πN *SCATTERING PARTIAL WAVES WITH ACCOUNT OF* $\pi\pi$ *INTERACTION*

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The effect of the $\pi\pi$ interaction in the T = J = 0 state on the πN scattering partial waves is investigated by the dispersion relations method. In the final expressions the static limit is taken and compared with experimental data. The most probable form of the δ_0^0 $\pi\pi$ scattering phase shift is discussed. Relations are obtained connecting the contributions of the $\pi\pi$ interaction to the s and p πN-scattering waves. The implications of these relations for the static limit are considered.

INTRODUCTION

THIS work is devoted to a study of the effect of the $\pi\pi$ interaction in the T = J = 0 state on the πN scattering partial waves and follows the previous work of the authors.[1] The analogous problem is dealt with in a number of papers.[2-4] In the work of Takahashi[2] the study is based on the one dimensional Cini-Fubini representation. The method used by that author to obtain the equations for the partial waves has been criticized by Efremov et al.[5] The small value of the $\pi\pi$ scattering length a_0 obtained in that paper is probably related to this last difficulty.

In the work of Hamilton et al.[3] the analysis of the $\pi\pi$ interaction in the state with T = J = 0 is based on a study of the so called "differences" $\Delta_{l,J}^{(\pm)}$, i.e., the sums of the contributions from the cuts $-\infty < s < 0$ and $|s| = M^2 - \mu^2$ in the s plane. The simultaneous analysis of the "differences" is simplified by the introduction of separate parameters for each, to take into account the effect of the distant singularities. The contribution of near singularities (the leading front of the circumference $|s| = M^2 - \mu^2$) is interpreted as the $\pi\pi$ interaction. In the present paper a number of relations is obtained (Sec. 4) which the $\pi\pi$ contributions to the πN scattering must satisfy. Based on these relations we may assert that in [3] the $\pi\pi$ contribution is extracted from the "differences" unsuccessfully.

Atkinson[4] makes use of dispersion relations for backward scattering in the variable $\nu = q^2$. In such an approach the differences are connected only with the $\pi\pi$ interaction, as follows from the work of Efremov et al.[5,6] The $\pi\pi$ scattering phase shifts are determined by analytic continuation of the "differences" from the region $\nu > 0$ to the cut $-\infty < \nu \leq -1$. The analytic continuation is accomplished with the help of a conformal mapping [see [4], Eq. (3.3)] which maps the ν plane with the cut $-\infty < \nu \leq -1$ into a 2n + 1 sheeted Riemann surface. Therefore for the values n = 1, 2 the Atkinson approach is not valid.

The method used in this paper for taking into account the $\pi\pi$ interaction allows one to choose between various forms for the energy dependence of the phase shift δ_0^0. It is shown that the s-wave dominant solution of Chew, Mandelstam, and Noyes[7] does not describe the energy dependence of the partial waves in πN scattering. We also discuss the approximations for the scattering length and the resonant behavior of the phase shift δ_0^0, taken from the work of Serebryakov and Shirkov on the solution of the set of equations for $\pi\pi$ scattering partial waves.[8] The conclusion is arrived at that the version with a resonant δ_0^0 phase shift is to be preferred.

1. UNITARITY CONDITION FOR THE $\pi\pi \rightarrow N\bar{N}$ PROCESS

The unitarity condition for the $\pi\pi \rightarrow N\bar{N}$ process is written with the help of the states with definite helicity J_{++}, J_{+-}, which are decomposed into partial waves as follows:

$$J_{++} = J_{--} = (p_3/8\pi p_3^0)\,[-A^{(+)} + 4Mq_3^2\,\beta\cos^2\theta_3]$$
$$= \frac{1}{p_3 p_3^0}\sum_l \left(l + \frac{1}{2}\right)(p_3 q_3)^l f_+^l P_l(\cos\theta_3);$$

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$$J_{+-} = -J_{-+} = \frac{q_3}{8\pi} B^+ \sin\theta_3$$

$$= q_3 \sum_l \frac{l + {}^1\!/_2}{\sqrt{l(l+1)}} (p_3 q_3)^{l-1} f_-^l P_l' (\cos\theta_3), \tag{1.1}$$

where p_3, q_3 and θ_3 are the momentum of the nucleon, the momentum of the pion and the scattering angle in the c.m. system of the reaction $\pi\pi \to N\bar{N}$; $\beta = B^+/(s - \bar{s})$.

