COULOMB COLLISIONS OF FAST ELECTRONS IN A POLARIZING MEDIUM

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Submitted to JETP editor May 24, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 899—902 (March, 1963)

Formulas for the cross section of inelastic scattering of electrons in a polarizing medium are derived in first-order quantum perturbation theory. The formulas can be employed for analyzing the characteristic energy losses of electrons in thin films.

STARTING with the classical work of Tamm, Frank, and Fermi the study of the influence of the polarization of a medium on electromagnetic processes accompanying the passage of fast particles through matter has attracted increasing attention. The Cerenkov radiation has been explained in detail, the slowing down of fast particles in condensed media with allowance for polarization effects has been studied, and various other electromagnetic phenomena associated with the polarization of the medium (for example, transition radiation) have been investigated. Recently, polarization phenomena have also been included in the description of Coulomb collisions of particles in plasma-like media. Thus Rukhadze and Silin, [1] in the calculation of the collision integral, have given an expression for the collision matrix element in which the polarization of the medium is taken into account and obtained the following expression for the scattering probability:

$$W_{01} = (2\pi/\hbar)\,|\,4\pi e_1 e_2 / k_{01}^i \varepsilon_{ij}^{01}(\omega_{01}) k_{01}^j\,|^2. \quad (1)$$

Here e_1 and e_2 are the charges of the colliding particles; $\mathbf{k}_{01} = \mathbf{k}_0 - \mathbf{k}_1$; $\omega_{01} = \omega_0 - \omega_0$; $\hbar\mathbf{k}_0$, $\hbar\omega_0$, and $\hbar\omega_1$ are the momenta and energy of the struck particle before and after the collision; and ϵ_{ij} is the dielectric tensor of the medium. In this paper, in particular, we shall derive the relativistic generalization of the collision integral with allowance for the polarization.

Theoretical studies of inelastic collisions in a medium usually employ the methods of classical physics. It turns out (see [2,3]) that the probability for an energy transfer $\hbar\omega$ to the medium is proportional to the quantity

$$\mathrm{Im}\,\varepsilon\cdot|\,\varepsilon\,|^{-2} \quad (2)$$

and bears a resonance character; there is peak at $\epsilon = 0$.

In quantum mechanics, judging from (1), we should obtain a similar result, but, as is shown below, the proportionality constant differs from the classical expression in a number of details. This has important meaning to the theory of characteristic energy losses by fast electrons based on the expression (2).

The probability of a quantum transition in first-order perturbation theory, as is known, has the form

$$d\dot{w} = \frac{2\pi}{\hbar^2}\,|\,W_{mn}^{01}(\omega_{01})\,|^2\,\delta(\omega_{01} - \omega_{mn})\,d\mathbf{k}_1. \quad (3)$$

Here W_{mn}^{01} is the transition matrix element; $\omega_{mn} = \omega_m - \omega_n$; the remaining notation is the same as in (1). We will need another modification of formula (3). For this, we note that in the derivation of this formula the attenuation of the atomic wave functions was neglected. Allowance for the attenuation, as is known, leads to the following expression for the transition probability:

$$d\dot{w} = \frac{2}{\hbar^2}\,|\,W_{mn}^{01}(\omega_{01})\,|^2\,\frac{\gamma_{mn}\,d\mathbf{k}_1}{(\omega_{01} - \omega_{mn})^2 + \gamma_{mn}^2}. \quad (4)$$

Here γ_{mn} is the half-width of the transition line $m \to n$. In the limit $\gamma_{mn} \to 0$, expression (4) goes over into (3).

