INVESTIGATION OF NUCLEON INDUCED DEUTERON DISINTEGRATION BY THE DIAGRAM SUMMATION TECHNIQUE

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Submitted to JETP editor January 11, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 214—227 (August, 1963)

Integral equations for the interaction amplitudes of three nucleons are derived by the diagram summation technique. The first diagrams corresponding to the first iterations of the equations are considered. Their contribution is calculated in analytic form. An exact correspondence is established between the contribution of the first diagrams and the results previously obtained by the authors in the first approximation of perturbation theory by taking into account particle interactions in the final state. The results are extended to the case of real nucleons with spin and isotopic spin.

1. INTRODUCTION

THE three-body problem with short-range forces has been investigated many times. The question of deuteron decay induced by a nucleon was considered by the authors of the present paper on the basis of perturbation theory, with account of the interaction of the particles in the final state [1]. Skornyakov and Ter-Martirosyan [2] and Danilov [3] have shown that the solution of the three-body problem can be reduced to an integral equation for a function that depends on a single variable. The equation was obtained with account of terms of zeroth [2] and first [3] orders in small parameters proportional to the effective radius of the forces.

A method of investigation of nuclear reactions on the basis of the diagram technique has been recently developed in several papers [1)]. The diagram method has the advantage that it shows readily during the course of its application the particular processes that are taken into account in the different approximations. It is of interest to ascertain which diagrams of the nucleon-deuteron interaction process yield, upon summation, the Skornyakov and Ter-Martirosyan equations (STM) in both the zeroth and the first approximations in the interaction radius r_0. We show below (Secs. 2 and 3) that the problem of obtaining an equation for the Nd-interaction amplitude can actually be simply and elegantly solved on the basis of the diagram technique.

[1)]Amado[4] considered the stripping reaction on the basis of the perturbation-theory pole diagram; Shapiro was the first to investigate direct nuclear reactions with account of more complicated diagrams, particularly the triangular one.[5]

We have shown earlier that many experimental data on deuteron disintegrations induced by protons [1] at a total reaction energy on the order of 5 MeV are in good agreement with the results of calculations on the basis of perturbation theory, with account of the particle interaction in the final state. In this connection, we clarify in Sec. 4 of the present article which of the diagrams correspond to the indicated approximation.

Before we proceed to the derivation of the integral equations for the amplitude of a nucleon-induced deuteron disintegration by a method wherein a series of diagrams is summed, we write out the equations corresponding to the vertices and to the several blocks from which the diagrams describing a particular process will be built up. Thus, the vertex corresponding to the transition from two nucleons into a deuteron (Fig. 1a) corresponds [2)] to the quantity $-i\sqrt{8\pi\alpha}/m$; the nucleon propagation line corresponds to a quantity $G_n^0 = [E - E(p) + i\tau]^{-1}$, where $\tau > 0$ and $\tau \to 0$; the nucleon-nucleon scat-

FIG. 1

[2)]This value corresponds to an interaction Hamiltonian in the form

$$V(t) = -g\sqrt{\frac{4\pi}{m}}\, f(k) \sum_{\mathbf{p},\mathbf{k}} \{b^+(\mathbf{p}, t)\, a(\mathbf{k}, t)\, a(\mathbf{p}-\mathbf{k}, t) + a^+(\mathbf{k}, t)\, a^+(\mathbf{p}-\mathbf{k}, t)\, b(\mathbf{p}, t)\},$$

where $b^+(\mathbf{p}, t)$ and $b(\mathbf{p}, t)$ are the deuteron creation and annihilation operators, while $a^+(\mathbf{k}, t)$ and $a(\mathbf{k}, t)$ are the nucleon creation and annihilation operators.

tering block (Fig. 1b) corresponds to $A = -4\pi i/m(\alpha + if)$ [6].

In the non-zero approximation[3)] in r_0, the vertex corresponding to the contact interaction between two nucleons (Fig. 1c) corresponds to $A = -r_0$. Any closed loop (Fig. 1d) corresponds to integration over the energy and the momentum in the form

$$J = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dE \int \frac{d\mathbf{p}}{(2\pi)^3} F(E, \mathbf{p}).$$

2. INTEGRAL EQUATIONS FOR THE INELASTIC Nd SCATTERING AMPLITUDE IN THE ZEROTH APPROXIMATION IN r_0

For the sake of simplicity we consider first the inelastic scattering of a nucleon by a deuteron, accompanied by the disintegration of the deuteron, in the zeroth approximation in the effective radius, and assume that all particles participating in the reaction are identical and spinless. The sum of the contributions of the diagrams to the inelastic scattering amplitude has in the zeroth approximation of the effective radius the form of the diagram shown in Fig. 2, where block a is the exact value of the Nd scattering amplitude $A(\mathbf{k}, \mathbf{k}_0)$.

These diagrams do not include the contact-interaction vertices and consequently correspond to the zeroth approximation in r_0. It is obvious that the entire sum of the diagrams in the right half of Fig. 2 can be represented in the form of Fig. 3, since the iteration of this last equation

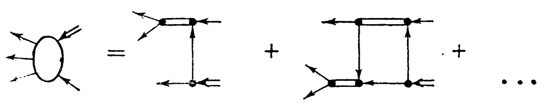

FIG. 2

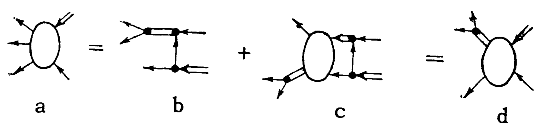

FIG. 3.

[3)]This result is the consequence of the fact that in the nucleon pair-interaction Hamiltonian

$$V(t) = -\sum_{\mathbf{k}',\mathbf{p}} a^+(\mathbf{k}')\, a^+(\mathbf{p}-\mathbf{k}')\, V(\mathbf{k}, \mathbf{k}')\, a(\mathbf{k})\, a(\mathbf{p}-\mathbf{k})$$

the value of

$$V(\mathbf{k}, \mathbf{k}') = \int e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} V(\mathbf{r})\, d\mathbf{r}$$

is equal to r_0 in the approximation linear in r_0.

makes it possible to reconstitute the aggregate of diagrams shown in Fig. 2. In fact, in view of the fact that the particles are identical, the pole diagram b of Fig. 3 is replaced by a sum of three diagrams, which differ only in the permutation of the particles in the final state (Fig. 4). The same pertains to the remaining diagrams. In other words, the amplitude of the nucleon-deuteron scattering with disintegration of the deuteron has the form of the diagram shown in Fig. 5.

