Letters to the Editor

MOBILITY OF SMALL-RADIUS POLARONS AT LOW TEMPERATURES

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WE know of semiconductors with a carrier mobility u that is very low ($u < 1\ cm^2/V\text{-sec}$) down to sufficiently low temperatures. Such small mobilities cannot be obtained by ordinary computation methods, which are suitable only for the case of weak interaction between the carriers and the scatterers. In several papers [1-4] it was shown for the case of strong interaction between the carriers and the polarization lattice vibrations, that at high temperatures ($T > T_0$) the mobility depends on the temperature like $\exp(-E_a/kT)$, and the principal role in the mobility mechanism is played by superbarrier classical jumps of the small-radius polarons from one site to the other. The value of T_0 is determined from the condition

$$kT_0 \approx \hbar\omega_0 / 2 \ln S_T.$$

Here ω_0 is the limiting frequency of the optical phonons and S_T is a dimensionless parameter characterizing the force binding the electrons with the phonons [see (14) of [3]], $S_T \gg 1$. It is of interest to obtain an expression for the mobility in the low-temperature region ($T < T_0$). We shall show in the present note that when $T < T_0$ the mobility increases sharply with decreasing temperature like $\exp(\hbar\omega_0/kT)$. The mobility thus has a minimum at $T \sim T_0$.

We shall use the procedure proposed in [3] and consider the temperature range $T_1 < T < T_0$ where T_1 is determined from the condition $\eta_4(T_1) = \Delta E_p/kT = 1$, and ΔE_p is the width of the polaron band. If the band is filled little, the condition $\eta_4 < 1$ guarantees that the carriers obey Boltzmann statistics. Incidentally, the technique proposed makes it possible to consider the case of Boltzmann statistics for arbitrary η_4.

It is shown in [3] that the electric conductivity is the sum of two terms, σ_H and σ_B, where σ_H is the contribution to the electric conductivity due to jumps from site to site, while σ_B must be determined by solving a transport equation in the form

$$r_{1\mathbf{k}}^x = F_{\mathbf{k}}^x W_{\mathbf{k}} - \sum_{\mathbf{p}} F_{\mathbf{p}}^x W_{\mathbf{pk}}, \tag{1}$$

where $r_{1\mathbf{k}}^x$ and $r_{2\mathbf{k}}^x$ are the left and right vertices, while $W_{\mathbf{k}}$ and $W_{\mathbf{pk}}$ are the probabilities of departure and arrival (see [3]). The contribution σ_B is equal to

$$\sigma_B = \frac{e^2\beta}{V} e^{\beta\mu} \sum_{\mathbf{k}} \mathrm{Re}\,(F_{\mathbf{k}}^x r_{2\mathbf{k}}^x), \tag{2}$$

where $\beta = 1/kT$, V —normalization volume, and μ —chemical potential. An analysis of the terms of the series for the vertices $r_{1\mathbf{k}}^x$ and $r_{2\mathbf{k}}^x$ for $T_1 < T < T_0$ yields [1)]

$$r_{1\mathbf{k}}^{x(0)} = r_{2\mathbf{k}}^{x(0)} = v_x(\mathbf{k}) = -\frac{i}{\hbar} \sum_{\mathbf{g}} J(\mathbf{g})\, g_x e^{-i\mathbf{kg}} e^{-S_T(\mathbf{g})}, \tag{3}$$

where $J(\mathbf{g})$ is the exchange integral*

$$S_T(\mathbf{g}) = \frac{1}{2N} \sum_{\mathbf{q}} |\gamma_{\mathbf{q}}|^2 (1 - \cos \mathbf{qg})\, \mathrm{cth}\, \frac{\hbar\omega_{\mathbf{q}}\beta}{2}$$

