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ON THE LIMITS WHICH DETERMINE THE POSSIBILITY OF MEASURING GRAVITATIONAL EFFECTS

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The conditions which must be satisfied in measuring gravitational effects on the basis of motion of test masses are formulated. The maximum accuracy of measurement of gravitational effects which can be obtained in principle is determined by the very low fluctuation level of the gravitational field. The possibility of diminishing non-gravitational fluctuations due to mechanical forces acting on the test masses is considered and the level of gravitational radiation which can be detected is estimated.

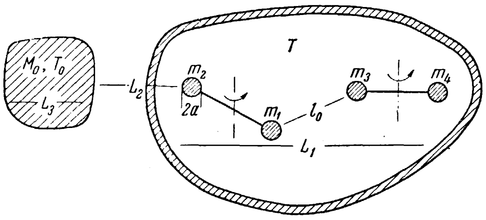
In recent years, a rather large number of gravitational experiments have been discussed in the literature; some of these have already been set up.[1-6] In the discussion of one or another of these experiments, estimates have been given of the possibility of the observation (in principle) of an effect or its achievable accuracy, usually starting out from the specific technical aspects of the experiment. In a whole series of such experiments, the detection of the gravitational effect reduces to the detection of a definite motion of certain test bodies.[1,3,5] The components of the intensity of the gravitational field or the components of the gradient of the field intensity (the components of the curvature tensor) can be determined from the motion of test masses. Limiting ourselves to the experimental arrangements of this type, we shall determine the fundamental requirements for the concrete technical realization of the experiment and the limiting possibilities of measurement of the small forces acting on the test masses by starting out from the already-achieved level of experimental technique.

In radio engineering, the fundamental requirement for apparatus designed for the detection of weak regular signals (for example, antennas and amplifiers) is that the receiving apparatus possess a noise power that is significantly less than the power P_{em} of the fluctuating electromagnetic radiation incident upon it. If this requirement is met, then the level of the small signals detected will be determined only by the value of P_{em}. The quantity P_{em} is usually characterized by the equivalent noise temperature: $P_{em} = kT_{eq}\Delta f$ (k is the Boltzmann constant, Δf is the band of frequencies within which the information is transmitted). The quantity T_{eq}, together with P_{em}, depends very strongly on the range of frequencies and the direction in which the energy is transmitted. For example, in certain parts of the centimeter wavelength range, T_{eq} of the zenith does not exceed 0.5°K.[7] We note that a receiving apparatus with small noise factor satisfying the above condition is almost entirely isolated from the thermal mechanisms of fluctuations in the laboratory which are equivalent to a noise temperature of about 300°K.

Similar requirements must be set forth for the "gravitational detection apparatus," which records the motion of test masses. In such a detecting system, interferences which occur in the motion of test masses by other (non-gravitational) forms of interaction must be significantly smaller than the interferences associated with fluctuations in the intensity of the gravitational field (or of the components of the curvature tensor) near the test masses. The fundamental technical requirement for apparatus recording the motion of test masses in this case is that the action of mechanical thermal fluctuations and fluctuations of the electromagnetic field on the test masses must be reduced as much as possible.[1)] We estimate the possibilities for the reduction of mechanical thermal fluctuations and fluctuations of the electromagnetic field under conditions which can be obtained with use of experimental technology currently available.

We consider a case (see the drawing) in which the test masses m_1 to m_4 are located inside a good conducting screen which strongly attenuates the external fluctuating electromagnetic field. Let

[1)]Other forms of noise for macroscopic objects under terrestrial conditions are much weaker.



the masses m_1 to m_4 be mechanically isolated from the walls of the screen. Under terrestrial laboratory conditions, this can be accomplished with the help of a so-called magnetic suspension, [8,9] with which one can suspend masses up to 25 kg in a vacuum without mechanical contact. If the temperature of the screen is lowered and a high vacuum is created inside of it, then the mechanical effect on the test masses of the fluctuations F_{mech} of the pressure of the residual gas and on the fluctuations F_{em} of the electromagnetic field will be weakened. There remains, in addition, the mechanical effect of fluctuations on the suspension.

Let us estimate the quantities $(\overline{F^2_{mech}})^{1/2}$ and $(\overline{F^2_{em}})^{1/2}$. For a highly rarefied gas, in the case of a spherical shape of the test masses,

$$(\overline{F^2_{mech}})^{1/2} = \left[\frac{64}{3\sqrt{2}}\pi^{1/2} a^2 \mu^{1/2} (kT)^{3/2} n \Delta f\right]^{1/2}. \quad (1)$$

Here a is the radius of the sphere, μ is the mass of the molecules of the gas, n is the concentration per unit volume, and Δf is the frequency bandwidth.

