REGGE POLES AND THE ASYMPTOTIC BEHAVIOR OF THE CROSS SECTIONS FOR SOME WEAK-INTERACTION PROCESSES

NGUYEN VAN HIEU

Joint Institute for Nuclear Research

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The asymptotic behavior of the cross sections for some weak-interaction inelastic processes is examined assuming the existence of Regge poles with $\alpha(0) = 1$.

1. INTRODUCTION

IN the universal theory of weak interactions[1,2] the interaction constant has the dimension of length squared, and the cross sections for the weak interaction processes increase with increasing energy so long as the second approximation of the perturbation theory can be neglected.[3,4] Thus, e.g., the cross sections for the elastic processes of the type $\nu + N \rightarrow l + N$ (where l is a lepton), calculated in the first approximation of perturbation theory neglecting the strong interaction of the baryons increase as E^2 with the c.m.s. energy E. The strong interaction is taken into account in these processes by introducing the form factors.

At large energies and small angles ($t \approx 0$), all form factors equal unity. The asymptotic behavior of the differential cross sections for inelastic processes at small angles is therefore fully determined by the nature of the weak interactions. Things are different for inelastic processes, e.g., $\nu + N \rightarrow l + \pi + N$. The asymptotic behavior of the cross sections for these processes is determined not only by the weak interactions, but by the properties of the strong interactions of the mesons and the baryons as well.

In the present article we consider the asymptotic behavior of the cross sections for several inelastic weak-interaction processes, assuming the existence of vacuum Regge poles with $\alpha(0) = 1$.[5-9] For simplicity, we shall consider first the production of an intermediate vector meson[1] by a π meson on a nucleus with zero spin:

$$\pi^- + A \rightarrow W^- + A, \tag{I}$$

and then the production of a π meson by a neutrino on a nucleus with zero spin:

$$\nu(\bar{\nu}) + A \rightarrow l^-(l^+) + \pi^+(\pi^-) + A. \tag{II}$$

2. CROSS SECTION FOR THE PRODUCTION OF AN INTERMEDIATE VECTOR MESON BY A π MESON

In the universal V − A theory of weak interactions involving intermediate vector mesons, the weak interaction Lagrangian is of the form

$$L = G(J_\mu^V + J_\mu^A) W_\mu + \text{herm. conj.} + \cdots \tag{1}$$

where J_μ^V and J_μ^A are the charged vector and axial currents, respectively, and W_μ is the annihilation operator of a positive W meson and the creation operator of a negative W meson. Only the terms containing annihilation and creation operators of charged W mesons are written explicitly in Eq. (1). We do not consider neutral W mesons.

The matrix element for the process (I) is

$$M_{\mathrm{I}} = (2\pi)^4 \delta^4 (p + P - q - Q)\, G\, (16 p^0 q^0 P^0 Q^0)^{-1/2} \xi_\mu^* \times \langle A \mid J_\mu^V + J_\mu^A \mid \pi^- A \rangle, \tag{2}$$

where p and P are, respectively, the four-momentum of the π meson and of the nucleus A in the initial state, q and Q are the four momenta of the W meson and of the nucleus A in the final state, and ξ_μ is the polarization vector of the W meson.

Let us introduce the invariant variables

$$s = -(p + P)^2, \qquad t = -(P - Q)^2.$$

It follows from invariance considerations that the matrix elements of the currents J_μ^V and J_μ^A can be written in the general form

$$\langle A \mid J_\mu^V \mid \pi^- A \rangle = F(s, t)\, \varepsilon_{\mu\nu\sigma\tau} p_\nu q_\sigma P_\tau, \tag{3}$$

$$\langle A \mid J_\mu^A \mid \pi^- A \rangle = G_1(s, t)\, [Q + P]_\mu + G_2(s, t)\, [Q - P]_\mu + G_3(s, t)\, q_\mu. \tag{4}$$

The amplitude G_3 does not contribute since $\xi_\mu q_\mu = 0$. To determine the asymptotic behavior of the amplitudes F(s, t) and $G_i(s, t)$ as $s \to \infty$, we must go over to the t-channel[6-9] of the reaction

$$\pi^- + W^+ \to A + \bar{A} \tag{III}$$

and expand these amplitudes in partial waves. It is then convenient to use the products $\xi_\mu^* \langle A | J_\mu^V | \pi^- A \rangle$ and $\xi_\mu^* \langle A | J_\mu^A | \pi^- A \rangle$. In channel (I) these products represent (to a numerical factor) the matrix elements for the production by π mesons on nucleus A of a vector or an axial meson, respectively, and in channel (III) as the matrix elements for the production of a pair $A + \bar{A}$ in the collisions of a π meson and a vector (or axial) meson. Expanding these products in terms of partial waves and comparing with Eqs. (3) and (4), we obtain the expansion of the amplitudes F(s, t) and $G_i(s, t)$.