In the nonrelativistic approximation of the perturbation energy the quantity $e\varphi$ appears, where φ is the Coulomb interaction potential. The matrix element of this quantity will be calculated first from the wave functions of the incident particle and then from the wave functions of the struck particle. The wave functions of the incident particle will be taken in the form of plane waves:

$$\psi = (2\pi)^{3/2} e^{i\,(\mathbf{kr} - \omega t)}. \quad (5)$$

The medium was taken into account by means of its dielectric constant ϵ. It is readily noted that the matrix element constructed from the incident-particle wave functions can be determined for the Coulomb field potential in the dielectric medium by the equation

$$\triangle^2 \varphi_{01} = -\frac{4\pi e_1}{\varepsilon} \psi_0^* \psi_1, \quad (6)$$

where the indices 0 and 1 denote the initial and final states of the incident particle. Solution of this equation, with allowance for (5), has the form

$$\varphi_{01} = \frac{e_1 \exp\{i(\mathbf{k}_{01}\mathbf{r} - \omega_{01}t)\}}{(2\pi)^2 \varepsilon(\omega_{01}) k_{01}^2} \quad (7)$$

(e_1 is the charge of the incident particle). By means of this expression we obtain the following expression constructed from the wave functions of the colliding particles:

$$W_{mn}^{01} = ee_1 f_{mn}(\mathbf{k}_{01}) / 2\pi^2 \varepsilon(\omega_{01}) k_{01}^2,$$

$$f_{mn}(\mathbf{k}_{01}) = \int e^{-i\mathbf{k}_{01}\mathbf{r}} \psi_m^* \psi_n \, d\mathbf{r}, \quad (8)$$

where ψ_m and ψ_n are the wave functions of the struck particle before and after the collision, respectively.

The probability of the transition is calculated by the substitution of (8) into (4). Noting that the cross section for the collision is expressed in our case through $d\dot{w}$ by means of the formula

$$d\sigma = (2\pi)^3 d\dot{w}/v,$$

we obtain

$$d\sigma = \frac{4e^2e_1^2 |f_{mn}(\mathbf{k}_{01})|^2}{\hbar^2 v |\varepsilon(\omega_{01})|^2 k_{01}^4} \frac{\gamma_{mn} \, d\mathbf{k}_1}{\pi [(\omega_{01} - \omega_{mn})^2 + \gamma_{mn}^2]}. \quad (9)$$

This expression can serve for the analysis of inelastic collisions in the medium. However, the quantities γ appearing in (9) are small, and hence we can bring (9) to a simpler form if we set all γ_{mn} equal to zero, including those which enter into ϵ. Noting that

$$\lim_{\gamma \to 0} (\mathrm{Im}\,\varepsilon) = 0, \qquad \lim_{\mathrm{Im}\,\varepsilon \to 0} \frac{\mathrm{Im}\,\varepsilon}{|\varepsilon|^2} = \pi\delta(\varepsilon),$$

we find that

$$d\sigma = \frac{4e^2e_1^2}{\hbar^2 v} \frac{|f_{mn}(\mathbf{k}_{01})|^2}{k_{01}^4} F(\omega_{01}) \delta[\varepsilon(\omega_{01})] \, d\mathbf{k}_1, \quad (10)$$

where

$$F(\omega_{01}) = \lim_{\gamma_{mn} \to 0} \frac{\gamma_{mn}}{[(\omega_{01} - \omega_{mn})^2 + \gamma_{mn}^2] \, \mathrm{Im}\,\varepsilon(\omega_{01})}. \quad (11)$$

This function has no poles for $\omega = \omega_{mn}$ and is nonzero for all ω for which $\epsilon = 0$. Hence, according to (10), the loss occurs in finite quantities $h\Omega_{mn}$, where Ω_{mn} are the roots of the equation $\epsilon(\Omega) = 0$.