For the isotopic index (+) the summation is over even l.

In the two-particle approximation the unitarity condition determines the phases of the partial amplitudes $f_\pm^l$ in terms of the s, d, etc., phase shifts for $\pi\pi$ scattering with T = 0. Since we shall be interested only in the region of small q_3 values we make the natural assumption that

$$\delta_0^l = 0 \text{ for } l \geqslant 2 \tag{1.2}$$

It follows from Eq. (1.1) that β is a real function on the interval $-\infty < \nu \leq -1$.

Approximating the higher partial waves by pole terms,[9] we obtain for $A^{(+)}$ and β the expressions

$$A^{(+)} = 4Mq_3^2\beta\cos^2\theta_3 + \gamma\exp(i\delta_0^0)$$
$$- 4M\cos^2\theta_3 q_3^2\Delta_\beta + 2Mq_3^2(\Delta_\beta)_1,$$
$$\beta = \beta_1\exp(i\delta_0^2) + \left[\Delta_\beta - \frac{1}{5\pi}\{(\Delta_\beta)_1 - (\Delta_\beta)_3\}\right]\sqrt{\frac{3}{2}};$$
$$(\Delta_\beta)_l = \int_{-1}^{+1} x\Delta_\beta(x) P_l(x)\,dx, \tag{1.3}$$

where Δ_β is the pole term of the function β; β_1 and γ are unknown real functions. On comparing Eq. (1.3) with the analogous expressions for α and $\beta^{(-)}$ from [1] [see Eq. (1.4) in [1]] we note in the first place that the lower terms in $A^{(+)}$ and α contain s and d waves, and in the second place that the assumption (1.2) gives rise to the appearance in $A^{(+)}$ of the unknown real function β_1. Consequently, the method for taking into account the $\pi\pi$ interaction as proposed in [5] must be modified.

2. DISPERSION RELATIONS FOR A^+ AND β, PARTIAL WAVE EQUATIONS

Below we shall obtain equations for πN scattering partial waves at low energies by making use of combinations of dispersion relations for the functions $A^{(+)}$ and β at $c = \cos\theta = \pm 1$. For forward scattering it is convenient[1] to consider dispersion relations in the variable s, i.e., in the conventional form:

$$\Phi(\nu, +1) = \frac{1}{\pi}\int_{(M+1)^2}^{\infty} \operatorname{Im}\Phi(\nu', +1)$$
$$\times\left[\frac{1}{s(\nu') - s(\nu)} + \frac{1}{\bar{s}(\nu') - \bar{s}(\nu)}\right] ds(\nu'). \tag{2.1}$$

At that the number of subtractions for the functions $A^{(+)}$ and β is different. Assuming, as is usual, the constancy of the total cross section at large energies we see easily that two subtractions are sufficient for $A^{(+)}$, whereas for β none are needed. If we take into account that the usual assumption $\operatorname{Im}\Phi \sim \operatorname{Im} f_3^3$ will be made in what follows, we can limit ourselves to one subtraction. At that the convergence of the integrals will be assured. The second subtraction will not give rise to the appearance of an additional constant because of the symmetry properties of the function $A^{(+)}$. The comparison of the final expressions with the experimental data will serve as the criterion for choosing the number of necessary subtractions.