(cf. (14) of [3]), and the succeeding terms of the series are small relative to the powers of the parameters η_1 and η_2':

$$\eta_1 = \frac{J}{\hbar\omega_0 S}, \qquad \eta_2' = \left(\frac{J}{\hbar\omega_0}\right)^2 \frac{1}{S} \ln S, \tag{4}$$

where $S = S_T(\mathbf{g})|_{T\to 0}$. In the series for the probability, as in the case when $T > T_0$, the main role is played by the second-order term $W^{(2)}$, apparently owing to the fact that the zeroth approximation chosen was not the best. For the zeroth and first terms of the expansion we have

$$W^{(0)} / W^{(2)} \approx e^{-2S} \ll 1, \qquad W^{(1)} / W^{(2)} \approx e^{-S} \ll 1, \tag{5}$$

and when $n > 2$ the succeeding terms $W^{(n)}$ of the series are small with respect to the powers of η_1 and η_2'. After cumbersome calculations we get [2)]

$$W_{\mathbf{pk}}^{(2)} = \frac{1}{N} \sum_{\Delta\mathbf{G}} e^{i(\mathbf{k}-\mathbf{p})\Delta\mathbf{G}} W(\Delta\mathbf{G}), \qquad W_{\mathbf{k}}^{(2)} = W(0), \tag{6}†$$

$$W(\Delta\mathbf{G}) = \sum_{\mathbf{g}_1\mathbf{g}_2} \frac{J^2(\mathbf{g}_1) J^2(\mathbf{g}_2)}{\hbar^4} \frac{1}{N^2} \sum_{\mathbf{q},\mathbf{q}'} |\gamma_{\mathbf{q}}|^2 |\gamma_{\mathbf{q}'}|^2 \frac{\omega_{\mathbf{q}}^2 \omega_{\mathbf{q}'}^2}{\mathrm{sh}\, p_{\mathbf{q}}\, \mathrm{sh}\, p_{\mathbf{q}'}}$$

$$\times \tilde{a}(\mathbf{q})\, \tilde{a}(\mathbf{q}') \cdot 2\pi\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}'}) \left\{ \left[\frac{1}{N} \sum_{\mathbf{q}} |\gamma_{\mathbf{q}}|^2 a_1(\mathbf{q})\, \omega_{\mathbf{q}} \right]^3 \right.$$

$$\left. \times \left[\frac{1}{N} \sum_{\mathbf{q}} |\gamma_{\mathbf{q}}|^2 a_2(\mathbf{q})\, \omega_{\mathbf{q}} \right]^3 \right\}^{-1} \approx \overline{W} = \eta_1^4 \frac{\omega_0^2}{\Delta\omega} \mathrm{sh}^{-2} p_0,$$

$$a_i(\mathbf{q}) = 1 - \cos \mathbf{q}\mathbf{g}_i, \quad i = 1, 2, \qquad p_{\mathbf{q}} = \hbar\omega_{\mathbf{q}}\beta / 2,$$

$$\tilde{a}(\mathbf{q}) = \tfrac{1}{2} [\cos \mathbf{q}(\Delta\mathbf{G} + \mathbf{g}_3 - \mathbf{g}_1) + \cos \mathbf{q}\Delta\mathbf{G} - \cos \mathbf{q}(\Delta\mathbf{G} - \mathbf{g}_1)$$

$$- \cos \mathbf{q}(\Delta\mathbf{G} + \mathbf{g}_3)], \tag{6a}$$

$\Delta\omega$ —width of dispersion of optical branch.

We seek the solution of (1) in the form

$$F_{\mathbf{k}}^{x} = \sum_{\mathbf{G}} f^{x}(\mathbf{G}) \exp(-i\mathbf{k}\mathbf{G}).$$

Substituting (3) and (6) in (1) we obtain

$$f^{x}(\mathbf{G}) = 0 \qquad \mathbf{G} \neq \mathbf{g},$$

$$f^{x}(\mathbf{G}) = \frac{-i\hbar^{-1}J(\mathbf{g})\, g_x \exp(-S_T(\mathbf{g}))}{W(0) - W(\mathbf{g})} \quad \mathbf{G} = \mathbf{g}, \tag{7}$$

where $\mathbf{g}$ is the vector joining the given atom with the nearest neighbor. We thus obtain ultimately[3)]

$$\sigma_B = \frac{ne^2\beta}{\hbar^2} \sum_{\mathbf{g}} \frac{J^2(\mathbf{g})\, g_x^2 \exp(-2S_T(\mathbf{g}))}{W(0) - W(\mathbf{g})}$$

$$\approx ne^2\beta \frac{J^2a^2}{\hbar^2} \frac{\Delta\omega}{\omega_0^2} \eta_1^{-4} e^{-2S} \operatorname{sh}^2 p_0. \tag{8}$$