Close to the resonance peak $\omega = \Omega_{mn}$ we can, neglecting spatial dispersion, approximate $\epsilon(\omega)$ by the function

$$\varepsilon(\omega_{01}) = (\omega_{01}^2 - \Omega_{mn}^2) / (\omega_{01}^2 - \omega_{mn}^2),$$

$$\Omega_{mn}^2 = \omega_{mn}^2 + \omega_0^2 f_{mn}, \qquad \omega_0^2 = 4\pi n e^2/m, \quad (12)$$

where n is the number of electrons in a unit volume, f_{mn} is the oscillator strength. Substituting this into (10) and taking into account the properties of the δ function, we find

$$d\sigma = \frac{e^2e_1^2}{\hbar^2 v} \frac{|f_{mn}(\mathbf{k}_{01})|^2}{k_{01}^4} \left(\frac{\Omega_{mn} + \omega_{mn}}{\Omega_{mn}}\right)^2 \delta(\omega_{01} - \Omega_{mn}) \, d\mathbf{k}_1. \quad (13)$$

For a vacuum $\omega_0 = 0$, and consequently $\Omega_{mn} = \omega_{mn}$. In this case (13) will coincide with the well-known expression (see [4]) for the scattering cross section for fast electrons on isolated atoms of the medium:

$$d\sigma = \frac{4e^2e_1^2}{\hbar^2 v} \frac{|f_{mn}(k_{01})|^2}{k_{01}^4} \delta(\omega_{01} - \omega_{mn}) \, d\mathbf{k}_1. \quad (14)$$

The influence of the medium roughly reduces not only to a difference in the finite quantities of energy lost by a fast particle in an individual collision (it is equal to $\hbar\Omega_{mn}$ in a medium and $\hbar\omega_{mn}$ in a vacuum), but also the quantities $f_{mn}(\mathbf{k}_{01})$. In fact, the wave functions ψ_m and ψ_n of the struck atomic electron in an isolated atom differ from the wave functions of an electron in a condensed medium. The energy spectrum of the atom is of a discrete character, while in a solid body the outer atomic electrons produce a band spectrum and are frequently described by nonlocalized wave functions such as plane waves modulated by the lattice period.

The angular distribution of the scattered electrons is described in (3) by a formfactor $f_{mn}(\mathbf{k}_{01})$ taking into account the structure of the struck atom before and after the collision and also by the quantity $\mathbf{k}_{01}$. In our case

$$k_{01}^2 = 2k_0^2 - 2m\Omega/\hbar - 2k_0 \sqrt{k_0^2 - (2m\Omega/\hbar)\cos\theta}, \quad (15)$$

where θ is scattering angle and $\hbar k_0$ is the momentum of the incident particle before the collision. For an elastic collision $\Omega = 0$ and therefore

$$k_{01}^2 = 4k_0^2 \sin^2(\theta/2), \qquad f_{mn} = 1;$$

here expression (14) goes over into the Rutherford formula.

Formula (9) can also be used for considering the ionization of the atoms of the medium and, in general, for collisions with an energy transfer $\hbar\omega_{01}$

such that $\epsilon(\omega_{01})$ is known not to vanish nor to diverge. In particular, it is applicable to the consideration of elastic collision of charges in a medium with allowance for the polarization of the latter. In such cases the transition to the limit $\gamma \to 0$ in (9), in view of the foregoing remarks, leads to the following expression:

$$d\sigma = \frac{4e^2e_1^2}{\hbar^2 v} \frac{|f_{mn}(k_{01})|^2}{k_{01}^4 |\varepsilon(\omega_{01})|^2} \delta(\omega_{01} - \omega_{mn})\, d\mathbf{k}_1. \quad (16)$$

For collisions of free electrons we have $f_{mn} = 1$, and hence (16), as is readily seen, is equivalent to expression (1.1) given by Rukhadze and Silin. [1]

It should be noted in conclusion that for near collisions (for which ω_{01} is large) the quantity $\epsilon(\omega_{01})$ differs little from unity, and hence the influence of the medium on such collisions is very small. The obtained expressions are therefore convenient not only for an analysis of distant collisions (for which the need to take into account the medium is known), but also for near collisions, for which there is no specific need to set ϵ equal to unity.

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[2] H. Frohlin and H. Pelzer, Proc. Phys. Soc. (London) **A68**, 525 (1955).

[3] V. P. Silin and A. A. Rukhadze, Elektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasma-Like Media) Gosatomizdat, 1961.

[4] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Gostekhizdat, 1948, Part I, Ch. 15.

Translated by E. Marquit
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