For backward scattering it is convenient to write the dispersion relations in the variable $\nu = q^2$. The sole nearby singularity is the cut $-\infty < \nu \leq -1$ from the reaction $\pi\pi \to N\bar{N}$. In view of the assumption (1.2) the dispersion relation for $\beta(\nu, -1)$ has the form

$$\beta(\nu, -1) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im}\beta(\nu', -1)}{\nu' - \nu} d\nu' + \Delta_\beta;$$
$$\operatorname{Im}\beta(\nu < 0, -1) = 0. \tag{2.2}$$

Let us consider the function $-8\pi p^0 J_{++}/p_3 = A^{(+)} - 4Mq_3^2\beta\cos^2\theta_3$. According to Eq. (1.3) this function has a simple structure on the cut $-\infty < \nu \leq -1$: the first term is the s-wave amplitude f_+^0, the second term is the sum of all higher partial waves in the pole approximation. Therefore the unphysical cut may be taken into account in a manner analogous to that used in [1], i.e., by considering instead of $-8\pi p^0 J_{++}/p_3$ the function $-8\pi p^0 J_{++}/p_3 F_0(\nu)$. The function $F_0(\nu)$ has no zeros in the complex ν plane. On the cut $-\infty < \nu \leq -1$ the phase of $F_0(\nu)$ coincides with δ_0^0.

In that case the dispersion relation for $A^{(+)}$ may be written in the form

$$A^{(+)}(\nu, -1) = A^{(+)}(0, -1)F_0(\nu) + 4M\beta(0, -1)$$
$$\times[F_0(\nu) - 1] + \frac{\nu}{\nu_0(\nu - \nu_0)}\left[1 - \frac{F_0(\nu)}{F_0(\nu_0)}\right]$$
$$\times \operatorname{Res}_{\nu_0 = -1 + 1/4M^2} 4M\omega^2\Delta_\beta + \frac{\nu}{\pi}\int_0^\infty \frac{\operatorname{Im}A^{(+)}(\nu', -1)}{\nu'(\nu' - \nu)} d\nu'$$
$$+ \frac{\nu}{\pi}\int_0^\infty \left\{\operatorname{Im}[A^{(+)}(\nu', -1) + 4M\omega'^2\beta(\nu', -1)]\left[\frac{F_0(\nu)}{F_0(\nu')} - 1\right]\right\}$$
$$\times \frac{d\nu'}{\nu'(\nu' - \nu)}. \tag{2.3}$$

The quantity $A^{(+)}(0, -1)$ is expressible in terms of the scattering lengths:

$$A^{(+)}(0,-1)/4\pi = (2M+1)\,a^+/2M - 2M\,(a_1^+ - a_3^+). \quad (2.4)$$

The function $F_0(\nu)$ is defined as follows:

$$F_0(\nu) = \exp\left\{-\frac{\nu}{\pi}\int_0^\infty \frac{\delta_0^0(k)}{(k^2+1)(k^2+1+\nu)}\,dk^2\right\}. \quad (2.5)$$

An arbitrary polynomial may be added into Eq. (2.5), however such a polynomial would not be determined by the phase δ_0^0. The function $F_0(\nu)$ is the only function whose behavior for $\nu \geq 0$ is fully determined by the value of the phase shift δ_0^0 on the physical (for $\pi\pi$ scattering) cut $-\infty < \nu \leq -1$. At that it is of small importance whether or not the phase shift δ_0^0 satisfies crossing symmetry relations for $\pi\pi$ scattering: in Eq. (2.3) $F_0(\nu)$ is needed for $\nu > 0$ only, therefore the details of the behavior of δ_0^0 for $\nu < 0$ are unimportant. If δ_0^0 approximates well the true function δ_0^0 then this is fully sufficient for a determination of $F_0(\nu)$ in the region $\nu > 0$. The true form of δ_0^0 is not known and therefore we assume below several concrete forms for the function $\tan\delta_0^0$.

