EMPIRICAL BEHAVIOR OF THE (n, p) CROSS SECTION FOR 14–15 MeV NEUTRONS

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The absolute values of the (n, p) and (n, α) cross sections for 14-MeV neutrons were measured by the activation method. An analysis of the literature data indicates that in the $12 < A < 150$ range $\sigma_{n,p}$ depends on Z and A of the target in a simple manner. Comparison of the obtained empirical formula with the semi-empirical relationship proposed by Gardner leads to a simple dependence of $\sigma_{n,p}$ on $Q_{n,p}$.

WE have shown earlier[1,2] that for 14-MeV neutrons the (n, p) cross sections of many isotopes of one element decrease monotonically and regularly with increasing A. The character of the variation of $\sigma_{n,p}$ is well represented by the formula

$$\frac{\sigma(A+\Delta A, Z)}{\sigma(A, Z)} \approx \exp\left[75\left(\frac{Z}{A+\Delta A} - \frac{Z}{A}\right)\right] \approx \exp\left[-33\frac{\Delta A}{A}\right]. \quad (1)$$

It was also proposed[2] that the rule (1) might be a reflection of a more general law relating the probability of emission of a proton from the struck nucleus [$\alpha_p = \sigma_{n,p}/\sigma_c(n)$] with a proton "concentration" Z/A. To check this proposition, we measured several absolute cross sections of (n, p) reactions in an extensive region of Z and A. We also measured simultaneously several (n, α) cross sections. The procedures used for the bombardment, radiochemical separation of the reaction products, measurement of their activity, and calculation of the cross sections are described in detail in [3-7]. The measurement results are listed in Table I.

An analysis of the data of Table I has shown that all the (n, p) cross sections [except for that of $S^{32}(n, p)P^{32}$] are well described by the relation

$$\sigma_{n,p} = \sigma_c(n)\,\alpha_p, \quad (2)$$

where $\sigma_c(n)$ is the geometrical cross section of the nucleus, equal to $45.2(A^{1/3}+1)^2$ mb, and $\alpha_p = \exp[-33(N-Z)/A]$.

TABLE I

Reaction	Cross section, mb	Reaction	Cross section, mb
$S^{32}(n, p)P^{32}$	220±40	$Zr^{94}(n, p)Y^{94}$	11±2
$Cl^{35}(n, \alpha)P^{32}$	100±20	$Zr^{92}(n, \alpha)Sr^{89}$	~10
$Ca^{42}(n, p)K^{42}$	160±30	$Zr^{94}(n, \alpha)Sr^{91}$	4.7±0.8
$Ca^{44}(n, p)K^{44}$	37±7	$Zr^{96}(n, \alpha)Sr^{93}$	2.3±0.5
$Zn^{64}(n, p)Cu^{64}$	210±40	$Ag^{109}(n, p)Pd^{109}$	11±2
$Zn^{66}(n, p)Cu^{66}$	75±10	$Cd^{106}(n, p)Ag^{106}$	76±24
$Zn^{67}(n, p)Cu^{67}$	48±8	$Cd^{111}(n, p)Ag^{111}$	15±4
$Zn^{68}(n, p)Cu^{68}$	~25	$Cd^{112}(n, p)Ag^{112}$	11±3
$Sr^{86}(n, p)Rb^{86}$	39±7	$Cd^{113}(n, p)Ag^{113}$	8±2
$Sr^{88}(n, p)Rb^{88}$	18±3	$Cd^{112}(n, \alpha)Pd^{109}$	1.3±0.3
$Zr^{90}(n, p)Y^{90}$	54±10	$Cd^{114}(n, \alpha)Pd^{111}$	0.6±0.1
$Zr^{91}(n, p)Y^{91}$	40±8	$Ba^{138}(n, p)Cs^{138}$	1.9±0.5
$Zr^{92}(n, p)Y^{92}$	25±5		

The summary Table II lists the mean values of the (n, p) cross sections for $E_n = 14-15$ MeV in the range $12 < A < 150$ as given by various sources published up to 1963[3,8]. It fails to include only some old data (principally the 1953 data of Pool and Clark), which either differ greatly from the mean $\sigma_{n,p}$ obtained in later work or are highly inaccurate, i.e., the error indicated by the authors themselves reaches 60–80%.