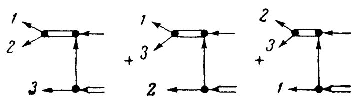

FIG. 4

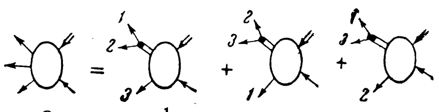

FIG. 5

The contribution of each diagram can be calculated with the aid of the values written out in Sec. 1 for its definite elements. To obtain the inelastic scattering amplitude it is necessary to multiply the contributions of these diagrams by $-i$ and by a factor $k/2\pi(dE/dk)$, the value of which can be readily obtained on going from the Feynman amplitudes (Figs. 3–5) to the amplitude whose square determines the inelastic scattering cross section. Here E is the total energy of the particles, which in the case of scattering has a value $E = -\alpha^2/m + k_{0d}^2/2\mu$, $\mu = 3m/2$. The quantities entering in both equations of Fig. 3 must be taken on the energy surface, that is, the value of the nucleon energy E_n and the deuteron energy E_d must be set equal to $E_n = k_0^2/2m$ and $E_d = -\alpha^2/m + k_{0d}^2/4m$ in the initial state (if we consider all the quantities in the c.m.s., then $\mathbf{k}_{0d} = -\mathbf{k}_0$).

For the sake of generality we assume that in the final state the NN system is formed in a continuous spectrum, that is, $E_d' = f^2/m + k^2/4m$, wherein the energy of the third nucleon must be set equal to $E_n' = k^2/2m$. Here f, k, and k_0 are connected by the energy conservation law, $3k_0^2/4 - \alpha^2 = 3k^2/4 + f^2$ (elastic nucleon-deuteron scattering corresponds to $f = -i\alpha$ and $k = k_0$).

The equation shown in Fig. 3 coincides directly with the STM equation for the nucleon-deuteron inelastic scattering. Indeed, the contribution to the inelastic scattering amplitude made by the pole diagram of Fig. 6, which takes into account

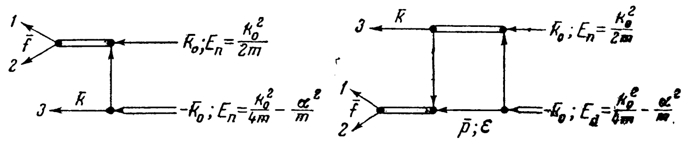

FIG. 6

the interaction of the particles in the final state, is

$$A_0(\mathbf{k},\ \mathbf{k}_0) = F\frac{k}{2\pi dE/dk} = \frac{m}{3\pi}F,$$

where

$$F = \frac{8\pi}{m}\sqrt{8\pi\alpha}\frac{1}{\alpha+if}\frac{1}{\alpha^2 - k_0^2/4 + k^2/2 + (\mathbf{k}+\mathbf{k}_0)^2/2},$$

or

$$A_0(\mathbf{k},\ \mathbf{k}_0) = \frac{8\sqrt{8\pi\alpha}}{3}\frac{1}{\alpha+if}\frac{1}{\alpha^2 - k_0^2/4 + k^2/2 + (\mathbf{k}+\mathbf{k}_0)^2/2}. \quad (1)$$

An analogous expression can be obtained for the contribution from exactly the same pole diagrams, but corresponding to the interaction of particles 2 and 3 or 3 and 1.

The contribution of the square diagram shown in Fig. 7 is determined by an integral of the form

$$A_1(\mathbf{k},\ \mathbf{k}_0) = \frac{m}{3\pi}\left(\frac{4\pi}{m}\right)^2\frac{i}{2\pi}\int d\varepsilon\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{\sqrt{8\pi\alpha}/m}{(\alpha+if)\left(\alpha-\sqrt{{}^3/_4p^2 - {}^3/_4k_0^2+\alpha^2}\right)}$$
$$\times\left\{\left[\varepsilon - \frac{p^2}{2m} + i\tau\right]\left[\frac{k_0^2}{4m} - \frac{\alpha^2}{m} - \varepsilon - \frac{(\mathbf{k}_0-\mathbf{p})^2}{2m} + i\tau\right]\right.$$
$$\left.\times\left[-\varepsilon + \frac{k^2}{4m} - \frac{\alpha^2}{m} - \frac{(\mathbf{p}+\mathbf{k})^2}{2m} + i\tau\right]\right\}^{-1}, \quad (2)$$

which after integration over the energy[4] E can be written in the form

$$A_1(\mathbf{k},\ \mathbf{k}_0) = \left(\frac{8}{3}\right)^2\sqrt{8\pi\alpha}\frac{1}{\alpha+if}4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}$$
$$\times\frac{{}^3/_4}{\alpha-\sqrt{{}^3/_4p^2 - {}^3/_4k_0^2+\alpha^2}}\left\{\left[\alpha^2 - \frac{k_0^2}{4} + \frac{p^2}{2} + \frac{(\mathbf{k}_0+\mathbf{p})^2}{2}\right]\right.$$
$$\left.\times\left[\alpha^2 - \frac{k^2}{4} + \frac{p^2}{2} + \frac{(\mathbf{k}+\mathbf{p})^2}{2}\right]\right\}^{-1}. \quad (3)$$

The integral term (diagram c on Fig. 3) has a perfectly analogous form and differs from (3) in that under the integral sign the term

$$\frac{8}{3}\sqrt{8\pi\alpha}\left\{\left(\alpha - \sqrt{\frac{3}{4}p^2 - \frac{3}{4}k_0^2 + \alpha^2}\right)\right.$$
$$\left.\times\left[\alpha^2 - \frac{k_0^2}{4} + \frac{p^2}{2} + \frac{(\mathbf{k}_0+\mathbf{p})}{2}\right]\right\}^{-1}$$

[4]This integration reduces to the calculation of the residue of the integrand at the point $\epsilon = p^2/2m$.

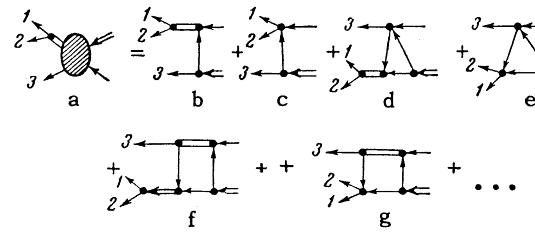

FIG. 7

is replaced by the exact value of the amplitude $A(\mathbf{p}, \mathbf{k}_0)$ on the energy surface and is therefore determined by an expression of the form

$$A_n(\mathbf{k},\ \mathbf{k}_0) = \frac{8}{3}\frac{1}{\alpha+if}4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{{}^3/_4A(\mathbf{p},\ \mathbf{k}_0)}{\alpha^2 - k^2/4 + p^2/2 + (\mathbf{k}+\mathbf{p})^2/2}. \quad (4)$$

