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ON THE CLASSIFICATION OF ELEMENTARY PARTICLES[1)]

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An attempt is made to classify elementary particles on the basis of nonlinear field theory and to derive the mass spectrum of baryons, mesons, and "resonons."

1. The recent discovery of resonant states [1] has greatly enriched the family of elementary particles. It has led an additional basis for their classification and new approaches to the mass spectrum.

There is a basis to suggest that the mass spectrum can be derived from a single expression through the assignment of definite values to some parameter. For example, if the fermions are arranged in a series of increasing mass and are numbered (as is done in Table I), then the straight line

$$E_n = 1 + n/6 \tag{1}$$

($E_0 \equiv E_{nucl} = 1$; $n = 2q = 0, 1, 2, \ldots$ is the number of the state) gives rather good agreement with the empirical mass-spectrum curve (see Fig. 1).

2. An expression for the mass spectrum was derived in [2] on the basis of nonlinear spinor theory. This expression contains parameters by means of which the states can be classified and, moreover, two free parameters, which, when suitably chosen, lead to a mass spectrum very close to the experimental one. The expression was derived from the nonlinear Lagrangian

$$\mathscr{L} = -\frac{1}{2}\left\{\left(\bar{\psi}\gamma_\mu \frac{\partial\psi}{\partial x_\mu} - \frac{\partial\bar{\psi}}{\partial x_\mu}\gamma_\mu\psi\right) + l^\beta(\bar{\psi}\psi)^\beta\right\}, \tag{2}$$

where l is a nonlinear parameter and β is the degree of nonlinearity.

Using the exact wave solutions $\psi = \chi(s) \times \exp(ik_\mu x_\mu)$, $\chi^*(s)\chi(s) = NL^3$ (for $N=1$), we determine the charge, spin, and energy of the field. Since the charge and spin are proportional to $(\chi^*\chi)L^3$ and the energy is proportional to $(\bar{\chi}\chi)^\beta L^3$, then we cannot eliminate the volume of integration L^3 from all the expressions by means of normalization.

Hence the energy (i.e., the rest mass for $k = 0$) of the field is essentially dependent on L. Thus [2]

$$E_n \equiv (k_0 l)^{\beta/(3\beta-4)} = \frac{1}{2}\left(4\pi\frac{n}{\beta}\right)^{(3\beta-3)/(3\beta-4)},$$

$$L = \frac{2\pi}{\omega}n = \frac{2\pi}{k_0}\frac{n}{\beta}, \tag{3}$$

where k_0 is the rest mass of the field contained in the volume L^3 and ω is the frequency. For $k = 0$ (k is the wave vector) and $N = 1$, we have $\omega = \beta k_0$.

3. To determine the state spectrum (i.e., the values of n), we set up a boundary value problem. Since the system has a characteristic length L and the solution is a periodic function, then requiring,

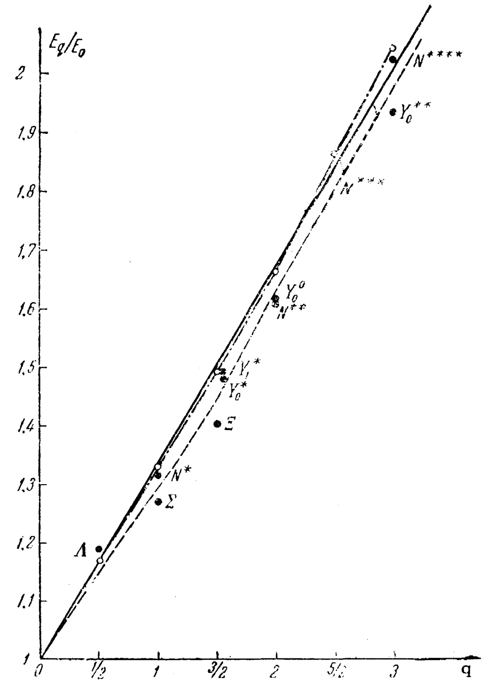

FIG. 1. Dependence of the baryon mass on the number $n = 2q$ ($E_0 = E_N = 1$, $q = p + r$). The black circles represent the experimental points; the dashed curve represents the average experimental data, the solid curve was obtained from formula (1); the open circles and the dash-dotted curve was obtained from formula (4) with $\beta = 7/2$.