An analysis of the data of Table II has shown that in the region $20 < N$ the $\sigma_{n,p}$ calculated from (2) coincide as a rule, within the limits of experimental scatter, with the $\sigma_{n,p}$ obtained in experiment. In the region $20 \geq N$ formula (2) likewise agrees well with experiment if

$$\alpha_p = \exp[-33(N-Z+1)/(A+1)].$$

The values of $\sigma_{n,p}$ calculated from (2) are listed in the fourth column of Table II. In the sixth column are given the $\sigma_{n,p}$ calculated with the empirical formula proposed by Gardner[8]. This formula, like ours, depends only on Z and A of the target, but is much more complicated (it contains four numerical coefficients in the exponent) and in general, as can be seen from Table II, is in worse agreement with experiment than (2).

Starting from highly simplified general expressions of the statistical theory of nuclear reactions, Gardner[8] arrived at relation (3), which connects the $\sigma_{n,p}$ in a series of stable isotopes of one element with the excitation energy $E = E_n + Q_{n,p} + \delta$:

TABLE II

Isotope	$\sigma_{n,p}$, exptl. (mean value)	Number of measurements	$\sigma_{n,p}$, calc. from (2)	$\sigma_{exp}/\sigma_{calc}$	$\sigma_{n,p}$, calc. after Gardner
$_{6}C^{12}$	17±2	2	40	0.4	40
$_{8}O^{16}$	39±2	4	80	0.5	64
$_{9}F^{19}$	16,5±2	1	21	0.8	40
$_{11}Na^{23}$	42±8	2	44	1.0	64
$_{12}Mg^{24}$	180±30	6	190	1.0	150
$_{12}Mg^{25}$	53±7	3	57	0.9	75
$_{12}Mg^{26}$	27±7	1	19	1.4	38
$_{13}Al^{27}$	77±20	16	72	1.1	87
$_{14}Si^{28}$	250±20	6	240	1.0	240
$_{14}Si^{29}$	100±30	1	70	1.4	120
$_{15}P^{31}$	82±15	4	65	1.3	140
$_{16}S^{32}$	240±40	5	280	0.9	72
$_{16}S^{34}$	85±40	1	50	1.7	36
$_{17}Cl^{35}$	130±15	4	130	1.0	210
$_{17}Cl^{37}$	32±5	3	30	1.1	53
$_{19}K^{39}$	350±50	1	370	1.0	320
$_{19}K^{41}$	80±30	1	80	1.0	80