The transition to dispersion relations for partial waves is accomplished with the help of the relations:

$$f_s^{(\pm)}(\nu) = \tfrac{1}{2}\,[f_1^{(\pm)}(\nu,+1) + f_1^{(\pm)}(\nu,-1)],$$

$$f_{p_{3/2}}^{(\pm)}(\nu) = \tfrac{1}{6}\,[f_1^{(\pm)}(\nu,+1) - f_1^{(\pm)}(\nu,-1)],$$

$$f_{p_{1/2}}^{(\pm)} - f_{p_{3/2}}^{(\pm)} = \tfrac{1}{2}\,[f_2^{(\pm)}(\nu,+1) + f_2^{(\pm)}(\nu,-1)]. \quad (2.6)$$

The connection between the functions $f_{1,2}^{(+)}$ and $A^{(+)}$, β is given by

$$f_1^{(+)}(\nu,c) = \frac{p^0+M}{8\pi W}\,[A^{(+)}(\nu,c) + 2(W-M)\,\beta(\nu,c)\,[2p^0q^0 + \nu(1+c)]],$$

$$f_2^{(+)}(\nu,c) = \frac{p^0-M}{8\pi W}\,[-A^{(+)}(\nu,c) + 2(W+M)\,\beta(\nu,c)\,[2p^0q^0 + \nu(1+c)]]. \quad (2.7$$

The functions $\mathrm{Im}\,A^{(+)}$ and $\mathrm{Im}\,\beta$ that appear in the integrands are expressible in terms of the amplitude $\mathrm{Im}\,f_{p_{3/2}}^{(+)}$ as follows:

$$\frac{\mathrm{Im}\,A^{(+)}(\nu,c)}{4\pi} = \left\{\frac{W+M}{p^0+M}\,3c + \frac{W-M}{p^0-M}\right\}\mathrm{Im}\,f_{p_{3/2}}^{(+)},$$

$$\frac{\mathrm{Im}\,\beta(\nu,c)}{4\pi} = \frac{1}{4p^0\omega + 2\nu(1+c)}\left[\frac{1}{p^0+M}\,3c - \frac{1}{p^0-M}\right]\mathrm{Im}\,f_{p_{3/2}}^{(+)}. \quad (2.8)$$

The set of relations (2.1)—(2.8) makes it possible to express the real parts of the partial waves $s^{(+)}$, $p_{1/2}^{(+)}$, $p_{3/2}^{(+)}$ in terms of the coupling constant, $\sin^2\alpha_{33}$—subtraction constants and the function $F_0(\nu)$. We note that so far no use was made of expanding in powers of $1/M$.

3. THE STATIC LIMIT ($1/M \to 0$)

Performing in the expressions (2.1)—(2.8) an expansion in powers of $1/M$ we obtain

$$\mathrm{Re}\,f_s^{(+)}(\nu) = \frac{a^+}{2}\,[1+F_0(\nu)] - \frac{\nu}{\pi}\,\mathrm{P}\int_0^\infty \frac{\mathrm{Im}\,f_{p_{3/2}}^{(+)}(\nu')}{\nu'(\nu'-\nu)}\left[\frac{F_0(\nu)}{F_0(\nu')} - 1\right]d\nu',$$

$$\mathrm{Re}\,(f_{p_{1/2}}^{(+)} - f_{p_{3/2}}^{(+)}) = -2\,\frac{\nu}{\omega}\,f^2 - \frac{\nu\omega}{\pi}\,\mathrm{P}\int_0^\infty \frac{\mathrm{Im}\,f_{p_{3/2}}^{(+)}(\nu')}{\nu'\omega'}\,\frac{d\nu'}{\nu'-\nu},$$

$$\mathrm{Re}\,(f_{p_{1/2}}^{(+)} + 2f_{p_{3/2}}^{(+)}) = \frac{a^+}{2}\,[1-F_0(\nu)] + \frac{2\nu}{\pi}\,\mathrm{P}\int_0^\infty \frac{\mathrm{Im}\,f_{p_{3/2}}^{(+)}(\nu')}{\nu'(\nu'-\nu)}\,d\nu' + \frac{\nu}{\pi}\int_0^\infty \frac{\mathrm{Im}\,f_{p_{3/2}}^{(+)}(\nu')}{\nu'(\nu'-\nu)}\,d\nu'\left[\frac{F_0(\nu)}{F_0(\nu')} - 1\right]. \quad (3.1)$$

