FIG. 2

= 0.34. The width Γ_1 of the p-wave differs then from the value of Γ introduced by Frazer and Fulco by a factor $\nu_r(\Gamma_1 = \nu_r\Gamma)$. This result coincides with the results of Serebryakov and Shirkov. It is seen from the curves that there exists a whole region of low cutoff ($\Lambda_{eff} = 9-15$), in which the Serebryakov-Shirkov solution with resonances in the A_0 and A_1 waves satisfies the Chew-Mandelstam equation. It is shown here that an account of the high-energy contributions (large Λ) changes radically the character of the solution. Apparently this limits the value of the cutoff of the left cut in the Chew-Mandelstam equation, if we wish to obtain a closed-form description of the low-energy scattering. It is interesting to note that the value of the parameter Λ coincides with the value of the cutoff parameter of the Chew-Mandelstam equation, which guarantees convergence of the expansion of the amplitude in Legendre polynomials. However, the question of the possibility of a closed-form description remains open. A probable experimental check of this question involves the question of the maximum of the width of the p-wave resonance.

Let us show how this maximum arises in the case of low cutoff. We consider the saturation of the A_0 wave on a large interval, i.e.,

$$\text{Im}\,A_0(\nu) = \sqrt{(\nu+1)/\nu}, \quad 0 < \nu < \Lambda,$$

$$\text{Im}\,A_0(\nu) = 0, \quad \nu > \Lambda; \qquad \text{Im}\,A_1(\nu) = \pi\Gamma_1\delta(\nu_r - \nu).$$

Condition (1) goes over when $\Lambda \gg 1$ into

$$\frac{\Gamma_1}{\nu_r} + \frac{3\Gamma_1}{\nu_r}\left[\frac{(4\Lambda + 2 - \nu_r)(\Lambda - \nu_r)}{\Lambda^2} - 2\ln\frac{\Lambda+1}{\nu_r+1}\right]$$

$$- \frac{1}{3\pi}\left(1 + \frac{1}{\Lambda}\ln 4\Lambda\right) = 0. \tag{3}$$

When $\lambda = 10-15$, we get $\Gamma_{max} = 0.43$, which corresponds to a bipion width of 43 MeV. No such maximum exists for large Λ.

Thus, if we confine ourselves to the low-energy region in the Chew-Mandelstam equation without subtraction, assuming that the contribution of the high-energy region is small, then the corresponding solution coincides with the Serebryakov-Shirkov solution for low-energy scattering.

In conclusion, I am deeply grateful to V. V. Serebryakov and D. V. Shirkov for formulating the problem and for continuous interest in it.

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NONMETALLIC NICKEL AT HIGH COMPRESSIONS

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IT is well known that, beginning from a certain density, dielectrics necessarily become metals on compression (see the final paragraph of this letter). It is natural to expect, therefore, that metals would retain their metallic properties on compression. Up to now, as far as we know, no one has noted the fact that on compression a metal may be transformed into a dielectric within a certain range of densities. Our calculations suggest that this unusual behavior is exhibited by nickel. Calculations similar to those carried out by one of the present authors [1] indicate that, beginning from a density corresponding to the compression $\delta = 6.5$, i.e., from a density of 60 g/cm^3 (obtained at a pressure of 250 × 10^6 atm), nickel becomes an insulator.

The reason for this lies in the fact that an atom of nickel has 28 electrons, i.e., the number

ε(k)

3
2   4s
1   4d, m=2
Gap 0   1   2   k   4d, m=1
ε_ф
-1   3p, m=1
-2   3p, m=0
-3   3d, m=2
-4   3d, m=1
-5   3d, m=0
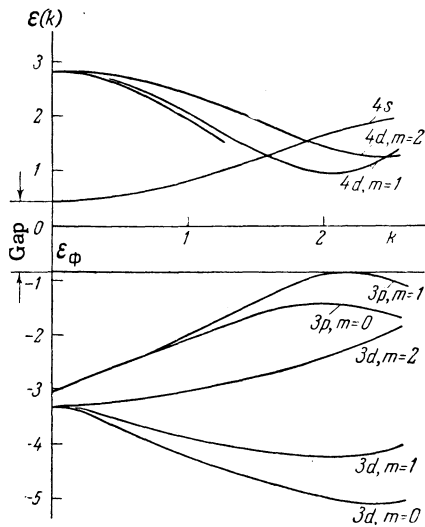
which would fill the levels with n = 1, 2, and 3 if the level distribution were hydrogen-like; nickel would then be an inert gas. In fact, under normal conditions in the metal, the 4s-band is lower than the 3d-band. Due to electron transitions from 3d to 4s we have the unfilled bands responsible for conduction. On increase of the density the level 4s rapidly rises and becomes higher than the levels 3s, 3p and 3d. We then have the situation when all the bands with n ≤ 3 are filled and nickel becomes a dielectric. At still higher densities we can expect overlapping of the 3p and 4d bands, making nickel again a metal (for δ > 15, ρ > 130 g/cm^3, p > 1500 × 10^6 atm).

The bands calculated for δ = 8.56 are shown in the figure, which gives the dependence of the energy (in atomic units) on the wave vector of the electron.[1] The spectroscopic index of the electrons is given for each branch and this index includes the magnetic number m which is the projection of the magnetic moment along the direction of the momentum k.

Caution is needed in calculating the critical density, since the calculations are based on the spherical approximation for the quasimomentum k. If the true form of the Brillouin zone for fcc nickel is allowed for, the upper edge of the 3p-band along some directions in k-space may be higher than in the spherical case. It is therefore desirable to estimate the possible increase of the energy maximum for the 3p-band. For this purpose the Slater-Koster interpolation method[2] was used. To find the constants we used the formulas for the energy ϵ(k) along the most symmetrical direction [010] in k-space. Along this direction the upper edge of the 3p-band was displaced upwards a little to −0.62, but it remained below the lower

edge of the 4s-band. Of the remaining directions the most likely to show an energy shift is the direction [111]. Although the spherical approximation is insufficient for finding all the nondiagonal constants in the Slater-Koster technique, it is nevertheless clear that in a wide range of values of these constants there is an energy gap between the 3p-band and the 4s-band.

In connection with the general rule for the transformation of any substance into a metal on unlimited increase of the density, we should note that the decisive factor is the deviation of the unit cell of the crystal (having the form of a polyhedron) from spherical shape. Let us consider a degenerate electron gas in the field of a uniformly distributed positive charge. For this gas E = k^2/2m. Replacement of the smeared-out charge distribution by separate ions at lattice sites may be regarded as a perturbation. It is known that in this case the hypersurface E (k) splits up into separate zones divided by gaps. On increase of the density the gap widths decrease but the gaps do not disappear! In this approximation substances with certain values of Z (in particular helium) would remain dielectrics at any density. However, since the unit cell shape is not spherical the limiting momentum depends on the direction with respect to the lattice and the limiting energy in the band depends on the direction of k. Consequently, at high densities the upper edge of the filled band along one direction of k moves below the bottom of the next unfilled band along another direction of k, and the substance becomes a metal in spite of the fact that the gap still remains.

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[1] The reciprocal radius of a Wigner-Seitz sphere is taken as the unit vector.

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