$_{20}Ca^{40}$	400±100	2	400	1.0	740
$_{20}Ca^{42}$	160±30	1	190	0.8	180
$_{20}Ca^{44}$	37±7	1	46	0.8	46
$_{22}Ti^{46}$	220±20	2	250	0.9	260
$_{22}Ti^{47}$	170±40	3	130	1.3	130
$_{22}Ti^{48}$	60±3	2	65	0.9	65
$_{22}Ti^{49}$	30±2	3	34	0.9	33
$_{23}V^{51}$	34±10	3	42	0.8	40
$_{24}Cr^{50}$	280±20	1	270	1.0	380
$_{24}Cr^{52}$	86±10	6	84	1.0	96
$_{24}Cr^{53}$	42±6	2	48	0.9	48
$_{25}Mn^{55}$	45±7	1	57	0.8	56
$_{26}Fe^{54}$	370±20	3	310	1.2	500
$_{26}Fe^{56}$	120±10	10	110	1.1	120
$_{26}Fe^{57}$	60±10	2	60	1.0	62
$_{26}Fe^{58}$	23±4	1	32	0.7	31
$_{27}Co^{59}$	75±10	3	72	1.0	80
$_{28}Ni^{58}$	400±100	6	400	1.0	750
$_{28}Ni^{60}$	140±10	2	140	1.0	190
$_{28}Ni^{61}$	88±2	2	77	1.1	94
$_{28}Ni^{62}$	56±3	1	48	1.2	47
$_{28}Ni^{64}$	6±2	2	16	0.4	13
$_{29}Cu^{63}$	100±20	2	94	1.1	110
$_{29}Cu^{65}$	22±7	8	35	0.6	28
$_{30}Zn^{64}$	210±50	6	160	1.3	310
$_{30}Zn^{66}$	75±20	5	63	1.2	78
$_{30}Zn^{67}$	45±5	2	40	1.1	39
$_{30}Zn^{68}$	~25	2	27	1.0	20
$_{32}Ge^{70}$	110±20	2	87	1.3	100
$_{32}Ge^{72}$	~32	1	34	1.0	25
$_{32}Ge^{73}$	~21	1	23	0.9	12
$_{33}As^{75}$	25±8	4	26	1.0	14
$_{34}Se^{77}$	45	1	45	1.0	76
$_{38}Sr^{86}$	39±7	1	38	1.0	76
$_{38}Sr^{88}$	18±1	3	17	1.0	19
$_{40}Zr^{90}$	44±8	3	38	1.2	100
$_{40}Zr^{91}$	36±4	2	28	1.3	50
$_{40}Zr^{92}$	21±1	3	21	1.0	25
$_{40}Zr^{94}$	11±1	4	11	1.0	6
$_{47}Ag^{109}$	11±2	2	17	0.6	10
$_{48}Cd^{106}$	76±24	1	71	1.0	370
$_{48}Cd^{111}$	15±4	1	19	0.8	12
$_{48}Cd^{112}$	11±3	1	15	0.7	6
$_{48}Cd^{113}$	8±2	1	12	0.7	3
$_{49}In^{115}$	17±3	2	13	1.3	32
$_{53}I^{127}$	12±2	1	8	1.5	16
$_{56}Ba^{136}$	43±10	2	6	7.2	—
$_{56}Ba^{138}$	2.3±0.3	3	3.8	0.6	—
$_{57}La^{139}$	5.5±0.5	2	5.4	1.0	—
$_{58}Ce^{140}$	11±2	2	7	1.6	—
$_{58}Ce^{142}$	7±2	2	5	1.4	—
$_{60}Nd^{142}$	13±3	1	12	1.0	—
$_{60}Nd^{143}$	11±2	1	10	1.1	—
$_{60}Nd^{148}$	3.5±1	1	4	0.9	—

