ANGULAR DISTRIBUTION OF COSMIC-RAY MUONS UNDER THICK ABSORBERS

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Submitted to JETP editor December 8, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 1618-1621 (May, 1963)

The angular distribution of cosmic-ray muons under thick layers of rock is calculated. The calculations are compared with the experimental data on the angular distribution at large depths underground. The results of the theory and experiment are in agreement. This seems to indicate that the cross sections for elementary interaction processes involving fast muons (with energy of 10^{12}—10^{13} eV) are satisfactorily described by present-day theory.

1. Cosmic-ray muons provide a unique possibility to study the interaction of these particles at high energies. The usual method of investigation (for more details see [1]) reduces to a comparison of the variation of muon intensity with the depth (so-called absorption curve) with the energy spectrum at sea level measured by means of a magnetic spectrometer. The absorption curve is thus determined by two factors: a) the spectrum at sea level, and b) the energy loss of muons in the rock, and such a comparison enables us to determine "experimentally" the energy loss, which can also be calculated theoretically (for electromagnetic interactions). Consequently, it is possible to check quantum electrodynamics at high energies.

In the present article we discuss a variant of this approach based on a measurement of the angular distribution of cosmic-ray muons under a thick absorber, which can be either rock or water. In the following, however, we shall discuss rock only.

The angular distribution at a given depth, recalculated taking the variation of the vertical intensity with depth into account, permits us to continue the absorption curve to depths unaccessible to direct measurements. The advantage of such an approach lies therefore in the possibility of studying the interaction of muons of higher energy than usual.

2. We shall deduce the general formula for the angular distribution at large depths.

Let $j(0, E, 0)$ be the differential spectrum of vertical muons at sea level. The muon intensity at an angle θ at sea level is then (see [2])

$$j(0, E, \theta) = j(0, E, 0)\, p(E, \theta). \tag{1}$$

The energy loss of muons in rock is given by

$$dE/dR = a + bE, \tag{2}$$

where a and b are constant, and R is the depth. We have then,

$$j(R_0, E, \theta) = j\left\{0, E + \frac{a}{b}\left[\exp\left(\frac{R_0 b}{\cos\theta}\right) - 1\right], 0\right\} p(E, \theta), \tag{3}$$

where R_0 is the depth measured vertically. Correspondingly, the integral muon flux is

$$I(R_0, \theta) = \int_0^\infty j\left\{0, E + \frac{a}{b}\left[\exp\left(\frac{R_0 b}{\cos\theta}\right) - 1\right], 0\right\} p(E, \theta)\, dE. \tag{4}$$

In general, $p(E, \theta)$ is a rather complicated function of its arguments. However, for sufficiently large energies ($> 10^{12}$ eV) it depends only on θ, and can be well approximated by the function

$$p(E, \theta) = \exp\left(2.3\frac{1-\cos\theta}{1+\cos\theta}\right). \tag{5}$$

We shall take the integral spectrum at sea level in the usual form

$$I(0, E, 0) = AE^{-\gamma}. \tag{6}$$

The angular distribution of muons at a depth R_0 is then given by the formula

$$I(R_0, \theta) = A\exp\left\{-\gamma\left[\ln\frac{a}{b} + \ln\left\{\exp\left(\frac{R_0 b}{\cos\theta}\right) - 1\right\}\right] + 2.3\frac{1-\cos\theta}{1+\cos\theta}\right\}. \tag{7}$$

3. Let us estimate the possible errors in $I(R_0, \theta)$ due to fluctuations in the energy loss of fast muons and their scattering in the rock. The influence of the fluctuations on the energy spectrum of high-energy muons was discussed earlier in [3] and by Zatsepin and Mikhal'chi.[4] The results obtained are similar, but in [4] they are given in a more convenient form.[1)] We shall give an approximate expression for the fluctuation correction to the integral spectrum which is usually measured in

[1)]See also the recent publication of Hayman et al.[5]

the experiment:

$$I_1(R_0, \theta) = I(R_0, \theta)\,\chi(R_0, \theta), \tag{8}$$

where $I_1(R_0, \theta)$ is the spectrum corrected for fluctuations,

$$\chi(R_0, \theta) = \exp\,[0.7\,b'(\gamma - 1.3)\,R_0/\cos\theta], \tag{9}$$

and b′ is the contribution to b due to the energy loss for bremsstrahlung and the production of nuclear showers. In the following we use $b' = 0.6\,b$.