Estimates show that when $T_1 < T < T_0$

$$\sigma_H / \sigma_B \approx \left[\frac{J^2}{S(\hbar\omega_0)^2} \operatorname{sh}^{-2} p_0\right]^2 \ll 1.$$

It follows from (8) that the expression for the mobility has the form

$$\bar{u} = u \frac{\Delta E_p}{kT} \frac{\Delta E_p}{\hbar \overline{W}} = \frac{e}{kT} \frac{\langle v_x^2 \rangle}{\overline{W}}, \tag{9}$$

where $\Delta E_p \approx J \exp(-S_T)$ —width of polaron band and $u = ea^2/\hbar = 0.1\,(a/a)^2$ cm²/V-sec has the dimension of mobility, where $a_0 = 10^{-8}$ cm. For broad bands ($\Delta E_p \gg kT$) we have $\langle v_x^2 \rangle / kT \approx 1/m^*$, i.e., (9) goes over into the usual expression for the mobility, suitable for the condition $\hbar\overline{W}/kT \ll 1$. The region of applicability of (9) is not confined to this condition, and (9) can be used if

$$\eta_1 \ll 1, \qquad \eta_2' \ll 1, \qquad T_1 < T < T_0.$$

We have $\Delta E_p/kT \ll 1$ by virtue of the condition $\eta_4 < 1$, while the ratio $\Delta E_p/\hbar\overline{W}$ can be arbitrary. This ratio increases sharply with decreasing temperature. When it exceeds unity the wave vector $\mathbf{k}$ becomes a "good" quantum number.

Thus, in the case of narrow bands and weak interaction with the scatterers ($\Delta E_p/\hbar\overline{W} > 1$, $\Delta E_p/kT < 1$) formula (9) applies to the mobility, as before. However, as the effective interaction between the polarons and the polarization phonons weakens, scattering by impurities or acoustic phonons may enter into play, and this changes the temperature variation of the mobility in the region of lower temperatures ($T \ll T_0$). We have left out of the Hamiltonian the terms responsible for the relaxation of the optical phonons, i.e., we have assumed that they always have a Planck distribution, and the effect of mutual dragging can be neglected.

1)The first line of Eq. (36) of [3], for $T \gg T_0$, contains an error. Actually the estimate of the ratio $r_{2k}^{x(1)}/r_{2k}^{x(0)}$ coincides with the result for $r_{1k}^{x(1)}/r_{1k}^{x(0)}$ in the second and third lines of (36).

*cth = coth.

2)It is stated mistakenly in [3] that when $T \gg T_0$ the probability W_{pk} is independent of p and k, i.e., the arrival terms in (1) are equal to zero. This, however, does not change the estimate of the order of smallness of σ_B when $T \gg T_0$, which was carried out in [3] without account of arrival.

†sh = sinh.

3)The estimate of σ_B in [2] is incorrect, since $W^{(2)}$ is mistakenly replaced by $W^{(0)} \ll W^{(2)}$.

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BARYON MOMENTUM SPECTRUM IN INELASTIC COLLISIONS BETWEEN FAST PIONS AND NUCLEONS

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It was established in recent investigations[1-2] that the momentum spectra of Λ and Σ hyperons, produced in inelastic π^-p collisions at energies $T \sim 10$ BeV, have two maxima: for $T = 7$ BeV one in the region $p \sim 0.8$ BeV/c and the other at $p \sim 1.6$ BeV/c (see Fig. 1). The recoil nucleons have similar spectra[3]. We shall show below that such a "double-hump" baryon spectrum is the direct consequence of resonant interaction between the primary π^- meson and the intermediate particle that transfers the main part of the interaction in primary πN collisions.

Let us consider the production of Λ hyperons