Setting $T = 3.2°K$, $\mu = 3.3 \times 10^{-24}$ g (molecule of H_2), and $n = 3 \times 10^4\ cm^{-3}$ (which corresponds to a pressure of 10^{-10} mm Hg [10]), we get for $\Delta f = 1$ cps the value $(\overline{F^2_{mech}})^{1/2} \approx 2 \times 10^{-14}$ dyne. If it is possible to increase the observation time of the effect to $\sim 10^5$ sec ($\Delta f = 10^{-5}$ cps) in a way similar to what was done in [5], then $(\overline{F^2_{mech}})^{1/2} \approx 6 \times 10^{-17}$ dyne. We note that the limits of values of T and n attainable under exceptional laboratory conditions were not used in obtaining the numerical estimate of $(\overline{F^2_{mech}})^{1/2}$ (see, for example, [10,11]).

The value of $(\overline{F^2_{em}})^{1/2}$ is much less than $(\overline{F^2_{mech}})^{1/2}$. The mechanical action of the fluctuations of the electromagnetic field on the test masses can be approximately reduced to two forces: $(\overline{F'_{em}})^2 + (\overline{F''_{em}})^2 = \overline{F^2_{em}}$. The force F'_{em} is the quasi-electrostatic action on the test mass brought about by the field of one or several low-frequency characteristic vibrations of the screen cavity, for which the dimensions of the test mass are comparable with the wavelength. This force must be taken into consideration in the case in which the band Δf of the receiver detecting the motion of the test mass contains one of the characteristic frequencies of the cavity formed by the screen. If the dielectric constant of the material of the masses $\epsilon \gg 1$, then the value of $[(\overline{F'_{em}})^2]^{1/2}$ can be estimated as

$$[(\overline{F'_{em}})^2]^{1/2} \approx a^2 kT/L_1^3, \quad (2)$$

here L_1 is a characteristic dimension of the cavity. For $a = 5$ cm, $T = 3°K$, and $L_1 = 10^2$ cm, we have $[(\overline{F'_{em}})^2]^{1/2} \approx 1 \times 10^{-20}$ dyne.

The force F''_{em} is brought about by the fluctuations of the pressure exerted on the test mass by the fluctuating thermal radiation of the walls of the screen (more precisely, by that part of the thermal radiation for which the wavelength is much less than a). In connection with the high level of the quantum fluctuations, [12] the total pressure of electromagnetic radiation should be taken as a principal estimate for F''_{em}:

$$[(\overline{F''_{em}})^2]^{1/2} \lesssim 2\pi\sigma T^4 a^2 c^{-1}; \quad (3)$$

here σ is the constant in the Stefan-Boltzmann law, c is the velocity of light, and it is assumed that $\epsilon \gg 1$, just as for F'_{em}. For $T = 3°K$ and $a = 5$ cm, we have $[(\overline{F''_{em}})^2]^{1/2} \lesssim 10^{-24}$ dyne.

One can get rid almost completely of the force F'_{em} if the gravitational effect has a spectrum of frequencies that is far from the very low characteristic frequencies of the screen cavity.

All the discussions above can be set forth on the basis of the general theory of Nyquist, which connects the fluctuation forces with the coefficient of friction H: $\overline{F^2} = 4kTH\Delta f$. Therefore, it is also important to estimate the friction (and consequently the mechanical fluctuating force) which the magnetic suspension introduces into the motion of the test mass. At the present time magnetic suspensions have been constructed which make it possible to maintain only "free" torsional vibrations (masses m_1, m_2 and m_3, m_4). A simple calculation on the basis of the data of [9] shows that, at least at a pressure of $p = 10^{-6}$ mm Hg and at $T = 300°K$, the magnetic suspension does not bring any appreciable damping into the torsional motion in comparison with the friction brought about by the residual pressure of the gas. Consequently, at least in these cases, the mechanical fluctuating action of the suspension is smaller than that from the residual pressure of the gas.

In all the calculations set forth, the fluctuating action of the external (relative to the screen) electromagnetic field and seismic oscillations have deliberately not been taken into account. These forms of interference can be greatly reduced by the use of a good conducting screen and special antiseismic suspensions. But it is a more significant question as to whether complete preliminary information (amplitude and phase of the spectrum) can be obtained from these two forms of interferences in the neighborhood of the test masses which, at least in principle, would make it possible to subtract the components associated with these interferences from the motion of the test masses.