Thus, the equation shown symbolically on Fig. 3, or the amplitude $A(\mathbf{k}, \mathbf{k}_0)$ for particles 1 and 2 produced in the final state in a continuous spectrum (diagram b of Fig. 5) can be written in the form

$$\frac{3}{8}A(\mathbf{k},\ \mathbf{k}_0) = \frac{1}{\alpha+if}\left[\sqrt{8\pi\alpha}\frac{1}{\alpha^2 + k^2/2 - k_0^2/4 + (\mathbf{k}+\mathbf{k}_0)^2/2}\right.$$
$$\left. + 4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{{}^3/_4A(\mathbf{p},\ \mathbf{k}_0)}{\alpha^2 - k^2/4 + p^2/2 + (\mathbf{k}+\mathbf{p})^2/2}\right]. \quad (5)$$

If we introduce the notation

$$A(\mathbf{k},\ \mathbf{k}_0) = [\sqrt{8\pi\alpha}/(\alpha^2 + f^2)]\,a(\mathbf{k},\ \mathbf{k}_0),$$

we obtain for $a(\mathbf{k}, \mathbf{k}_0)$ the equation

$$\frac{3}{8}\frac{a(\mathbf{k},\ \mathbf{k}_0)}{\alpha - if} = \frac{1}{\alpha^2 + k^2/2 - k_0^2/4 + (\mathbf{k}+\mathbf{k}_0)^2/2} + 4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}$$
$$\times\frac{a(\mathbf{p},\ \mathbf{k}_0)}{[\alpha^2 - k^2/4 + p^2/2 + (\mathbf{k}+\mathbf{p})^2/2]\,[p^2 - k_0^2 + i\tau]}. \quad (6)$$

It is obvious that the value of $a(\mathbf{k}, \mathbf{k}_0)$ obtained by summation of the diagrams coincides fully with the corresponding value of $a(\mathbf{k}, \mathbf{k}_0)$ in the STM equation.

The total scattering amplitude $A(\mathbf{k}, \mathbf{f}; \mathbf{k}_0, \alpha)$ has in accordance with Fig. 5 the form

$$A(\mathbf{k},\ \mathbf{f};\ \mathbf{k}_0,\ \alpha) = A(\mathbf{k},\ \mathbf{k}_0) + A(\mathbf{f} - {}^1/_2\mathbf{k},\ \mathbf{k}_0)$$
$$+ A(-\mathbf{f} - {}^1/_2\mathbf{k},\ \mathbf{k}_0). \quad (7)$$

3. CONTRIBUTION OF FIRST DIAGRAMS TO THE INELASTIC Nd SCATTERING IN THE APPROXIMATION LINEAR IN r_0

We now proceed to calculate the inelastic scattering amplitude with account of terms linear in r_0. The nucleon-nucleon scattering amplitude with account of the terms linear in r_0 can be written in the form

$$a(f) = (f\,\mathrm{ctg}\,\delta - if)^{-1}, \quad (8)^*$$

*ctg = cot.

where

$$f \operatorname{ctg} \delta = -\alpha + {}^1/_2 r_0 (f^2+\alpha^2).$$

If we now separate in the expression for a(f) the term corresponding to the vertex of the contact interaction, proportional to r_0, namely

$$a(f) = r_0/2 - (1+\alpha r_0)/(\alpha + if),$$

then the blocks in the diagrams of Fig. 2, which describe the scattering of two nucleons, must be represented in the form of a sum of two diagrams, one corresponding to the direct contact interaction, and the other including the multiple scattering of the two nucleons. Then the nucleon-deuteron inelastic scattering amplitude with disintegration of the deuteron and with interaction of nucleons 1 and 2 in the final state, $A'(\mathbf{k}, \mathbf{k}_0)$, can be represented in the form of contributions from the sum of the following diagrams (see Fig. 8).

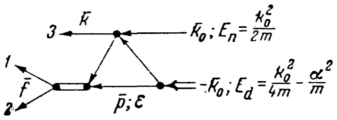

FIG. 8

We shall henceforth prime all the quantities determined in the approximation linear in r_0 to distinguish them from the quantities in the zeroth approximation in the nuclear-force radius. Since any pair of particles can interact in the final state, the expression for $A'(\mathbf{k}, \mathbf{k}_0)$ must be symmetrized with respect to the permutation of any pair of particles.

Let us see what are the contributions from the first four diagrams. The contribution to the amplitude of the process from the first two pole diagrams, b and c of Fig. 7, obviously is of the form

$$A_0'(\mathbf{k}, \mathbf{k}_0) = \frac{8}{3}\sqrt{8\pi\alpha}\, a(f)\left[\alpha^2 - \frac{k_0^2}{4} + \frac{k^2}{2} + \frac{(\mathbf{k}_0+\mathbf{k})^2}{2}\right]^{-1},$$

$$a(f) = \frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}. \tag{9}$$

We now calculate the summary contribution to the scattering amplitude from the two triangular diagrams d and e of Fig. 7, which is determined by an integral of the form[5)]

$$A_T'(\mathbf{k}, \mathbf{k}_0) = \left(\frac{8}{3}\right)^2 \sqrt{8\pi\alpha}\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right] 4\pi\, \frac{r_0}{2}\int_{-\infty}^{\infty} d\varepsilon \int \frac{d\mathbf{p}}{(2\pi)^3}$$
$$\times\left[\varepsilon - \frac{p^2}{2m} + i\tau\right]^{-1}\left[\frac{k_0^2}{4m} - \alpha^2 - \varepsilon - \frac{(\mathbf{k}_0+\mathbf{p})^2}{2m} + i\tau\right]^{-1}$$
$$\times\left[\frac{k_0^2}{4m} - \frac{k^2}{2m} - \varepsilon - \alpha^2 - \frac{(\mathbf{k}+\mathbf{p})^2}{2m} + i\tau\right]^{-1}. \tag{10}$$

After calculating the integral (10) with respect to the energy ϵ, which obviously reduces to the determination of the residue of the integrand at the point $\epsilon = p^2/2m$, we obtain

$$A_T'(\mathbf{k}, \mathbf{k}_0) = \frac{8}{3}\sqrt{8\pi\alpha}\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right]\cdot 4\pi \frac{r_0}{2}\int \frac{d\mathbf{p}}{(2\pi)^3}$$
$$\times\left[\alpha^2 - \frac{k_0^2}{4} + \frac{p^2}{2} + \frac{(\mathbf{k}_0+\mathbf{p})^2}{2}\right]^{-1}$$
$$\times\left[\alpha^2 - \frac{k^2}{4} + \frac{p^2}{2} + \frac{(\mathbf{k}+\mathbf{p})^2}{2}\right]^{-1}$$
$$= \frac{8}{3}\sqrt{8\pi\alpha}\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right] 4\pi\, \frac{r_0}{2} I_\Delta. \tag{11}$$

This quantity can be calculated analytically. Using the well-known Feynman relation