Let us find whether it is possible to neglect the scattering in the rock. Coulomb scattering in the rock is given by the expression

$$\theta_C < \frac{1}{5}\frac{E_C}{E}\sqrt{\frac{R_0}{\cos\theta}}, \tag{10}$$

where $E_C = 21$ MeV, and R_0 is taken in g/cm². From Eqs. (3) and (6) it follows that the effective value of muon energy at the depth R_0 and angle θ is

$$E_e = \frac{a}{b}\left[\exp\left(\frac{R_0 b}{\cos\theta}\right) - 1\right]. \tag{11}$$

Coulomb scattering can be neglected if

$$\frac{E_C b}{5a\,[\exp\,(R_0 b/\cos\theta) - 1]}\sqrt{\frac{R_0}{\cos\theta}} \ll \theta \tag{12}$$

or

$$\frac{E_C}{5a}\sqrt{\frac{\cos\theta}{R_0}} \ll \theta. \tag{13}$$

Since $a \sim 2$ MeV/g-cm^{-2}, then for depths of interest $R_0 \geq 1.5 \times 10^5$ g/cm² and we can neglect Coulomb scattering if

$$\theta \gg 0.005. \tag{14}$$

The deflection of muons due to other processes (bremsstrahlung, pair production, and the production of electron-nuclear showers) is much smaller than for Coulomb scattering.

The main contribution is that of the deflection θ_r due to bremsstrahlung. An estimate gives the following result:

$$\frac{\theta_r}{\theta_C} \sim 5\cdot 10^{-2}\sqrt{\ln\frac{E}{\mu}}, \tag{15}$$

where μ is the muon mass.

4. In order to estimate the magnitude of b we have used the data on the angular distribution of muons obtained by Bollinger [6] ($R_0 = 1.5 \times 10^5$ g-cm^{-2}, muon energy interval $\sim 5 \times 10^{11}$–10^{13} eV). Unfortunately, there are no experimental data on the spectrum at sea level for energies greater than 10^{12} eV. According to most recent measurements,[7] in the energy range 3×10^{11}–10^{12} eV the spectrum is described by an exponential function with the exponent $\gamma \approx 2.6$. If we assume that γ is constant also at higher energies then the data of Bollinger are best described for $b = 4 \times 10^{-6}$ g^{-1} cm² (Fig. 1). However, the analysis of data [7] encompassing a wide energy range indicates that the exponent γ tends to increase slightly with increasing energy. The increase in γ leads to a decrease in b for a given absorption curve. Therefore the value $b = 4 \times 10^{-6}$ g^{-1} cm² (Fig. 1) obtained from our analysis gives the upper limit of b. To find the lower limit we used the value $\gamma = 3.3$. In that case, the best agreement is obtained for $b = 3 \times 10^{-6}$ g^{-1} cm².

A theoretical estimate of b, which depends on three processes (bremsstrahlung, direct pair production, and nuclear shower production) gives $(3.8$–$4) \times 10^{-6}$ g^{-1} cm². Within the limits of experimental errors we find a good agreement which, nevertheless, cannot be considered as a final justification of quantum electrodynamics at very high energies for two reasons: a) the accuracy of the calculations of the direct pair production and nuclear cascade production is unknown, and b) the final result depends on γ.