The relations (3.1) contain one parameter—the scattering length a^+. They satisfy the conditions of crossing symmetry:[1]

$$f_s^{(+)}(\omega) - f_s^{(+)}(-\omega) = 0,$$

$$[f_{p_{1/2}}^{(+)}(\omega) - f_{p_{3/2}}^{(+)}(\omega)] + [f_{p_{1/2}}^{(+)}(-\omega) - f_{p_{3/2}}^{(+)}(-\omega)] = 0,$$

$$[f_{p_{1/2}}^{(+)}(\omega) + 2f_{p_{3/2}}^{(+)}(\omega)] - [f_{p_{1/2}}^{(+)}(-\omega) + 2f_{p_{3/2}}^{(+)}(-\omega)] = 0. \quad (3.2)$$

We note that the $\pi\pi$ and πN terms of the Eqs. (3.1) separately satisfy the Eqs. (3.2).

It is an interesting peculiarity of the dispersion relations (3.1) that the expression for $\mathrm{Re}\,[f_{p_{1/2}}^{(+)} - f_{p_{3/2}}^{(+)}]$ does not involve $\pi\pi$ terms. The same is true for the difference $\mathrm{Re}\,[f_{p_{1/2}}^{(-)} - f_{p_{3/2}}^{(-)}]$, as follows from Eqs. (5.4) – (5.6) of [1]. This property is a manifestation of a certain symmetry of the contribution of the $\pi\pi$ interaction to πN scattering.

4. SYMMETRY PROPERTIES OF THE $\pi\pi$ TERMS IN πN SCATTERING

Let us denote by $G_{lJ}^{(\pm)}(W)$ the contribution of the $\pi\pi$ terms to the πN scattering partial wave with given values of l, J and isotopic index ($\pm$). Since the $\pi\pi$ terms enter only into the dispersion relations for backward scattering it follows from Eq. (2.6) that

$$G_s^{(\pm)}(W) = -3G_{p_{3/2}}^{(\pm)}(W). \quad (4.1)$$

The relation (4.1) is valid in s and p approximation, i.e., in the low energy region. For backward scattering the replacement $s \to \bar{s}$ corresponds to replacing the variable $W = p^0 + \omega$ by the variable $W' = p^0 - \omega$, so that

$$WW' = M^2 - 1. \tag{4.2}$$

At that $\nu(W) \equiv \nu(W')$ and $A^{(+)}[\nu(W)] = A^{(+)}[\nu(W')]$.

In the approximation (1.2) the terms responsible for the $\pi\pi$ contributions to the functions $\Delta^{(+)}$ and β have the form

$$A_{\pi\pi}^{(+)}(\nu) = -\frac{1}{\pi}\int\limits_{-\infty}^{-1}\frac{\gamma\sin\delta_0^0}{\nu'-\nu}d\nu', \quad \beta_{\pi\pi}(\nu) = 0, \tag{4.3}$$

which gives rise to the additional relation

$$G_{p_{3/2}}^{(+)}(W) - G_{p_{1/2}}^{(+)}(W) = \frac{p_0 - M}{p_0 + M}G_s^{(+)}(W). \tag{4.4}$$

Making use of (4.3), (2.6), and (2.7) we easily find that

$$W[G_s^{(+)}(W) - 3G_{p_{3/2}}^{(+)}(W)] = W'[G_s^{(+)}(W') - 3G_{p_{3/2}}^{(+)}(W')],$$

$$W[G_{p_{1/2}}^{(+)}(W) - G_{p_{3/2}}^{(+)}(W)] = W'[G_{p_{1/2}}^{(+)}(W') - G_{p_{3/2}}^{(+)}(W')]. \tag{4.5}$$

The relations (4.1) and (4.5) give rise to the symmetry properties of the function $G^{(+)}$ first established by Lovelace (private communication to Hamilton, see [3]):

$$WG_s^{(+)}(W) = W'G_s^{(+)}(W'), \quad WG_{p_{1/2}}^{(+)}(W) = W'G_{p_{1/2}}^{(+)}(W'),$$
$$WG_{p_{3/2}}^{(+)}(W) = W'G_{p_{3/2}}^{(+)}(W'). \tag{4.6}$$

They are a direct consequence of crossing symmetry and the second of Eqs. (4.3).