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[a + b(1-x)^2]^2},$$

we represent the integral (11) in the form

$$I_\Delta = \int_0^1 dx \frac{1}{2\pi^2}\int \frac{d\mathbf{p}}{[p^2 + \mathbf{p}[x\mathbf{k}_0 + (1-x)\mathbf{k}] + xk_0^2 + (1-x)k^2 - \lambda^2]^2}$$
$$= \int_0^1 dx \frac{2}{\pi}\int_0^\infty \frac{q^2\, dq}{[q^2 - \xi]^2};$$

$$q^2 = {}^1/_4 (k^2 + k_0^2 - 2\mathbf{k}\mathbf{k}_0),$$

$$\xi = f^2 - (\alpha^2 + f^2 + q^2)\, x + q^2 x^2$$
$$= q^2\Big\{[x - (\alpha^2 + f^2 + q^2)/2q^2]^2$$
$$- (\alpha^2 + f^2 + q^2)/2q^2 + \frac{f^2}{q^2}\Big\}. \tag{12}$$

After integrating with respect to dq we get

$$I_\Delta = \frac{i}{2q}\int_0^1 \frac{dx}{\sqrt{\xi + i\tau}} = \frac{1}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)}. \tag{13}$$

Consequently, the contribution of the triangular diagram to the scattering amplitude is of the form

$$A_T'(\mathbf{k}, \mathbf{k}_0) = \left(\frac{8}{3}\right)^3\sqrt{8\pi\alpha}\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right]\frac{r_0}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)}. \tag{14}$$

The contributions of the first two diagrams are of the form

[5)]In the nonrelativistic case the contribution of the triangular diagrams was calculated for the first time in general form by Blokhintsev et al,[7] by a method somewhat different from that used by the authors.

$$A_0'(\mathbf{k}, \mathbf{k}_0) + A_T'(\mathbf{k}, \mathbf{k}_0)$$
$$= \frac{8}{3}\sqrt{8\pi\alpha}\, a(f)\left\{\frac{1}{\alpha^2 - k_0^2/4 + k^2/2 + (\mathbf{k}+\mathbf{k}_0)^2/2} + \frac{r_0}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)}\right\}. \quad (15)$$

4. COMPARISON WITH THE PERTURBATION-THEORY MATRIX ELEMENTS, CONSTRUCTED WITH ACCOUNT OF THE PARTICLE INTERACTION IN THE FINAL STATE

The authors have previously investigated [1-3] the energy distributions of the neutrons emitted at different angles, on the basis of perturbation theory with account of interaction of the particle pair in the final state, for the decay of a deuteron induced by protons at a total reaction energy ~ 5 MeV. In this approximation the amplitude of the reaction $p+d \rightarrow p+p+n$ (without account of the spin and isospin) is of the form

$$B(\mathbf{k}, \mathbf{k}_0) = \frac{1}{4\pi}\frac{2\mu}{\hbar^2}\int e^{-i\mathbf{k}\mathbf{r}_3}\varphi_f(\rho_{12})\,[V_{13} + V_{23}]\, e^{i\mathbf{k}_0\mathbf{r}_2}\varphi_d(\rho_{13})\, d\rho_{13}\, d\mathbf{r}_2. \quad (16)$$

In the approximation of zero effective radius, the potential of the interaction can be represented in the form $V_{ij} = -4\pi\hbar^2\mu^{-1}\rho_{ij}\delta(\rho_{ij})$; the wave function $\varphi(\rho_{12})$ of the two interacting nucleons in the final state has then the form

$$\varphi_f(\rho_{12}) = e^{-if\rho_{12}} + a(f)\, e^{if\rho_{12}}/\rho_{12}.$$

The wave function of the deuteron is

$$\varphi_d(\rho_{13}) = \sqrt{\alpha/2\pi}\, e^{-\alpha\rho_{13}}/\rho_{13}.$$

Substituting these expressions into the integral (16), we obtain the scattering amplitude in the form $B = J_1 + J_2$, where

$$J_1 = \frac{4}{3}\int e^{i\mathbf{k}_0\mathbf{r}_2}\sqrt{\frac{\alpha}{2\pi}}\frac{1}{\rho_{13}}e^{-\alpha\rho_{13}}\rho_{13}\delta(\rho_{13})\, e^{-ikr_3}\left[e^{-if\rho_{12}} + \frac{a(f)}{\rho_{12}}e^{if\rho_{12}}\right] \times d\rho_{13}\, d\mathbf{r}_2,$$

$$J_2 = \frac{4}{3}\int e^{i\mathbf{k}_0\mathbf{r}_2}\sqrt{\frac{\alpha}{2\pi}}\frac{1}{\rho_{13}}e^{-\alpha\rho_{13}}\rho_{23}\delta(\rho_{23})\, e^{-ikr_3}\left[e^{-if\rho_{12}} + \frac{a(f)}{\rho_{12}}e^{if\rho_{12}}\right] \times d\rho_{13}\, d\mathbf{r}_2. \quad (17)$$

The second term of J_2 is proportional to the integral $\int \rho_{23}\delta(\rho_{23})d\rho_{23}$, which vanishes in the approximation of zero effective radius. If terms of the first order in r_0 are taken into account, the value of the analogous integral $(-\mu/4\pi\hbar^2)\int V(\rho_{23})d\rho_{23}$ will be of the order of the effective radius of the nuclear forces. We denote it by r_0.

After calculating the integrals J_1 and J_2 we obtain the expressions [6)]

$$J_1 = \frac{4}{3}\sqrt{\frac{\alpha}{2\pi}}\, 4\pi\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right]\frac{1}{(\mathbf{k}+\mathbf{k}_0/2)^2 + \alpha^2},$$

$$J_2 = \frac{4}{3}\sqrt{\frac{\alpha}{2\pi}}\, 4\pi\left\{\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right]\frac{r_0}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)} + \frac{1}{(\mathbf{q}-\mathbf{f})^2 + \alpha^2}\right\}, \quad (18)$$

where

$$\left[\left(\mathbf{k} + \frac{\mathbf{k}_0}{2}\right)^2 + \alpha^2\right]^{-1} = \left[\alpha^2 - \frac{k_0^2}{4} + \frac{k^2}{2} + \frac{(\mathbf{k}+\mathbf{k}_0)^2}{2}\right]^{-1} = [q_0^2 + \alpha^2]^{-1}.$$

Thus, the inelastic nucleon-deuteron scattering amplitude is obtained in this approximation in the form

$$B(\mathbf{k}, \mathbf{k}_0) = \frac{16}{3}\pi\sqrt{\frac{\alpha}{2\pi}}\left\{\left[\frac{r_0}{2} - \frac{1+\alpha r_0}{\alpha + if}\right] \times \left[\frac{1}{q_0^2 + \alpha^2} + \frac{r_0}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)}\right] + \frac{r_0}{(\mathbf{q}-\mathbf{f})^2 + \alpha^2}\right\}. \quad (19)$$

The term $r_0/[(\mathbf{q}-\mathbf{f})^2 + \alpha^2]$ with $f \rightarrow 0$ is a small addition to $B(\mathbf{k}, \mathbf{k}_0)$ and can be neglected in the calculation of the corresponding sections of the nucleon spectrum; then expression (19) coincides exactly with the contribution of the first two perturbation-theory diagrams (15).

