EFFECT OF JOULE HEAT ON THE QUENCHING OF SUPERCONDUCTIVITY BY A CURRENT

S. Ya. BERKOVICH and G. M. LAPIR

Institute of Precision Mechanics and Computer Technology, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 13, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 1329—1332 (April, 1963)

The alteration in the form of the temperature and current dependences of the resistance of superconducting cylindrical samples resulting from internal Joule heating is considered. From the results thus obtained it is possible to explain the departure of the experimental data from the well-known London formula (1) [1].

THE current and temperature dependence of the resistance R of a cylindrical superconducting specimen was treated theoretically by London as long ago as 1937 [1]. The relation

$$R = 0 \quad \text{for } I < I_c,$$
$$R = \tfrac{1}{2} R_n [1 + \sqrt{1 - (I_c/I)^2}] \quad \text{for } I > I_c, \qquad (1)$$

was derived, where I is the current in the sample, I_c is the critical current at the given temperature, and R is the resistance of the sample in the normal state.

The derivation of this relation is based upon the assumption that for $I > I_c$ an interior portion of the sample, whose radius tends to zero as the current increases, is in the intermediate state. The structure of the intermediate state must consist of alternating normal and superconducting layers perpendicular to the axis of the cylinder; the thickness of the normal layers increases linearly with distance from the axis, but they do not completely fill the length of the cylinder at the boundary of the intermediate region. Experimental investigations [2,3] have not contradicted this model of the intermediate state. As regards the relations (1), measurements (cf. [3-8]), despite qualitative agreement, have revealed systematic departures from the curve (1): the jump in resistance at $I = I_c$ is found to exceed 0.5 R_n; R reaches R_n at finite values of the current, and the dependence of R upon I shows hysteresis.

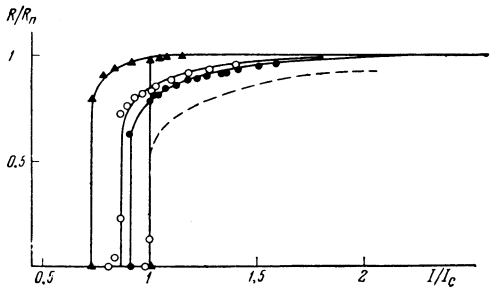
Various hypotheses have been advanced in order to explain these discrepancies: distortion of the intermediate state structure due to surface tension at the interfaces between the normal and superconducting layers [6], or a change in the resistivity of the sample due to scattering of electrons at these boundaries [9]; a dynamical model of the intermediate state differing from that just described has also been suggested [10]. The majority of these authors have failed to associate these discrepancies with the effects of Joule heating; that this is, however, their source has recently been demonstrated by Troĭnar [3]. A systematic accounting for this effect makes it possible to explain to a considerable degree the divergence of theory and experiment.

If the resistance per unit length of the sample is known as a function of temperature and current, $R = F(T, I)$, then for a given current and helium bath temperature T_b the measured value of the resistance R will be determined by the equation

$$R = F(T_b + I^2 R/2\pi a h,\ I), \qquad (2)$$

where h is the coefficient for heat transfer through the sample-liquid helium boundary, and a is the cross-sectional radius of the sample. We consider the temperature within the specimen to be constant, since the thermal resistance of a sample of the usual dimensions is much less than $(1/2)\pi ah$; we neglect the heat transfer through the ends of the specimen.

Solving Eq. (2), we transform the functional dependence of R upon (T_b, I) into the experimentally observable dependence of R upon (T_b, I). The



Dependence of resistance upon current for various specimens of tin: ▲ – No. 12[3], T_b = 2.87° K; ○ – No. 12[3], T_b = 1.85° K; ● – No. 6[7], T_b = 3.27° K. The solid curves are constructed according to Eq. (3); the dashed curves represent London's Eq. (1).[1]

solution to Eq. (2) is determined from the aggregate of points of intersection between the surface $R = F(T, I)$ and the family of surfaces $R = 2\pi ahI^{-2}(T - T_b)$; choosing either I or T_b, we obtain the observable transition curves with respect to either current or temperature. By differentiating (2), we find that if at any given intersection point $\partial R/\partial T > 2\pi ah/I^2$, then $dR/dI < 0$ and $dR/dT_b < 0$ and, in consequence, hysteresis will be observed along the corresponding transition curves.