TABLE III

Element	$B(Z)^e_e$	$B(Z)^e_o$	$B(Z)^o_o$	$B(Z)^o_e$	Element	$(BZ)^e_e$	$B(Z)^e_o$	$B(Z)^o_o$	$B(Z)^o_e$
$_{24}$Cr	4.20	4.51	—	—	$_{33}$As	—	—	4.78	5.16
$_{25}$Mn	—	—	4.50	4.80	$_{34}$Se	4.41	4.81	—	—
$_{26}$Fe	4.16	4.48	—	—	$_{35}$Br	—	—	4.84	5.16
$_{27}$Co	—	—	4.52	4.83	$_{36}$Kr	4.47	4.84	—	—
$_{28}$Ni	4.18	4.51	—	—	$_{37}$Rb	—	—	4.84	5.18
$_{29}$Cu	—	—	4.64	4.97	$_{38}$Sr	4.58	4.88	—	—
$_{30}$Zn	4.40	4.78	—	—	$_{39}$Y	—	—	4.94	5.26
$_{31}$Ga	—	—	4.70	5.04	$_{40}$Zr	—	5.05	—	—
$_{32}$Ge	4.40	4.81	—	—	$_{41}$Nb	—	—	5.01	5.32

TABLE IV

Element	$B(Z)^e_e$	$B(Z)^e_o$	$B(Z)^o_o$	$B(Z)^o_e$	Element	$B(Z)^e_e$	$B(Z)^e_o$	$B(Z)^o_o$	$B(Z)^o_e$
$_{37}$Rb	—	—	5.03	5.28	$_{48}$Cd	4.81	5.14	—	—
$_{38}$Sr	4.70	5.01	—	—	$_{49}$In	—	—	5.18	5.48
$_{39}$Y	—	—	5.11	5.32	$_{50}$Sn	4.82	5.14	—	—
$_{40}$Zr	4.82	5.03	—	—	$_{51}$Sb	—	—	5.36	5.67
$_{41}$Nb	—	—	5.20	5.38	$_{52}$Te	5.09	5.39	—	—
$_{42}$Mo	4.80	5.03	—	—	$_{53}$I	—	—	5.36	5.64
$_{43}$Tc	—	—	5.12	5.38	$_{54}$Xe	5.08	5.38	—	—
$_{44}$Ru	4.77	5.04	—	—	$_{55}$Cs	—	—	5.42	5.67
$_{45}$Rh	—	—	5.13	5.39	$_{56}$Ba	5.15	5.40	—	—
$_{46}$Pd	4.79	5.10	—	—	$_{57}$La	—	—	5.44	5.70
$_{47}$Ag	—	—	5.16	5.44	$_{58}$Ce	5.20	5.41	—	—

$$\sigma (A + \Delta A, Z)/\sigma (A, Z) = \exp \{2 [\sqrt{aE (A + \Delta A)} - \sqrt{aE (A)}]\}, \quad (3)$$

where a, the empirical parameter widely used in statistical-theory calculations, is assumed equal to A/20. It has been shown in [9] that relation (3) describes well the variation of $\sigma_{n,p}$ in series of isotopes of Fe, Cu, Zn, and Cd.

Equation (3) can be compared with our general empirical formula (2):

$$\frac{\sigma (A, Z)}{\sigma (2Z, Z)} = \exp\left[-33 \frac{N-Z}{A}\right] = \exp \{2 \sqrt{a} [\sqrt{E (A)} - \sqrt{E (2Z)}]\},$$

from which it follows that (for $\delta = 0$ and $a = A/20$)

$$\sqrt{14.5 + Q_{n,p}} = B (Z) - 74 (N - Z) A^{-1/3}. \quad (4)$$

It turns out that in the region $28 \leq N < 50$ formula (4) represents very well the character of the variations of $Q_{n,p}$ in the series of isotopes of one element: for different isotopes of one element having the same parity, i.e., even-even, even-odd, odd-even, or odd-odd, B(Z) is constant. Table III gives the values of B(Z) in the region $28 \leq N \leq 50$, obtained by substituting in (4) the tabulated values of $Q_{n,p}$[10].

When $N \geq 50$ all the values of B(Z) [calculated from (4)] decrease sharply and are no longer constant. However, B(Z) is again constant, up to N = 82, in the series of isotopes of one element if the coefficient a = A/20 is replaced by a = A/40. Table IV lists the values of B(Z) in the range $50 \leq N < 82$, calculated from (4) with a = A/40 (this leads to a replacement of the coefficient 74 of (4) by 105 [1)]).

It follows from Tables III and IV that the values of B(Z) increase with increasing Z for all four types of nuclei in both intervals under consideration. This increase, however, is not monotonic: in the regions Z = 23—28, 30—37, 40—45, and 51—56 all the B(Z) remain practically constant, with $B(Z)^e_o = B(Z)^o_o$ [with the exception of Z = 40—44, where $B(Z)^o_o$ is somewhat larger than $B(Z)^e_o$]. All the B(Z) increase sharply when Z > 28 and 50, and less noticeably as N approaches 50 and 82.

Table V illustrates the constancy of B(Z) by listing the $Q_{n,p}$ taken from the latest nuclear mass tables[10] and calculated from (4), for several elements in both regions under consideration. The values of B(Z) are taken from Tables III and IV. The use of averaged values of B(Z) in each region also yields in general good agreement between the experimental and calculated $Q_{n,p}$, but in many

[1)]The coefficient a = A/40 is also used sometimes in the statistical theory of nuclear reactions, but a = A/10 is used more frequently.