[1)]This paper was first presented at the Theoretical Conference, Joint Institute of Nuclear Research, May, 1962.

Table I

$q=p+r$	(p, r)	Heavy fermions (nucleon, hyperons, and resonons): $J=(2r+1)/2$ theory	$J=(2r+1)/2$ experiment	State theory	State experiment	Mass theory $\beta=7/2$	Mass theory $\beta=4$	Mass experiment	Leptons: State theory	State experiment	Mass theory $\beta=7/2$	Mass experiment
0	(0,0)	$1/2$	$1/2$	N_0	N_0	1	1	1	ν_0	ν_0	0	0
$1/2$	($1/2$, 0)	$1/2$	$1/2$	Λ_0	Λ_0	1.17	1.18	1.187	μ_0	μ_0	0.106	0,11
1	($2/2$, 0)	$1/2$	$1/2$	Σ_0	Σ_0	1.33	1.28	1,27	l_0	—	0.287	—
	(0,1)	$3/2$	$3/2$	N_1^*	N *			1,318	ν_1^*	—		
$3/2$	($3/2$, 0)	$1/2$	$1/2$	Ξ^0	Ξ^0			1.399	$\varkappa_0$	—		
	($1/2$, 1)	$3/2$	$3/2$	Λ_1^*	Y_1^*	1.49	1.39	1.486	μ_1^*	—	0.375	—
			?		Y_0^*			1.491				
2	($4/2$, 0)	$1/2$	—	A_0	—			—	a_0	—		
	($2/2$, 1)	$3/2$	$3/2$	Σ_1^*	Y_0^0	1.66	1.58	1,614	l_1^*	—	0.522	—
	(0,2)	$5/2$	$3/2$	N_2^*	N **			1.608	ν_2^*	—		
$5/2$	($5/2$, 0)	$1/2$	—	B_0	—			—	b_0	—		
	($3/2$, 1)	$3/2$	—	Ξ_1^*	—	1.86	1.71	—	$\varkappa_1^*$	—	0.689	—
	($1/2$, 2)	$5/2$	$5/2$	Λ_2^*	N ***			1.800	μ_2^*	—		
3	($6/2$, 0)	$1/2$	—	C_0	—			—	c_0	—		
	($4/2$, 1)	$3/2$	—	A_1^*	—			—	a_1^*	—		
	($2/2$, 2)	$5/2$	$>3/2$	Σ_2^*	Y_0^{**}	2.04	1.93	1.933	l_2^*	—	0,830	
	(0,3)	$7/2$	?	N_3^*	N ****			2.02	ν_3^*	—		

for example, that $\psi(0) = \epsilon\psi(L\omega)$, $\epsilon = \pm 1$, we obtain $L\omega = 2\pi r$ for $\epsilon = +1$ and $L\omega = \pi r$ for $\epsilon = -1$ ($r = 0, 1, 2, \ldots$), i.e., $L\omega = 2\pi n$, $n = 0, 1/2, 1, \ldots$. As is seen, the introduction of a nonlinear interaction in the field equation is equivalent, in a certain sense, to the introduction of a "nonlinear potential well" of width L and depth $d = d(\beta)$. The values of L and β determine the character of the state spectrum.

We now fix the ground state (the nucleon in the case of heavy fermions, and the neutrino in the case of leptons) by taking $n = n_0$. The deviations from the ground state $\Delta n = n' - n_0$ should separate into two groups, one of which is characterized by half-integral intervals and the other by integral intervals Δn:

$$\Delta\tilde{n} = 0, 1/2, 1/2 + 1/2, \ldots \equiv p; \qquad \Delta n^* = 0, 1, 2, \ldots \equiv r.$$

States obtained in the first way will be called hyperons (p is the hyperon quantum number). States obtained in the second way will be called resonons (r is the resonon quantum number). Then (3) takes the form ($q = p + r$)

$$E_{n_0+q} = E_{n_0}\left\{1 + \frac{q}{n_0}\right\}^{(3\beta-3)/(3\beta-4)},$$

$$E_{n_0} = \frac{1}{2}\left(4\pi\frac{n_0}{\beta}\right)^{(3\beta-3)/(3\beta-4)}. \qquad (4)$$

Although the energy depends only on one number ($q = p + r$), the state is determined by two numbers (p, r); we can therefore classify the states in terms of these numbers (see Table I).