In the present case, proceeding from (1), we arrive at the relations

$$R = 0 \quad \text{for} \quad 0 < I < I_c,$$
$$R = \frac{R_n}{2 + (I/I_c)^2\delta^2/2} \times \left[1 + \frac{\delta}{2} + \sqrt{1 + \delta - (I_c/I)^2}\right] \text{for} \frac{I_c}{\sqrt{1+\delta}} < I < \frac{I}{\sqrt{\delta}},$$
$$R = R_n \quad \text{for} \quad I > I_c/\sqrt{\delta}. \tag{3}$$

Here

$$\delta = H_c\,|\,dH_c/dT\,|\,\rho/8\pi^2 ha,$$

ρ is the resistivity of the normal phase, and H_c is the critical field ($H_c = 2I_c/a$).

In the derivation of Eqs. (3) it was assumed that the change in the magnitude of the critical field ΔH_c and the temperature rise of the specimen relative to the helium bath ΔT are linearly related: $\Delta H_c = (dH_c/dT)\Delta T$, and that h is independent of ΔT (the latter condition will prevail for sufficiently large thermal loadings both in He II [11] and in He I [8]). The quantity δ has a simple physical significance: $\delta \sim \Delta T/(T_c - T_b)$ (T_c is the critical temperature of the specimen).

We see that in the region $[I_c/\sqrt{1+\delta}, I_c]$ the dependence is not single-valued; i.e., hysteresis arises. The upward jump in the resistance takes place at $I = I_c$, the return at $I = I_c/\sqrt{1+\delta}$. The full resistance is present for a current $I_n = I_c/\sqrt{\delta}$. If, for example, the initial jump in resistance is taken to be $0.8\,R_n$, then $I_n = 2\,I_c$, in full agreement with the facts as established long since by Shubnikov and Alekseevskiĭ [4].

The figure shows R(I) curves plotted for various samples according to Eq. (3). The parameter h as well as δ, which depends upon the experimental conditions and the surface state of the sample, has been selected appropriately for each case. For tin with a relatively high residual resistance, greater than $\rho \sim 10^{-8}$ Ωcm, the values of h obtained in this way agree for various samples, and amount, in He I, to 0.07 – 0.13 W cm^{-2} deg^{-1}, and in He II, to 0.6 – 0.8 W cm^{-2} deg^{-1} (cf. the table).

For He II ($T_b \ll 2.19°$ K) these results agree well with the data from direct measurements [11]. In the case of He I, where the heat evolved depends strongly upon the experimental circumstances, the data obtained lie within the range of established values for h [8,12]. It may therefore be suggested that for such samples, the departures from curve (1) are almost entirely due to the effects of Joule heating.

Sample	T_b, °K	δ	h, W cm^{-2} deg^{-1}
	3.47	0.6	0.09
№ 12 [3]	2.87	0.9	0.13
	1.85	0.32	0.67
№ 5 [7]	3.48	0.14	0.09
№ 6 [7]	3.27	0.20	0.07
[5]	1.95	0.25	~0.8

For tin samples of higher purity ($\rho \sim 10^{-9}$ Ωcm; for example, Nos. 1–5 [3] and 7 [7]), however, the resistance values calculated from Eq. (3) do not agree as closely with the experimental data as for specimens with larger residual resistances. This means that the isothermal transition curve for "pure" specimens departs somewhat from the transition curve defined by London's equation (1). This departure evidently occurs when the mean free path of the electrons in the normal state becomes greater than the periodicity of the intermediate state structure, which can lead (see [9]) to an additional increase in the resistance due to the scattering of electrons at the domain boundaries.

[1] F. London, Une conception nouvelle de la supraconductibilité, Paris, 1937; D. Shoenberg, Superconductivity, Cambridge, 1952.

[2] B. V. Makeĭ, JETP **34**, 312 (1958), Soviet Phys. JETP **7**, 217 (1958).

[3] E. Troĭnar, JETP **38**, 654 (1960), Soviet Phys. JETP **11**, 473 (1960); Dissertation, Moscow State University, 1961.

[4] L. V. Shubnikov and N. E. Alekseevskiĭ, Nature **138**, 804 (1936).

[5] N. E. Alekseevskiĭ, JETP **8**, 342 (1938).

[6] R. B. Scott, J. Res. N. B. S. **41**, 581 (1948).

[7] L. Rinderer, Helv. Phys. Acta **29**, 339 (1956).

[8] H. Meissner, Phys. Rev. **109**, 668 (1958).

[9] C. G. Kuper, Phil. Mag. **43**, 1264 (1952).

[10] C. J. Gorter, Physica **23**, 45 (1957).

[11] Kuang Wey-yen, JETP **42**, 921 (1962); Soviet Phys. JETP **15**, 635 (1962).

[12] T. Frederking and P. Grassmann, Bull. Inst. Intern. Froid, 1958 Annexe **1**, p. 317.

Translated by S. D. Elliott
213