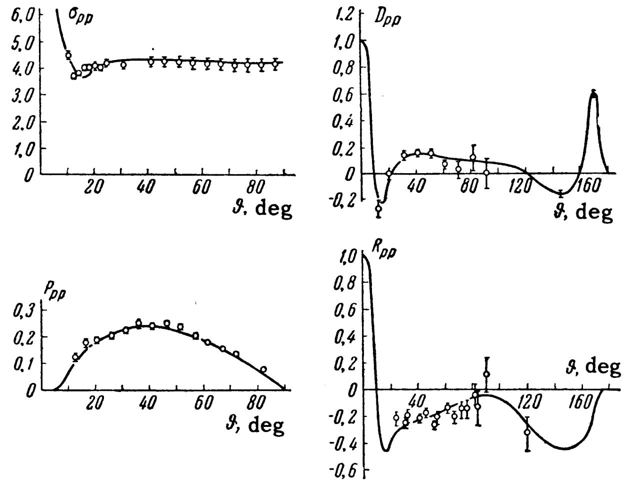

FIG. 3

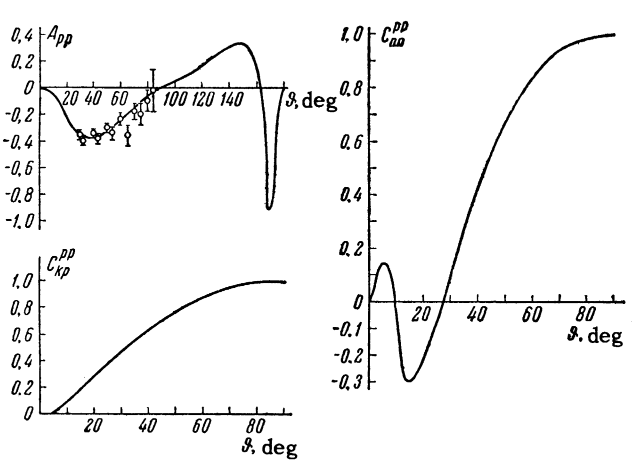

FIG. 4

The results agree, within the limits of errors, with the results of the authors' earlier analysis [6] as well as with the results of Perring [3]. The solution was found to be unusually stable on going from $l_{max} = 3$ to $l_{max} = 4$; the phase shifts themselves and the errors remain practically unchanged in the overwhelming majority of cases (Table II). The solution is also stable under small changes in the energy (147–143 MeV). The ratio $\chi^2/\overline{\chi^2} = 1.24$ for $l_{max} = 3$ and $\chi^2/\overline{\chi^2} = 1.04$ for $l_{max} = 4$. The coupling constants are $f^2 = 0.056 \pm 0.006$ and $f^2 = 0.060 \pm 0.009$ for $l_{max} = 3$ and $l_{max} = 4$, respectively.

It must be noted here that in checking on the computer program previously set up for the phase shift analysis, an error was noted in one of the formulas and this resulted in a slight overestimate of the coupling constant [1)]. To obtain the correct values of f^2 it is necessary to multiply the coupling constants given in [6,18] by $(1 + T/2m)^{1/2} \times (1 + T/m)^{-1}$, where m is the nucleon mass, and T is the kinetic energy. After correcting for f^2, set 1 consists of the values 0.070 ± 0.008, 0.069 ± 0.005, and 0.073 ± 0.007 for $l_{max} = 3$ and for 95, 210, and 310 MeV, respectively. In this energy interval the average value of f^2 is 0.067 ± 0.003 and thus, assuming that the mass of the virtual meson is 140 MeV, differs slightly from 0.08. The value $f^2 = 0.08$ corresponds to a somewhat larger value of the mass of the virtual meson (~ 150 MeV) [16].

Comparison of the results given in Table II with the phase shifts obtained by Breit et al [8,9] where the first Stapp solution [19] was extrapolated from 350 MeV towards the lower energies, as well as with the phase shifts calculated with the Hamada-Johnston potentials [10], shows that the solution obtained differs noticeably from those results. Whereas the difference in the phase shifts of waves with large orbital angular momenta can apparently be attributed to some degree to different methods of utilization of the one-meson approximation [2)], the difference in the S and P phases can hardly be due to this cause. It seems to us that the difference in the values of $\delta_{^3S_1}$ can, in particular, be connected with different values of the mixing parameter ϵ_1 ($\epsilon_1 = 5.0$ in the paper of Breit et al [9]; $\epsilon_1 = 3.9$ in the paper of Hamada and Johnston [10], and $\epsilon_1 = -0.93 \pm 1.1$ in the present paper).