Expression (19) differs from the total reaction amplitude $A'(\mathbf{k}, \mathbf{k}_0)$ by an amount equal to the contributions of the large number of the remaining diagrams. There are no theoretical grounds for assuming that the main contribution is made by the diagrams taken into account here. It is curious to note, however, that a comparison of the cross section calculated on the basis of (19) (or more accurately on the basis of analogous formulas calculated with account of the spin and of the isospin) with the experimental data [1-3] offers evidence that the contribution of all the remaining diagrams is negligibly small, at least in the region of very low energies of the pair of produced nucleons.

5. ACCOUNT OF THE SPIN AND ISOSPIN

For a comparison of the experimental data it is essential to write down the amplitude of nucleon-

6)These expressions were obtained by the authors together with V. K. Voĭtovetskiĭ in the investigation of the experimental energy distribution of the protons from the (n, d) reaction.

deuteron inelastic scattering with neutron disintegration for real particles, with account of the spin and isospin variables. In the diagram method such an account reduces to a procedure wherein each block, describing the scattering of two nucleons (if) with production of an intermediate state in the zero effective radius approximation, is set in correspondence with an operator

$$a + b\hat{P}^{\sigma}_{ij},$$

$$a = \frac{1}{2}\left[\frac{-1}{\alpha_t + if_{ij}} + \frac{-1}{\alpha_s + if_{ij}}\right], \quad b = \frac{1}{2}\left[\frac{-1}{\alpha_t + if_{ij}} - \frac{-1}{\alpha_s + if_{ij}}\right],$$

where f_{ij} has the relative momentum of the particles of i and j. The vertex of the decay of the deuteron into nucleons or the production of a deuteron from two nucleons corresponds, as in the case of bosons, to a number equal to $-i\sqrt{8\pi\alpha_t}/m$.

The vertex operators presented above have been written out for s-scattering of nucleons, that is, for the case when the contribution to the scattering amplitude is made only by the s-waves of the expansion of the coordinate wave functions in partial waves, and consequently, when $\hat{p}^r_{ij} = 1$. The contribution of each diagram will now be determined by a matrix element of some operator between the wave functions of the spin and isospin of the initial and final states.

Let us construct first the matrix elements of the diagrams describing the process of three-nucleon scattering, when particles 1 and 2 can interact in the final state. Inasmuch as at low reaction energies the particles 1 and 2 interact only in the s-state, the wave functions of the spin and isospin $\Phi(12;3)$ should be antisymmetrical with respect to permutation of the variables of particles 1 and 2. These functions, as is well known, are for the case $T = 1/2$ and $S = 1/2$ the functions $\chi^{1/2}_{\text{sym}}(12;3)\,\vartheta^{1/2}_{a}(12;3)$ and $\chi^{1/2}_{a}(12;3)\vartheta^{1/2}_{\text{sym}}(12;3)$, where

$$\begin{aligned}
\chi^{1/2}_{\text{sym}}(12;3) &= 6^{-1/2}(2b_3a_1a_2 - a_3a_1b_2 - a_3b_1a_2),\\
\chi^{1/2}_{a}(12;3) &= 2^{-1/2}(a_1b_2 - a_2b_1)\,a_3,\\
\vartheta^{1/2}_{\text{sym}}(12;3) &= 6^{-1/2}(2\beta_3\alpha_1\alpha_2 - \alpha_3\alpha_1\beta_2 - \alpha_3\beta_1\alpha_2),\\
\vartheta^{1/2}_{a}(12;3) &= 2^{-1/2}(\alpha_1\beta_2 - \alpha_2\beta_1)\,\alpha_3
\end{aligned} \tag{20}$$

(χ are the spin and ϑ the isospin functions), while for the case $T = 1/2$ and $S = 3/2$ this is the function $\chi^{3/2}_{\text{sym}}(12;3)\vartheta^{1/2}_{a}(12;3)$, where

$$\chi^{3/2}_{\text{sym}}(12;3) = 3^{-1/2}(b_3a_1a_2 + a_3a_1b_2 + a_3b_1a_2). \tag{21}$$

The case of states with $T = 3/2$ is not considered, in view of the fact that the isotopic spin of the initial state of the nucleon-deuteron system is $1/2$.

Simple calculation shows that the contribution of the pole diagram can be represented in the form

$$A_0 = (\Phi^*(12;3)\hat{A}_{01}\Phi(23;1)) + (\Phi^*(12;3)\hat{A}_{02}\Phi(31;2)). \tag{22}$$

Here $\Phi(12;3)$ denotes one of the wave functions of the spin and isospin of the final state, while $\Phi(23;1)$ and $\Phi(31;2)$ denote the analogous functions of the spin and isospin of the particles in the initial state. The operators $\hat{A}_{01}$ and $\hat{A}_{02}$ are of the form

$$\begin{aligned}
\hat{A}_{01} &= \tfrac{4}{3}\,[a + b\hat{P}^{\sigma}_{12}]\,\gamma^{-1}(\mathbf{k},\mathbf{k}_0)\,[a_1 + b_1\hat{P}^{\sigma}_{23}],\\
\hat{A}_{02} &= \tfrac{4}{3}\,[a + b\hat{P}^{\sigma}_{12}]\,\gamma^{-1}(\mathbf{k},\mathbf{k}_0)\,[a_1 + b_1\hat{P}^{\sigma}_{31}];\\
\gamma(\mathbf{k},\mathbf{k}_0) &= \alpha^2 - k_0^2/4 + k^2/2 + (\mathbf{k}+\mathbf{k}_0)^2/2.
\end{aligned} \tag{23}$$