The analogous relations for the functions $G_{lJ}^{(-)}(W)$ are considerably more complicated since in this case $A_{\pi\pi}^{(-)}$ and $B_{\pi\pi}^{(-)}$ are different from zero. They reflect only the crossing symmetry properties of these functions. Since these properties for the partial waves are simplest in the static approximation we go to the limit $M \to \infty$.

Let

$$g_{lJ}^{(\pm)}(\omega) = \lim_{M\to\infty} G_{lJ}^{(\pm)}(W). \tag{4.7}$$

For the functions $g_{lJ}^{(\pm)}(\omega)$ it is easy to show that

$$g_{lJ}^{(\pm)}(\omega) = \pm g_{lJ}^{(\pm)}(-\omega). \tag{4.8}$$

A comparison of Eq. (4.8) with the crossing symmetry relations (5.1) from [1] shows that one and the same function $g_{p_{1/2}}^{(\pm)}(\omega) - g_{p_{3/2}}^{(\pm)}(\omega)$ must be both symmetric and antisymmetric in ω, i.e. it must vanish identically, or

$$g_{p_{1/2}}^{(\pm)}(\omega) \equiv g_{p_{3/2}}^{(\pm)}(\omega). \tag{4.9}$$

The Eqs. (3.1) satisfy the conditions (4.1), (4.8) and (4.9). The Eqs. (4.1), (4.4), (4.6), (4.8), and (4.9) are convenient for checking the consistency of the $\pi\pi$ contributions to different partial waves if the latter are evaluated independently. In the paper of Hamilton et al.[3] the relations (4.6) are satisfied. However the equality (4.1) is valid for the "differences" $\Delta^{(\pm)}$ themselves, which in addition to the $\pi\pi$ contributions contain, for example, also integrals from the crossed reaction to πN scattering. If the proposed decomposition of $\Delta^{(+)}$ into $\pi\pi$ terms and contributions due to distant singularities is accepted, then Eq. (4.1) is valid for the $\pi\pi$ contributions neither in magnitude nor in sign. Therefore the extraction of the $\pi\pi$ terms from the differences $\Delta_{lJ}^{(\pm)}$ must be viewed as unsuccessful.

Within statistical error limits the Eq. (4.9) is satisfied for $\Delta_{1/2\,3/2}^{(-)}$. For the quantities $\Delta_{lJ}^{(-)}$ none of the above mentioned relations is satisfied, which cannot be blamed on the neglect of the d waves since the $\Delta_{lJ}^{(-)}$ are calculated only up to energies of 100 MeV. While the failure of relations (4.9) and (4.8) could be blamed on terms of order 1/M, the equality (4.1) should nevertheless be satisfied.

5. COMPARISON WITH EXPERIMENTAL DATA

The system of equations (3.1) contains one subtraction parameter a^+. Its numerical value is small ($a^+ = -0.005$) and we therefore set in the calculations $a^+ = 0$. Since the subtraction parameters take into account the high energy behavior of the functions, it follows that in this case the low energy region is little influenced by the behavior of the scattering amplitude at high energies.

In order to obtain the energy dependence of Re $f_{l,J}^{(\pm)}$ it is necessary to know the functional form of δ_0^0. Below we consider the following versions:

$$\text{a) } \operatorname{tg}\delta_0^0 = a_0 k, \qquad \text{b) } \operatorname{tg}\delta_0^0 = \frac{a_0 k}{1 + k^2/3},$$

$$\text{c) } \operatorname{tg}\delta_0^0 = \frac{a_0 k}{1 + k^2/3}\frac{1}{1 - b_0 k^2}. \tag{5.1*}$$

Making use of the method described in [1] for the calculation of the function $F_0(\nu)$ we obtain the general formula:

$$F_0(\nu) = \Pi\frac{i\omega - k_i}{i\omega + k_i}\frac{i + k_j}{i - k_j},$$

$$\operatorname{Im} k_i > 0, \quad \operatorname{Im} k_j < 0, \quad \omega = \sqrt{\nu + 1},$$

*tg = tan.