It must be noted that refinement of the phase shift analysis at 147 MeV has made it clear that

[1)]This error did not influence the other parameters, since the coupling constant was varied during the search for the solutions.

[2)]To be sure, changing l_{max} from 3 to 4 makes practically no difference in the phase shifts (Table II).

Table II

	Phase shifts of waves,* deg			Phase shift in waves,* deg	
	$l_{max}=3$	$l_{max}=4$		$l_{max}=3$	$l_{max}=4$
χ^2	132.7	106.7	$^3D_3$	1.777±0.727	−1.150±0.952
f^2	0.057±0.006	0.063±0.009	ε_2	−2.644±0.153	−2.654±0.146
$^1S_0$	17.021±0.667	17.090±0.708	$^3F_2$	−0.223±0.327	−0.004±0.322
$^3S_1$	28.409±0.717	28.483±0.843	$^1F_3$	−1.375±0.496	−1.425±0.863
$^3P_0$	6.541±0.563	6.288±0.623	$^3F_3$	−1.644±0.239	−1.721±0.227
$^1P_1$	−20.003±1.368	−12.798±3.016	$^3F_4$	0.312±0.191	0.452±0.201
$^3P_1$	−18.243±0.232	−18.226±0.231	ε_3		2.289±0.803
$^3P_2$	14.538±0.152	14.505±0.159	$^3G_3$		−3.842±0.655
ε_1	−0.961±1.108	−2.178±1.166	$^1G_4$		0.706±0.147
$^3D_1$	−14.668±0.598	−15.207±0.787	$^3G_4$		4.296±0.128
$^1D_2$	5.736±0.183	5.404±0.242	$^3G_5$		−0.507±0.359
$^3D_2$	20.218±1.108	23.597±1.295			

*Parametrization of Stapp et al.[19]

the energy dependence of the mixing parameter ϵ_1, obtained in the direct phase shift analysis in the energy interval 40—310 MeV, differs from the dependence $\epsilon_1(T)$ obtained in the referred to papers of the Breit and Hamada-Johnston group to an extent much greater than allowed for by the errors (Fig. 5). The parameter ϵ_1 of Breit and of Hamada-Johnston increases monotonically with the energy and remains positive all the time. The mixing parameter obtained in the work on the phase shift analysis is positive at 300 MeV, decreases rapidly with decreasing energy, and passes through zero at an energy close to 150 MeV. At lower energies the parameter ϵ_1 is less reliably determined, but

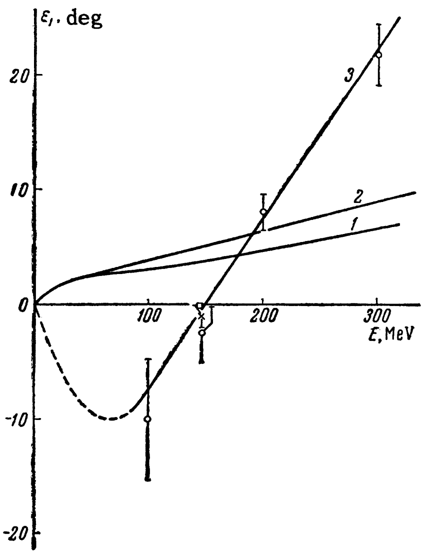

FIG. 5. Plot of mixing parameter against the energy: curve 1 – according to [10], curve 2 – according to [9], curve 3 and o – according to [6], □ – according to Perring[3], x – present work.

the negative values of ϵ_1 are apparently more probable. Thus, for example, at 95 MeV, only one out of the five solutions with low values of χ^2 [6], corresponding to Stapp's set No. 2 at T = 315 MeV, has $\epsilon_1 < 0$. At 40 MeV, where the experimental data are even more skimpy, one can find a solution with positive ϵ_1. This solution, however, gives a positive value for the polarization correlation coefficient C_{nn}^{np} at 180°, and consequently, contradicts the fact that the scattering observed at angles close to 180° is predominantly singlet.[6,20] This difference in the behavior of ϵ_1 could be the result of the fact that Hull et al[9] and Hamada and Johnston[10], in determining $\epsilon_1(T)$ at low energies from the deuteron parameters (the unphysical region on the energy axis), apparently assume that ϵ_1 increases monotonically with increasing energy, as should be the case as $T \to 0$[21]. The latter possibly does not occur in the energy region 10—20 MeV.

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