TABLE V

Element	A	Q, MeV [10]	Q, MeV after (4)	Element	A	Q, MeV [10]	Q, MeV after (4)
	57	4,2	4,3		89	3,6	3,6
	58*	0,4	0,3		90	—1,5	—1,6
	59	1,8	1,7		91	—0,8	—0,7
$_{28}$Ni	60	2,0	2,0	$_{40}$Zr	92	—2,8	—2,9
	61	—0,5	—0,5		93	—2,1	—2,2
	62	—4,4	—3,9		94	—4,2	—4,2
	77	3,7	3,8		105	3,8	3,7
	78	0,1	0,1		106	0,4	0,3
	79	2,4	2,4		107	2,2	2,3
	80	—1,2	—1,2		108	—1,0	—0,9
$_{36}$Kr	81	1,0	1,0		109	0,9	1,0
	82	—2,3	—2,2	$_{48}$Cd	110	—2,1	—2,0
	83	—0,2	—0,2		111	—0,3	—0,2
	84	—3,9	—3,4		112	—3,3	—3,0
					113	—1,2	—1,2
					114	—3,8	—3,9
					115	—2,1	—2,1

*The stable isotopes are underlined.

cases, particularly near magic Z and N, the disparity between Q_{exp} and Q_{calc} is already appreciable, reaching 1—1.5 MeV.

It follows from the foregoing that in the region N = 28—82 the (n, p) cross sections can be expressed in terms of the thermal effects $Q_{n,p}$ by means of the equation

$$\sigma_{n,p} = \sigma_c(n) \exp\{2\sqrt{a}[\sqrt{E} - B(Z)]\}, \qquad (5)$$

where $E = 14.5 + Q_{n,p}$; $a = A/20$ for $28 \leq N < 50$ and $a = A/40$ for $50 \leq N < 82$, while the B(Z) are listed in Tables III and IV. In the regions Z = 24—28, 30—37, 40—50, and 51—56 the use of the mean values $\bar{B}(Z)$ also gives good agreement with experiment.

In the region of light nuclei, N < 28, the thermal effects of the (n, p) reactions do not vary in general monotonically over a series of isotopes of one element. Monotonicity is violated in the regions of magic N (14, 20, 28, and apparently 24). The number of isotopes known in this region is small, making the analysis of the dependence of B(Z) on Z and A difficult. However, it is always possible here, too, to choose the coefficients B(Z) and a such that relation (5) is satisfied. For example, for nuclei with the same number of protons and neutrons, in the region Z = 6 – 14 ($_6C^{12}$, $_8O^{16}$, $_{12}Mg^{24}$, $_{14}Si^{28}$), very good agreement with experiment is gotten with $\bar{B}(Z) = 3.76$ and a = A/20. In a broad interval of Z (11—28), perfectly satisfactory results are obtained with $\bar{B}(Z) = 4.52$ for even-odd and odd-odd nuclei and $\bar{B}(Z) = 4.16$ for even-even nuclei (cf. Table III).

CONCLUSIONS

1. It is shown as a result of an analysis of the author's own data and those in the literature, concerning the (n, p) cross sections with a neutron energy of 14—15 MeV, that in the range 12 < A < 150 the (n, p) cross sections are well described by Eq. (2). A feature of Eq. (2) is that it contains only one empirical coefficient, which is the same for even-even, even-odd, and odd-odd nuclei, and does not vary in the region of magic Z and N. This result contradicts in general the statistical theory of nuclear reactions, according to which the $\sigma_{n,p}$ are highly sensitive to fluctuations in $Q_{n,p}$.

2. It is shown further that Eq. (2) can be related with the thermal effects $Q_{n,p}$ by using Gardner's semi-empirical equation (3). This leads to Eq. (5), which now expresses the $\sigma_{n,p}$ in terms of $Q_{n,p}$. Unlike (2), Eq. (5) contains an explicit dependence on the parity of the target nucleus and on the shell effects, as is manifest in the different values of B(Z) for nuclei with different parity, and in jump-like changes of B(Z) and a in the region of magic Z and N.

Although Eq. (5) has been formally derived using the function $\omega(E)$ ("level density" of the nucleus), usually employed in calculations of the statistical theory of nuclear interactions, it would hardly be justified to regard it as a confirmation of the theory, since, first, Gardner's relation (3) has been derived, using very crude approximations, from general expressions of statistical theory, which are themselves, as testified by their authors, quite approximate ([11], pp 292–294), and, second, Eq. (5) does not contain a term that can be regarded with sufficient justification as a function of the Coulomb barrier of the nucleus.

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