The contribution of the square diagram is in this case of the form

$$A_1 = (\Phi^*(12;3)\hat{A}^1_1\Phi(23;1)) + (\Phi^*(12;3)\hat{A}^2_1\Phi(31;2)) + (\Phi^*(12;3)\hat{A}^3_1\Phi(12;3)) + (\Phi^*(12;3)\hat{A}^4_1\Phi(12;3)), \tag{24}$$

where

$$\begin{aligned}
\hat{A}^1_1 &= 2\left(\frac{4}{3}\right)^2[a + bP^{\sigma}_{12}]\left\{4\pi\int[a_2 + b_2\hat{P}^{\sigma}_{31}]\times\frac{d\mathbf{p}}{(2\pi)^3}\gamma^{-1}(\mathbf{p},\mathbf{k})\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\right\}[a_1 + b_1\hat{P}^{\sigma}_{23}],\\
A^2_1 &= 2\left(\frac{4}{3}\right)^2[a + bP^{\sigma}_{12}]\left\{4\pi\int[a_2 + b_2\hat{P}^{\sigma}_{23}]\times\frac{d\mathbf{p}}{(2\pi)^3}\gamma^{-1}(\mathbf{p},\mathbf{k})\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\right\}[a_1 + b_1\hat{P}^{\sigma}_{31}],\\
A^3_1 &= 2\left(\frac{4}{3}\right)^2[a + bP^{\sigma}_{12}]\left\{4\pi\int[a_2 + b_2\hat{P}^{\sigma}_{23}]\times\frac{d\mathbf{p}}{(2\pi)^3}\gamma^{-1}(\mathbf{p},\mathbf{k})\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\right\}[a_1 + b_1\hat{P}^{\sigma}_{12}],\\
A^4_1 &= 2\left(\frac{4}{3}\right)^2[a + bP^{\sigma}_{12}]\left\{4\pi\int[a_2 + b_2\hat{P}^{\sigma}_{31}]\times\frac{d\mathbf{p}}{(2\pi)^3}\gamma^{-1}(\mathbf{p},\mathbf{k})\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\right\}[a_1 + b_1\hat{P}^{\sigma}_{12}].
\end{aligned}$$

If the two nucleons were in a bound state in the final state (deuteron), then the operator corresponding in the diagrams to the block of scattering of two nucleons ij in the initial state $a_1 + b_1P^{\sigma}_{ij}$ must be replaced by the number $\sqrt{8\pi\alpha_t}$.

Using the indicated values of the contributions of the pole and quadratic diagrams, we can obtain integral equations for the nucleon-deuteron scattering amplitudes for a total system spin $S = 1/2$ and isospin $T = 1/2$, or $S = 3/2$ and $T = 1/2$. We first obtain the equations for the nucleon-deuteron scattering amplitude in the zero effective radius approximation for $S = 1/2$ and $T = 1/2$.

We denote all the terms of the nucleon-nucleon scattering amplitude with particle-pair interaction in the singlet state by the subscripts s, while in the triplet state by the subscripts t. The contribution of the pole diagram to the nucleon-deuteron inelastic scattering amplitude in the case of interaction of particles 1 and 2 with antiparallel spins in the final state will be of the form

$$A_{0s}^{1/2}(\mathbf{k},\mathbf{k}_0) = (\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,{}^4/_3\,[a+b\hat{P}_{12}^\sigma] \times \gamma^{-1}(\mathbf{k},\mathbf{k}_0)\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(23;1)\,\vartheta_a^{1/2}(23;1)) + (\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,{}^4/_3\,[a+b\hat{P}_{12}^\sigma]\,\gamma^{-1}(\mathbf{k},\mathbf{k}_0) \times \sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(31;2)\,\vartheta_a^{1/2}(31;2)) = 2\sqrt{8\pi\alpha_t}\,[\alpha_s+if]^{-1}\gamma^{-1}(\mathbf{k},\mathbf{k}_0). \quad (25)$$

In the case of interaction of particles 1 and 2 in the triplet state

$$A_{0t}^{1/2}(\mathbf{k},\mathbf{k}_0) = (\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\,{}^4/_3\,[a+b\hat{P}_{12}^\sigma] \times \gamma^{-1}(\mathbf{k},\mathbf{k}_0)\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(23;1)\,\vartheta_a^{1/2}(23;1)) + (\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\,{}^4/_3\,[a+b\hat{P}_{12}^\sigma]\,\gamma^{-1}(\mathbf{k},\mathbf{k}_0) \times \sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(31;2)\,\vartheta_a^{1/2}(31;2)) = \frac{2}{3}\sqrt{8\pi\alpha_t}\,(\alpha_t+if)^{-1}\gamma^{-1}(\mathbf{k},\mathbf{k}_0). \quad (26)$$

The contribution from the square diagram for the case of interaction of particles 1 and 2 in the final singlet state is determined by the expression

$$\begin{aligned} A_{1s}^{1/2}(\mathbf{k},\mathbf{k}_0) = {} & \Big(\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{31}^\sigma]\,\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\} \\ & \times\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(23;1)\,\vartheta_a^{1/2}(23;1)\Big) \\ & +\Big(\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{23}^\sigma] \\ & \times\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\}\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(31;2)\,\vartheta_a^{1/2}(31;2)\Big) \\ & +\Big(\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{31}^\sigma] \\ & \times\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\}\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\Big) \\ & +\Big(\chi_a^{1/2}(12;3)\,\vartheta_{sym}^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{23}^\sigma] \\ & \times\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\}\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\Big) \\ = {} & \left(\tfrac{4}{3}\right)^2\frac{4\pi}{\alpha_s+if}\sqrt{8\pi\alpha_t}\int\frac{d\mathbf{p}}{(2\pi)^3}\Big[\frac{1}{2}A_{0s}^{1/2}(\mathbf{p},\mathbf{k}_0) \\ & +\frac{3}{2}A_{0t}^{1/2}(\mathbf{p},\mathbf{k}_0)\Big]\gamma^{-2}(\mathbf{k},\mathbf{p}). \end{aligned} \quad (27)$$

For the case of interaction of particles 1 and 2 in the triplet state, the contribution from the square diagram to the scattering amplitude will be of the form

$$\begin{aligned} A_{1t}^{1/2}(\mathbf{k},\mathbf{k}_0) = {} & \Big(\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{31}^\sigma]\,\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\} \\ & \times\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(23;1)\,\vartheta_a^{1/2}(23;1)\Big)+\Big(\chi_{sym}^{1/2}(12;3) \\ & \times\vartheta_a^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma]\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{23}^\sigma] \\ & \times\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\}\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(31;2)\,\vartheta_a^{1/2}(31;2)\Big) \\ & +\Big(\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{31}^\sigma]\,\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\} \\ & \times\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\Big) \\ & +\Big(\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\,2\left(\tfrac{4}{3}\right)^2[a+b\hat{P}_{12}^\sigma] \\ & \times\Big\{4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}[a_2+b_2\hat{P}_{23}^\sigma] \\ & \times\gamma^{-1}(\mathbf{p},\mathbf{k}_0)\,\gamma^{-1}(\mathbf{p},\mathbf{k})\Big\}\sqrt{8\pi\alpha_t}\,\chi_{sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3)\Big) \\ = {} & \left(\tfrac{4}{3}\right)^2\frac{4\pi}{\alpha_t+if}\sqrt{8\pi\alpha_t}\int\frac{d\mathbf{p}}{(2\pi)^3}\Big[\frac{1}{2}A_{0t}^{1/2}(\mathbf{p},\mathbf{k}_0) \\ & +\frac{3}{2}A_{0s}^{1/2}(\mathbf{p},\mathbf{k}_0)\Big]\gamma^{-1}(\mathbf{p},\mathbf{k}). \end{aligned} \quad (28)$$