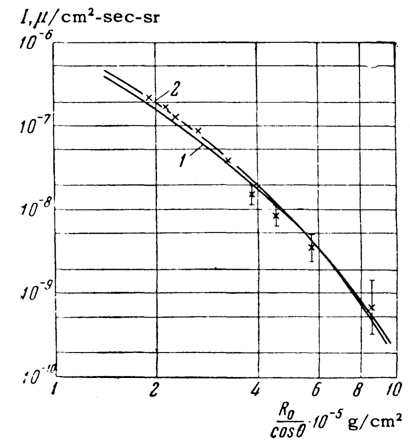

FIG. 1. Absorption curves according to Eq. (5). Experimental data from [6]. Curve $1-\gamma = 2.6$, $b = 4 \times 10^{-6}$ g^{-1} cm²; curve $2-\gamma = 3.3$, $b = 3 \times 10^{-6}$ g^{-1} cm².

5. Using Eqs. (7)–(9) we can write the angular distribution of muons, normalized to unity for vertical incidence, in the form

$$I_1(R_0, \theta) = \exp\left\{-\gamma\ln\left[\frac{\exp\,(R_0 b\sec\theta) - 1}{\exp\,(R_0 b) - 1}\right] + 2.3\frac{\sec\theta - 1}{\sec\theta + 1} + 0.42\,R_0 b\,(\gamma - 1.3)\,(\sec\theta - 1)\right\}. \tag{16}$$

For sufficiently large depths ($R_0 \geq 1.5 \times 10^5$ g/cm²) the function (16) can be approximated by

$$I_1(R_0, \theta) \approx (\cos\theta)^{2+5\cdot 10^{-6} R_0}. \tag{17}$$

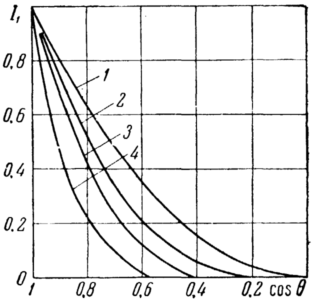

FIG. 2. Angular distribution of muons at different depths normalized to unity for the vertical direction. The minimum muon energy equals zero. Curve 1 – at sea level; curve 2 – at the depth of 1.5×10^5 g/cm²; curve 3 – at the depth of 4.5×10^5 g/cm²; curve 4 – at the depth of 8.25×10^5 g/cm².

The angular distributions of muons at different depths are shown in Fig. 2.

The discrepancies in the absorption curves obtained by different authors decrease if the angular distribution is taken into account. Thus, the values of the intensity given by Barton[8] for 3.28 and 5.05×10^5 g/cm should be increased by a factor of 1.5 and 1.8 respectively. This change improves the agreement with other data.[9]

[1] A. I. Nikishov and I. L. Rozental', Nekotorye voprosy fiziki élementarnykh chastits i atomnogo yadra (Certain Problems of Elementary-Particle and Nuclear Physics), Atomizdat, 1962, p. 48.

[2] G. T. Zatsepin and V. A. Kuz'min, JETP **39**, 1677 (1960), Soviet Phys. JETP **12**, 1171 (1961).

[3] I. L. Rozental' and V. N. Strel'tsov, JETP **35**, 1440 (1958), Soviet Phys. JETP **8**, 1007 (1959).

[4] G. T. Zatsepin and E. D. Mikhal'chi, J. Phys. Soc. Japan, Suppl. **17**, AIII, 356 (1962).

[5] Hayman, Palmer, and Wolfendale, Proc. Phys. Soc. (London) **80**, 800 (1962).

[6] Pine, Davison, and Greisen, Proc. Cosmic-Ray Conf. IUPAP, Moscow **1**, 293 (1960).

[7] P. I. Hayman and A. W. Wolfendale, Proc. Phys. Soc. (London) **80**, 710 (1962).

[8] J. C. Barton, Phil. Mag. **6**, 1271 (1961).

[9] Miyake, Narasimhan, and Ramanamurthy, J. Phys. Soc. Japan, Suppl **17**, AIII, 318 (1962).

Translated by H. Kasha
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