We now estimate the level of the interferences associated with fluctuations of the gravitational field near the test masses. Let us find the fluctuating mechanical force F_{gr} brought about by fluctuations of the gravitational field acting on the test mass m_2. Let F_{gr} be produced by thermal vibrations of the body M_0 located outside the screen. If the bandwidth Δf contains one of the low natural frequencies f_0 of the vibrations M_0, then the expression for $(\overline{F_{gr}^2})^{1/2}$ for $L_2 \approx L_3$ has the form

$$(\overline{F_{gr}^2})^{1/2} \approx [2\gamma^2\rho^2a^6kT_0M_0/\pi^2L^6f_0^2]^{1/2}. \quad (4)$$

Here ρ is the density of the mass m_2, γ is the gravitational constant. For $L_{2,3} \approx 2 \times 10^2$ cm, $f_0 \approx 10^3$ cps, $M_0 = 10^8$ g, $T_0 = 300°$K, $\rho = 10$ g/cm, we get $(\overline{F_{gr}^2})^{1/2} \approx 3 \times 10^{-18}$ dyne. As is seen from a comparison of the estimates for $(\overline{F_{gr}^2})^{1/2} \approx 3 \times 10^{18}$ dyne and of the very large value from the interferences $(\overline{F_{mech}^2})^{1/2} \approx 2 \times 10^{-14}$ dyne, even under the conditions described above, the level of the interferences brought about by fluctuations of the gravitational field, under ordinary laboratory conditions, is much lower than the level of non-gravitational interferences. However, measurement of the gravitational effects even at the level of interferences of the order of $(\overline{F_{mech}^2})^{1/2}$ can give interesting results.

As an example, we estimate the value of the gravitational radiation power which can be recorded by starting out from the estimates obtained for $(\overline{F_{mech}^2})^{1/2}$. If a gravitational radiation power flux t with frequency ω_0 is incident on the system of test masses, then the difference of forces acting on the test masses (for example, m_1 and m_3) is equal to $F_t \approx mc^2 l_0 R_{i0j0}$,[1] where R_{i0j0} is the variable component of the Riemann tensor associated with the radiation. By using the well-known relations connecting R_{i0j0} with the components of the metric tensor $h_{\mu\nu}$ and the power flux t (see, for example, [13]), it is not difficult to obtain the following expression for t:

$$t \approx c^3F_t^2/8\pi\gamma l_0^2m^2\omega_0^2. \quad (5)$$

Setting $F_t^2 = \overline{F_{mech}^2} = 4 \times 10^{-28}$ dyne^2, $l_0 = 10^2$ cm, $\omega_0 = 2\pi \times 10^2$ rad/sec, $m = m_1 = m_3 = 1 \times 10^4$ g, we get $t \approx 2 \times 10^{-8}$ erg/sec-cm^2, and for an observation time $\sim 10^5$ sec, the value $(\overline{F_{mech}^2})^{1/2} \approx 6 \times 10^{-17}$ dyne, $l_0 = 10^3$ cm, $t \approx 2 \times 10^{-15}$ erg/sec-cm^2.

These elements are already sufficiently close to the level of the power of the radiator suggested by Weber[1] ($P_{gr} \approx 10^{-13}$ erg/sec). Thus, at least in principle, the circuit arrangement "receiver-transmitter of gravitational radiation" is possible under laboratory conditions.

We note that the estimate obtained by Weber[2)] (with account of the arithmetic error noted by Gertsenshteĭn and Pustovoĭt[4]) for the upper limit of detected gravitational radiation $t \approx 10^{-1}$—10^{-2} erg/sec-cm^2, was obtained for a specific system of test masses which are interconnected by a piezo-converter. The very appreciable mechanical friction in such a system is a source of fluctuations of non-gravitational origin, and determines such a high level of minimum observable power. The friction in such a system is much greater than in the systems described above.

In conclusion, we shall show that the basic technical difficulty, after decreasing all forms of "non-gravitational" noises, reduces to the construction of a highly sensitive apparatus measuring small periodic changes in the distance l_0 between the test masses. Thus for $(\overline{F_{mech}^2})^{1/2} = 2 \times 10^{-14}$ dyne and $\omega_0 = 2\pi \times 10^2$ rad/sec, we have $\Delta l_0 = 2 \times 10^{-23}$ cm.

Making use of a high sensitivity capacitative displacement pickup developed by us[5] allows one to measure periodic mechanical motions $\sim 10^{-11}$ cm. Its sensitivity was determined by the rather high level of amplitude fluctuations in the supply circuit. The methods developed for reducing these fluctuations[14] apparently make it possible to lower the level of the resolved displacements to $\sim 10^{-14}$—10^{-15} cm. The application of optical methods for solution of this problem is also possible.[4] The amplification of the quantity Δl_0 can be achieved by lowering the frequency f_0, and also, if one employs the well-known techniques of parametric regeneration.

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