The integral terms of the equations for the nucleon-deuteron scattering amplitude with singlet or triplet interaction of particles 1 and 2 in the final state can be obtained from (27) and (28) by replacing the zero-order terms $A_{0s}^{1/2}(\mathbf{k},\mathbf{k}_0)$ or $A_{0t}^{1/2}(\mathbf{k},\mathbf{k}_0)$ under the integral signs with the exact values of the scattering amplitudes $A_s^{1/2}(\mathbf{k},\mathbf{k}_0)$ and $A_t^{1/2}(\mathbf{k},\mathbf{k}_0)$. We introduce the notation

$$A_s^{1/2}(\mathbf{k},\mathbf{k}_0) = a_s^{1/2}(\mathbf{k},\mathbf{k}_0)\sqrt{8\pi\alpha_t}/(\alpha_s^2+f^2),$$
$$A_t^{1/2}(\mathbf{k},\mathbf{k}_0) = a_t^{1/2}(\mathbf{k},\mathbf{k}_0)\sqrt{8\pi\alpha_t}/(\alpha_t^2+f^2).$$

We then obtain the following expressions for $a_s^{1/2}(\mathbf{k},\mathbf{k}_0)$ and $a_t^{1/2}(\mathbf{k},\mathbf{k}_0)$:

$$\begin{aligned} \frac{\alpha_s+if}{k^2-k_0^2}a_s^{1/2}(\mathbf{k},\mathbf{k}_0) &= \frac{3}{2}\gamma^{-1}(\mathbf{k},\mathbf{k}_0) + 4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{{}^3/_2\,a_t^{1/2}(\mathbf{p},\mathbf{k}_0)+{}^1/_2\,a_s^{1/2}(\mathbf{p},\mathbf{k}_0)}{\gamma(\mathbf{p},\mathbf{k})\,(p^2-k_0^2)}, \\ \frac{\alpha_t+if}{k^2-k_0^2}a_t^{1/2}(\mathbf{k},\mathbf{k}_0) &= \frac{1}{2}\gamma^{-1}(\mathbf{k},\mathbf{k}_0) + 4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{{}^1/_2\,a_t^{1/2}(\mathbf{p},\mathbf{k}_0)+{}^3/_2\,a_s^{1/2}(\mathbf{p},\mathbf{k}_0)}{\gamma(\mathbf{p},\mathbf{k})\,(p^2-k_0^2)}. \end{aligned} \quad (29)$$

It is obvious that these equations correspond exactly to the STM equations with the spin and isospin taken into account. Analogous equations can be obtained for the scattering amplitude in the case when particles 2 and 3 or 3 and 1 interact in the final state. Then we have for the total nucleon-deuteron scattering amplitude

$$A(\mathbf{k},\mathbf{f};\mathbf{k}_0,\alpha) = A_s^{1/2}(\mathbf{k},\mathbf{k}_0)\,\chi_a^{1/2}(12;3)\,\vartheta_{\rm sym}^{1/2}(12;3) + A_t^{1/2}(\mathbf{k},\mathbf{k}_0)\,\chi_{\rm sym}^{1/2}(12;3)\,\vartheta_a^{1/2}(12;3) + A_s^{1/2}(\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_a^{1/2}(23;1)\,\vartheta_{\rm sym}^{1/2}(23;1) + A_t^{1/2}(\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_{\rm sym}^{1/2}(23;1)\,\vartheta_a^{1/2}(23;1) + A_s^{1/2}(-\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_a^{1/2}(31;2)\,\vartheta_{\rm sym}^{1/2}(31;2) + A_t^{1/2}(-\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_{\rm sym}^{1/2}(31;2)\,\vartheta_a^{1/2}(31;2). \quad (30)$$

We now obtain expressions for the contributions of the first two diagrams to the nucleon-deuteron scattering amplitude in the approximation linear in r_0. If particles 1 and 2 interact in the final state then the contribution of the pole term in the indicated approximation, for singlet interaction of particles 1 and 2 in the final state, will be of the form

$$A_{0s}'^{1/2}(\mathbf{k},\mathbf{k}_0) = 2\sqrt{8\pi\alpha_t}\left[\frac{r_{0s}}{2} - \frac{1+\alpha_s r_{0s}}{\alpha_s + if}\right]\gamma^{-1}(\mathbf{k},\mathbf{k}_0), \quad (31)$$

and for triplet interaction of particles 1 and 2

$$A_{0t}'^{1/2}(\mathbf{k},\mathbf{k}_0) = 2\sqrt{8\pi\alpha_t}\left[\frac{r_{0t}}{2} - \frac{1+\alpha_t r_{0t}}{\alpha_t + if}\right]\gamma^{-1}(\mathbf{k},\mathbf{k}_0). \quad (32)$$

The contribution of the triangular diagram for the case of interaction of particles 1 and 2 in the singlet state is of the form

$$A_{Ts}'^{1/2}(\mathbf{k},\mathbf{k}_0) = \sqrt{8\pi\alpha_t}\left[\frac{r_{0s}}{2} - \frac{1+\alpha_s r_{0s}}{\alpha_s + if}\right]\frac{r_{0s}+r_{0t}}{2iq} \times \ln\frac{\alpha_t - i(f-q)}{\alpha_t - i(f+q)}, \quad (33)$$

and for the case of interaction of particles 1 and 2 in the triplet state

$$A_{Tt}'^{1/2}(\mathbf{k},\mathbf{k}_0) = \sqrt{8\pi\alpha_t}\left[\frac{r_{0t}}{2} - \frac{1+\alpha_t r_{0t}}{\alpha_t + if}\right]\frac{9r_{0s}+r_{0t}}{2iq} \times \ln\frac{\alpha_t - i(f-q)}{\alpha_t - i(f+q)}. \quad (34)$$

The total amplitude of the process, with account of the interaction of any pair of particles in the final state, must be written in a form analogous to (30), where $A_s^{1/2}$ and $A_t^{1/2}$ are replaced by $A_s'^{1/2}$ and $A_t'^{1/2}$.

The scattering amplitude in the real case can be obtained by multiplying the expression for the total scattering amplitude by the spin and isospin functions of the final state of the system under consideration. For example, in the case of the disintegration of a deuteron induced by a neutron, the spin and isospin functions of the final state for $S = 1/2$ and $T = 1/2$ can be

$$\chi_{\rm sym}^{1/2}(12;3)\,\alpha_1\alpha_2\beta_3 \quad \text{or} \quad \chi_a^{1/2}(12;3)\,\alpha_1\alpha_2\beta_3,$$

where $\alpha_1\alpha_2\beta_3$ — isospin state of neutron 1, neutron 2, and proton 3.