In order to obtain numerical values, it is necessary to fix n_0. For this purpose, in the case of the ground state, we set $L = 2\pi/k_0$, i.e., $n_0 = \beta\theta$ ($\theta = 0$ corresponds to the neutrino and $\theta = 1$ corresponds to the nucleon). Then β is restricted so that $\beta = \kappa/2$, $\kappa = 0, 1, 2, \ldots$. It can be shown that a reasonable spectrum is obtained only for $\beta = 7/2$ [2] or $\beta = 4$ (see Table I) (until now β has usually been taken to be equal to 2 in the theory).

With the given normalization of the solution ($N = 1$), the spin of the field is $S = \hbar/2$. However, we can change the normalization ($N \neq 1$) so as to obtain other desirable values of the spin without changing the state spectrum (only the energy scale changes).

4. We now consider transitions between two states (in the special case $\beta = 7/2$). For the transition energy we have

$$\omega_{p'-p;\, r'-r} = E_{p',\, r'} - E_{p,\, r} = E_{\mu}\{(7\theta' + 2q')^{15/13} - (7\theta + 2q)^{15/13}\}$$
$$\approx E_{\pi}\{7(\theta' - \theta) + 2(q' - q)\} = E_{\pi}\{7\theta^* + 2q^*\},$$
$$\theta^* \equiv \theta' - \theta, \quad q^* = q' - q, \quad q^* = p^* + r^*,$$
$$p^* = p' - p, \quad r^* = r' - r, \qquad (5)$$

[2]In the special case $\beta = 7/2$, we obtain from (4) the expression [see (1)]:

$$E_{n_0+q}(\beta = 7/2) = E_{n_0}\{\theta + 2q/7\}^{15/13} \approx E_{n_0}\{\theta + 15/13 \cdot 2q/7 + \ldots\}$$
$$= E_{n_0}\{\theta + 2q/6 + \ldots\}.$$

Table II

$q^* = p^* + r^*$	(p^*, r^*)	Mesons: State, theory	State, experiment	Mass, experiment	Transition frequency* $\omega_{q_1+q^*\to q_1}$ $(\beta = 7/2)$	
0	(0.0)	Ω_0	γ	0	0,0	—
$1/2$	$(1/2, 0)$	π_0	π_0	0,147	0,16	0,20
1	$(2/2, 0)$	ϑ_0	?	?		
	(0.1)	Ω_1^*	—	—	0,33	0,37
$3/2$	$(3/2, 0)$	$K_0^{(1)}$	$K^{(1)}$	0,528	0,50	0,54
	$(1/2, 1)$	$K_0^{(2)} \equiv \pi_1^{(*)}$	$K^{(2)}$			
2	$(4/2, 0)$	α_0	—	—		
	$(2/2, 1)$	ϑ_1^*	η	0,584	0,66	0,73
	(0.2)	Ω_2^*	ζ	0,639		
$5/2$	$(5/2, 0)$	β_0	—	—		
	$(3/2, 1)$	$K_1^{(2)*} \equiv \pi_2^*$	ρ	0,834	0,86	0.88
	$(1/2, 2)$	$K_1^{(1)*}$	ω	0,848		
3	$(6/2, 0)$	Υ_0	—	—		
	$(4/2, 1)$	α_1^*	—	—	1,04	—
	$(2/2, 2)$	ϑ_2^*	$K^* \equiv (K\pi)$	0,94		
	(0.3)	Ω_3^*	ξ (?)	1,06 (?)		