Following the well-known method[5-9] we have to write this expansion as a Sommerfeld-Watson integral and then modify the integration contour. Taking only the contributions from poles into account, and remembering the factor $1 + e^{-i\pi\alpha(t)}$, we obtain the following asymptotic expressions:

$$F(s, t) \to f(t) \frac{1 + \exp\{-i\pi\alpha(t)\}}{\sin \pi\alpha(t)} s^{\alpha(t)-1}, \tag{5}$$

$$G_1(s, t) = g_1(t) \frac{1 + \exp\{-i\pi\alpha(t)\}}{\sin \pi\alpha(t)} s^{\alpha(t)-1}, \tag{6}$$

$$G_2(s, t) = g_2(t) \frac{1 + \exp\{-i\pi\alpha(t)\}}{\sin \pi\alpha(t)} s^{\alpha(t)}. \tag{7}$$

The asymptotic expressions (5) – (7) for the amplitudes F(s, t), $G_1(s, t)$, and $G_2(s, t)$ are symmetrical ($s \to \infty$):

$$F(s, t) = -F(u, t),$$
$$G_1(s, t) = -G_1(u, t), \qquad G_2(s, t) = G_2(u, t), \tag{8}$$

Crossing symmetry requires

$$F(s, t) = -\tilde{F}(u, t),$$
$$G_1(s, t) = -\tilde{G}_1(u, t), \qquad G_2(s, t) = \tilde{G}_2(u, t), \tag{9}$$

where $\tilde{F}$, $\tilde{G}_1$, and $\tilde{G}_2$ are the production amplitudes of a W meson by a π meson on an antinucleus $\bar{A}$, i.e., the process

$$\pi^- + \bar{A} \to W^- + \bar{A}. \tag{I$'$}$$

Relations (8) and (9) give together ($s \to \infty$)

$$F(s, t) = \tilde{F}(s, t),$$
$$G_1(s, t) = \tilde{G}_1(s, t), \qquad G_2(s, t) = \tilde{G}_2(s, t), \tag{10}$$

i.e., the matrix elements of processes (I) and (I′) are equal in the high-energy range.

According to the well-known hypothesis[7-9,11-14] we have for the vacuum trajectory $\alpha(0) = 1$. In that case, the differential cross section for process (I) has the following asymptotic behavior at large energies and small angles:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{s\to\infty \\ \theta=0}} = \left(\frac{G}{8\pi}\right)^2 \frac{s}{M^2} \left[\frac{M^2 - m^2}{2} g_2(0) - g_1(0)\right]^2, \tag{11}$$

where M and m are the masses of the W meson and the π meson respectively. As follows from Eq. (3), the vector current does not contribute to the cross section at small angles. The cross section $d\sigma/d\Omega_{\theta=0}$ increases like s if $\frac{1}{2}(M^2 - m^2)g^2(0) - g_1(0) \neq 0$.

3. CROSS SECTION FOR THE PRODUCTION OF A π MESON BY A NEUTRINO

Let us now consider process (II). To be specific, we consider the case when the leptons ν and l^- are involved in the process. We assume that the mass of l^- equals zero. The cross section for the process involving antileptons has the same asymptotic expressions as the cross section for the process involving leptons. In the universal V – A theory of four-fermion weak interaction[1,2] the matrix element for the process under consideration is

$$M_{11} = (2\pi)^4 \delta^4 (k_1 + P - k_2 - q - Q)\, g\, (16 q^0 P^0 Q^0)^{-1/2}$$
$$\times\, \bar{u}_l \gamma_\mu (1 + \gamma_5) u_\nu \langle \pi^+ A | J_\mu^V + J_\mu^A | A \rangle, \tag{12}$$

The matrix elements of the currents J_μ^V and J_μ^A can be easily obtained from Eqs. (3) and (4). k_1 and P are, respectively, the 4-momenta of the neutrino and of the nucleus in the initial state and k_2, q, and Q—the four-momenta of the lepton, π meson, and the nucleus in the final state.

Let us write $\mathscr{E}$ for the total energy in the c.m.s., E for the total energy of the πA system in the center of mass of this system, $k = k_1 - k_2$, and θ for the angle between the momenta k and q in the center of mass of the πA system. The differential cross section for the process under consideration at an angle θ has the following asymptotic behavior (E^2, $\mathscr{E}^2 \to \infty$):

$$\left.\frac{\partial^3\sigma}{\partial E^2\, \partial k^2\, \partial\Omega}\right|_{\theta=0} = \frac{g^2}{4(2\pi)^4} \frac{E^4}{(E^2 + k^2)^2} \left(1 - \frac{E^2 + k^2}{\mathscr{E}^2}\right)$$
$$\times \left[\frac{k^2}{2} g_2(0) + \left(1 + \frac{k^2}{2E^2}\right) g_1(0)\right]^2. \tag{13}$$

It should be noted that Eq. (13) is correct for $k^2 \ll E^2$ only. To estimate the total cross section we have therefore to integrate not over the whole interval of k^2, but only over the interval from 0 to the nucleon mass.[1)] For such k^2 we have

$$\left.\frac{\partial^3\sigma}{\partial E^2\, \partial k^2\, \partial\Omega}\right|_{\theta=0} = \frac{g^2}{4(2\pi)^4} \left(1 - \frac{E^2}{\mathscr{E}^2}\right) \left[\frac{k^2}{2} g_2(0) + g_1(0)\right]^2. \tag{14}$$

Let us assume that in a small range of angles near $\theta = 0$ the trajectory $\alpha(t)$ is linear with t. Let us denote by $\Delta\sigma$ the contribution from the

[1)]The author is grateful to B. L. Ioffe for this remark.

corresponding range of angles θ and k^2. After integrating, we obtain the estimate

$$\Delta\sigma \to \frac{1}{4} C g^2/(2\pi)^4 \ln s, \qquad s = \mathscr{E}^2, \tag{15}$$

where C is a constant of the order of unity.

If weak interactions propagate through intermediate vector mesons, then in the matrix element (12) one should write $g m_W^2 (m_W^2 + k^2)$ instead of g. However, in estimating $\Delta\sigma$ we took into consideration the range $k^2 \ll E^2$ only. Equation (15) is therefore correct also in that case.

In conclusion it should be noted that we have considered processes (I) and (II) on a nucleus without spin. The results obtained are therefore applicable to corresponding processes on a π meson. The results are, however, also correct for corresponding processes on a nucleon.[2)]

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97