We now obtain the integral equations for the nucleon-deuteron scattering amplitude in the case $S = 3/2$ and $T = 1/2$. Assume that in the final state particles 1 and 2 have a triplet interaction and in the initial state the deuteron consists of particles 2, 3 or 3, 1 or 1, 2. Then the spin function of the final state is $\chi_{\rm sym}^{3/2}(12;3)$. The spin function of the initial state is $\chi_{\rm sym}^{3/2}(12;3)$ or $\chi_{\rm sym}^{3/2}(23;1)$ or $\chi_{\rm sym}^{3/2}(31;2)$, while the isospin function of the initial state is $\vartheta_a^{1/2}(12;3)$ or $\vartheta_{\rm sym}^{1/2}(23;1)$ or $\vartheta_a^{1/2}(31;2)$.

In the zeroth approximation in r_0, the contribution of the pole term to the scattering amplitude will be of the form

$$A_0^{3/2}(\mathbf{k},\mathbf{k}_0) = -{}^3/_4\,[\alpha_t + if]^{-1}\sqrt{8\pi\alpha_t}\,\gamma^{-1}(\mathbf{k},\mathbf{k}_0). \quad (35)$$

The contribution of the square diagram to the scattering amplitude will be determined by the integral

$$A_1^{3/2}(\mathbf{k},\mathbf{k}_0) = -\frac{3}{4}\sqrt{8\pi\alpha_t}\,\frac{1}{\alpha_t + if}\,4\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\,\frac{A_0^{3/2}(\mathbf{p},\mathbf{k}_0)}{\gamma(\mathbf{p},\mathbf{k})}. \quad (36)$$

For the integral term in the equation for the scattering amplitude we obtain an expression similar to (3.6), in which $A_0^{3/2}(\mathbf{p},\mathbf{k}_0)$ is replaced by $A^{3/2}(\mathbf{p},\mathbf{k}_0)$. If we introduce the notation

$$A^{3/2}(\mathbf{k},\mathbf{k}_0) = \sqrt{8\pi\alpha_t}\,(k^2 - k_0^2)^{-1}a^{3/2}(\mathbf{k},\mathbf{k}_0),$$

we obtain the following integral equation for $a^{3/2}(\mathbf{k},\mathbf{k}_0)$:

$$\frac{\alpha_t + if}{k^2 - k_0^2}\,a^{3/2}(\mathbf{k},\mathbf{k}_0) = \gamma^{-1}(\mathbf{k},\mathbf{k}_0) + 4\pi\int\frac{a^{3/2}(\mathbf{p},\mathbf{k}_0)}{\gamma(\mathbf{k},\mathbf{p})\,(p^2 - k_0^2)}\,\frac{d\mathbf{p}}{(2\pi)^3}. \quad (37)$$

The expression for the scattering amplitude with account of the interaction of each pair of particles will have in the final state the form

$$A^{3/2}(\mathbf{k},\mathbf{f};\mathbf{k}_0,\alpha) = A^{3/2}(\mathbf{k},\mathbf{k}_0)\,\chi_{\rm sym}^{3/2}(12;3)\,\vartheta_a^{1/2}(12;3) + A^{3/2}(\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_{\rm sym}^{3/2}(31;2)\,\vartheta_a^{1/2}(31;2) + A^{3/2}(-\mathbf{f}-{}^1/_2\mathbf{k};\mathbf{k}_0)\,\chi_{\rm sym}^{3/2}(23;1)\,\vartheta_a^{1/2}(23;1). \quad (38)$$

Using the same arguments as before, we can readily show that the contribution of the pole and triangular diagrams to the amplitude of the inelastic scattering of a nucleon by a nucleon is, for $S = 3/2$ and $T = 1/2$ in the approximation linear in r_0,

$$A'^{3/2}(\mathbf{k},\ \mathbf{k}_0) = -\frac{3}{2}\sqrt{8\pi\alpha_t}\left[\frac{r_{0t}}{2} - \frac{1+\alpha_t r_{0t}}{\alpha_t + if}\right]$$

$$\times\left[\frac{1}{\gamma(\mathbf{k},\ \mathbf{k}_0)} + \frac{r_{0t}}{2iq}\ln\frac{\alpha - i(f-q)}{\alpha - i(f+q)}\right], \qquad (39)$$

and the total nucleon-deuteron inelastic scattering amplitude in the approximation linear in r_0 will have a form similar to (38).

In order to obtain the scattering amplitude in a real case, it is necessary to project the expression for the total amplitude on the real final spin and isospin states of the system. Thus, in the case of the decay of a deuteron induced by neutrons, the spin and isospin states of the system with $S = 3/2$ and $T = 1/2$ are described by the function $\chi^{3/2}_{\mathrm{sym}}(12;\ 3)\,\alpha_1\alpha_2\beta_3$. It is easy to show that the expressions (30) and (38) obtained for the contributions of the pole and triangular diagrams in the approximation linear in r_0, with account of the spin and isospin, correspond fully to the expressions obtained within the framework of perturbation theory, with account of the interaction of real particles with spins in the final state.

6. CONCLUSION

The investigations carried out in the preceding sections show that in the zeroth approximation in the interaction radius r_0 (corresponding to the well-known Bethe-Peierls theory of the deuteron), the Nd interaction amplitude corresponds only to ladder-type diagrams, the summation of which is quite easy to carry out and leads to the STM equation. The use of the diagram summation procedure makes it possible to obtain also relatively simply equations for the Nd-interaction amplitude in the first approximation in r_0, as investigated by Danilov.

Comparison of the matrix elements of perturbation theory with account of particle interaction in the final state and the contributions of the series of diagrams in the approximation linear in r_0 shows that these matrix elements correspond to the contributions from the pole and triangular diagrams.

Inasmuch as a comparison of the experimentally obtained energy distributions of the neutrons from the reaction p + d = p + p + n with those calculated on the basis of the Born approximation with account of the particle interaction in the final state [1-3] shows that they agree well, we can assume that the greatest contribution to the amplitude of the process is made by the pole and triangular diagrams.

It must be noted that the diagram summation procedure considered yields integral equations for the scattering amplitude of more complicated systems consisting of four or five nucleons. The kernels of these equations include the three-nucleon interaction amplitudes, just as the kernal of the STM equations for three nucleons includes the two-nucleon interaction amplitude.

In conclusion, the authors express their sincere gratitude to K. A. Ter-Martirosyan for interest in the work and for useful discussions. The authors are also grateful to G. S. Danilov, who reviewed the work and made several remarks.

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Translated by J. G. Adashko
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