In view of the fact that the transition energy depends not only on two numbers (p, r*) but also on other parameters, a separate frequency spectrum appears about each state (p*, r*).

where

$$E_\mu = E_{n_0} \cdot 7^{-15/13} \approx 0.106,$$

$$E_\pi = 1/7 E_{n_0} \approx 0{,}147, \qquad E_{n_0} \equiv E_{\text{nucl}} = 1.$$

If we roughly identify the radiation field with the meson field (to an accuracy of the kinetic energy), we find that the mesons, in a first approximation, can be characterized by two numbers (p*, r*) (taking $\theta^* = 0$) and they can be classified correspondingly. Since in the general case the mesons are characterized by four numbers (taking $\theta^* = 0$), then a fine structure occurs [4] close to each state (p*, r*) (see Table II).

5. The lepton group is obtained with $\theta = 0$. It is quite possible that most of these states are not realized in practice, in view of the extremely small lifetime. In Table I, we have limited ourselves to a state of fermions with $E \leq 2E_0$, i.e., states whose energy is less than the deuteron mass. If we allow for heavier fermions, then we should allow for bosons that are heavier than the nucleon.

In view of the fact that the theory is invariant relative to homologous (scale) transformations, we can introduce the following quantum numbers: [2,3] $\hat{S}_\Gamma$ ($s_\Gamma = -3/2$) is the analog of the spin operator, $\hat{R}$ ($\hat{R}\psi = -r\psi$, r = 0, 1, 2) is the analog of the orbital angular momentum operator, and $\hat{J}' = \hat{R} + \hat{S}_\Gamma$ [$J = -(J' + 1) = (2r + 1)/2$] is the analog of the total angular momentum operator. The eigenvalues of $\hat{R}$ can be compared with the "resonon quantum number" r. The quantity $J = (2r + 1)/2$ turns out to coincide with the orbital angular momentum of the fermions, [1] and for the energy $E^{(J)}$ we obtain

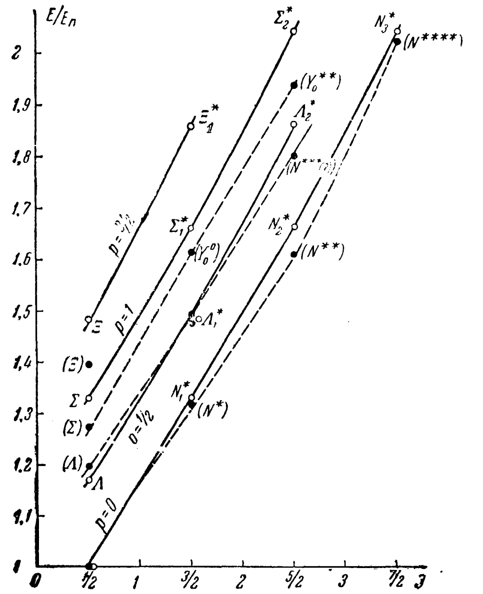

FIG. 2. Dependence of the baryon mass on J [J = (2r + 1)/2]. The black circles and the dashed curves represent the experimental data, the open circles and the solid curves were obtained from formula (6) with $\beta = 7/2$.

(see Fig. 2) (for the nucleon resonons the values of J were chosen on the basis of poorly known experimental values.

$$E^{(J)} = E_{n_0}\left\{\frac{5+2p}{7} + \frac{4}{7}J\right\}^{15/13} \approx E_\mu\{5+2p+4J\}. \quad (6)$$

6. If we now assume that the width of the nonlinear potential well L contains almost, but not exactly, integral (or half integral) numbers of wavelengths, i.e., $L\omega \pm \alpha = 2\pi n$, $\alpha = 2\pi\alpha_0$, we obtain an additional hyperfine structure. If, here, α_0 is determined from the requirement that $E_0(\alpha_0) - E_0(\alpha_0 = 0) \equiv E_e - E_\nu = 0.51$ MeV, then we obtain $\alpha_0 \approx 1/190$ and, further, $E_{n_0}(\alpha_0) - E_{n_0}(\alpha_0 = 0) \equiv E_{N^+} - E_{N^0} = 1.6$ MeV (N is the nucleon) with the experimental value 1.3 MeV.

Hence, simple analysis of the conclusions of the nonlinear theory permits us to arrive at a rather satisfactory semiempirical formula (1) for the mass of elementary particles which apparently reflects certain deeper basic properties.

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[4] P. S. Isaev, Joint Institute of Nuclear Research, Preprint, 1961.

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