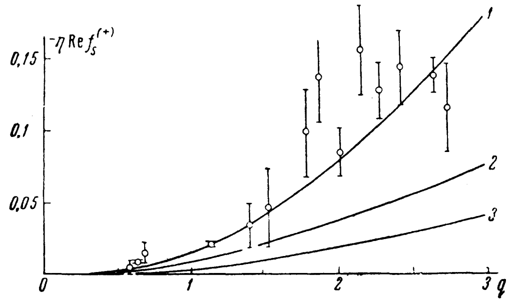

FIG. 1

where $k_{i,j}$ are the roots of the equation $1 + i \tan \delta_0^0 = 0$. The versions a) and b) differ in the asymptotic behavior for $k \to \infty$. Calculation shows that for values $1 < a_0 < 3$ and $1 < \omega < 3$ the ratio $F_0^{(a)}/F_0^{(b)}$ lies in the interval (1; 0.75). The same can be said of the $\pi\pi$ contribution: in the low energy region ($\omega \leq 3$) it varies by less than 20%. Consequently the assumptions about the asymptotic behavior of δ_0^0 have little effect on the low energy region $\omega < 3$.

Version b) is more realistic than version a) since in the scattering length approximation $\tan \delta_0^0 = a_0 k/\sqrt{1 + k^2}$. The presence of the root $\sqrt{1 + k^2}$ makes the calculation of $F_0(\nu)$ more difficult and we therefore approximate it by $1 + k^2/3$ in the region $k^2 \leq 10$. In version b) it is necessary that $a_0 > 3$ in order to explain the energy dependence of $\mathrm{Re}\, f_S^{(+)}$. Since such large values for the scattering length are improbable we consider below the version c).

It follows from a comparison of the solutions of the $\pi\pi$ scattering equations with the experimental data that the parameters a_0 and b_0 lie in the intervals $0.5 \leq a_0 \leq 1$ and $0.05 \leq b_0$.[8] We have therefore taken the extreme values of the scattering length and determined from them the parameter b_0. The following intervals are not in contradiction with experimental data on πN scattering:

$$a_0 = 0.5;\quad 0.04 \leqslant b_0 \leqslant 0.08 \quad (1030\ \text{MeV} \leqslant t_r^{1/2} \leqslant 1430\ \text{MeV});$$
$$a_0 = 1;\quad 0.07 \leqslant b_0 \leqslant 0.11 \quad (890\ \text{MeV} \leqslant t_r^{1/2} \leqslant 1095\ \text{MeV}).$$

The version $a_0 = 1$, $b_0 = 0.05$ (Fig. 1, curve 1) should be considered as best.[2)]

It is interesting to analyze the s-wave dominant solution of Chew, Mandelstam and Noyes[7] from the point of view of its correspondence to the πN scattering data. Up to values $k^2 = 9$ the phase shift δ_0^0 may be approximated to a high accuracy by the expressions

$$\begin{aligned} \mathrm{tg}\, \delta_0^0 &= 0.63\, k/(1 + 0.7\, k^2) \quad (\lambda = -0.1), \\ \mathrm{tg}\, \delta_0^0 &= 4.2\, k/(1 + 3.08\, k^2) \quad (\lambda = -0.3) \end{aligned} \tag{5.2}$$

(Fig. 1, curves 2, 3). The corresponding curves for $\mathrm{Re}\, f_S^+$ lie below all experimental points (see Fig. 1). Let us note that after the assumption $a^+ = 0$ is made the quantity $\mathrm{Re}\, f_S^{(+)}$ is determined by the $\pi\pi$ contribution only. Therefore the s^+ wave is most sensitive to the $\pi\pi$ interaction parameters.

The experimental data on the $p_{1/2}^+$ wave are known with large errors. Consideration of this wave adds nothing new to our knowledge of the parameters a_0 and b_0, although it does indicate agreement with experiment in the low energy region. The behavior of the $p_{3/2}^+$ wave is determined in essence by the resonant f_3^3 wave; the role played by the $\pi\pi$ terms is small.

The relations (4.1) and (4.9) connect the contributions into the s and p πN-scattering waves due to the $\pi\pi$ interaction. Given the $\pi\pi$ terms in the s waves these relations may be used to calculate the quantity $g_{3/2}^{3/2}(\omega)$. Its magnitude, as was to be expected, is small:

q	$g_{3/2}^{3/2}$	$\mathrm{Re} f_{3/2}^{3/2}$
0.5	+0.00025	0.05
1.0	−0.0006	0.23
1.5	−0.0054	0.24
2.0	−0.0120	−0.24
2.5	−0.0216	−0.18
3.0	−0.043	−0.095

Thus the low energy data on πN scattering is not in contradiction with the chosen values of the parameters $a_0 = 1$ and $b_0 = 0.05$ ($t_r^{1/2} = 1250$ MeV). The value of the scattering length $a_0 = 1$ is in agreement with the results of Hamilton et al.[3]

CONCLUSIONS

The Mandelstam double dispersion representation connects the πN scattering problem to that of the $\pi\pi$ interaction. Equations (3.1) and (5.6) from [1] explicitly display this connection. They have been obtained by one and the same method, which was proposed by Efremov et al.[5] The same method has been used by Serebryakov and Shirkov[8] to analyze $\pi\pi$ scattering. The manner in which Eqs. (3.1) are deduced from Eq. (5.6) of [1] allows the use of various assumptions for the form of the $\pi\pi$ scattering phase shifts in order to explain πN scattering. At that it turns out that the best description of πN scattering in s and p waves at low energies is obtained by using the solutions from [8]. The s-wave dominant solution of Chew,

2)Note added in proof (July 1, 1963). An analysis based on the χ^2 test allowed the extraction of regions of two minima for $b_0 < 0.09$. The first has borders $0.05 < b_0 < 0.07$, $0.7 < a_0 < 1.3$ and corresponds to a resonant form for the phase shift. Borders of the second region are much broader ($b_0 < 0.05$, $a_0 > 1.4$) and it corresponds to version b) with $a_0 \approx 3$ and $b_0 = 0$. The parameters a_0 and b_0 are strongly correlated. For $b_0 = 0.07$ we have $a_0 = 1 \pm 0.12$.

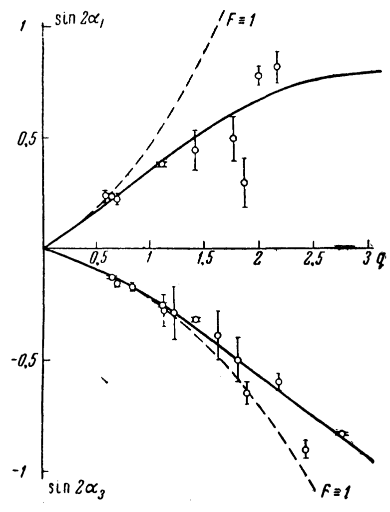

FIG. 2

Mandelstam and Noyes[7] does not correspond to the experimental data on πN scattering. From our point of view the result obtained proves the selfconsistency of the method for describing phenomena in the low energy region.

A comparison with the experimental data is carried out in the static approximation. The following conclusions are obtained:

1) For a correct description of the scattering of pions on nucleons the $\pi\pi$ interaction must be taken into account (Fig. 2).

2) The $\pi\pi$ contributions to πN scattering satisfy the conditions (4.1), (4.4), (4.6), (4.8), and (4.9).

3) s and p $\pi\pi$ scattering waves were taken into account independently. The assumption that these phase shifts have a resonant character leads to a satisfactory description of the experimental data on πN scattering.

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